In The Name of God The Most Compassionate, The Most Merciful

Electric Machines I

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Chapter 6
Direct Current (DC) Motors

Different Types of DC Motors

Base on the magnetic field production, DC Motors are classified as

1. **Separately excited DC motors**: a separate voltage source (from the armature voltage source) is required for field production.
2. **Shunt DC motors**: field winding is connected in parallel to the armature winding.
3. **Series DC motors**: field winding is connected in series with the armature winding.
4. **Cumulative compound DC motors**: both series and parallel field windings are used and their magnetic fields are added together. It is further divided as **long shunt** and **short shunt**.
5. **Differential compound DC motors**: both series and parallel field windings are used and their magnetic fields are subtracted from each other. It is divided as **long shunt** and **short shunt**.
6. **Permanent magnet DC motors**: field is produced by PM.

Specifications of DC Motors

DC motors have the following specifications:

1. Nominal rotational **speed**

2. Nominal output **power** (nominal torque can be obtained from the first two items)

3. **Efficiency**

4. Nominal terminal **voltage**

5. **Speed regulation** (SR)

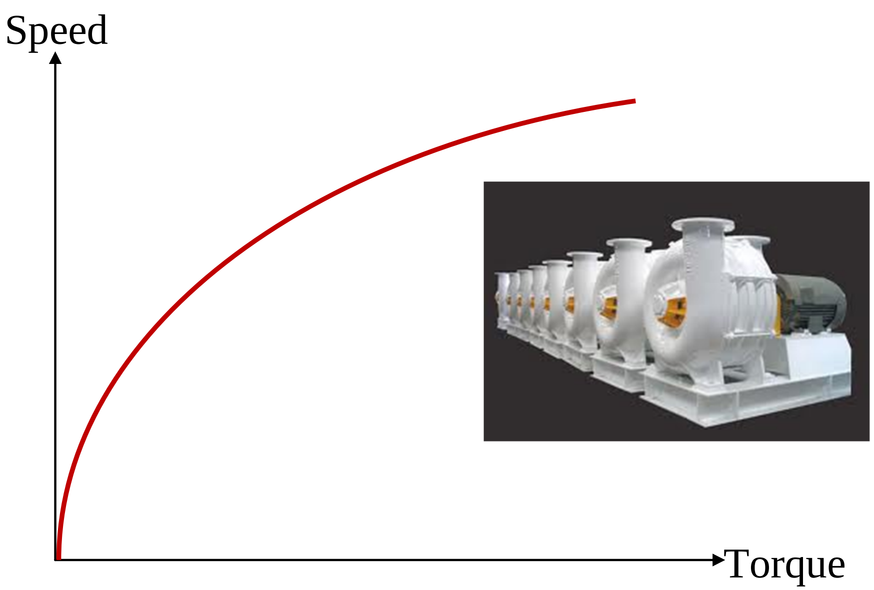
No-load speed

Full-load speed

$$SR = \frac{\omega_{nl} - \omega_{fl}}{\omega_{fl}} \times 100$$

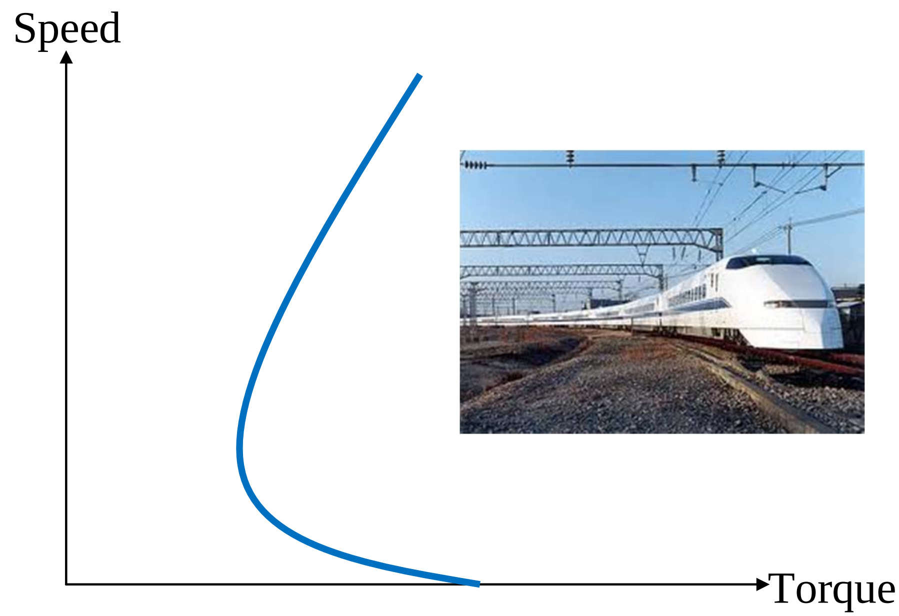
Examples of Mechanical Load

Fans, blowers and centrifugal pumps



- In any loads involving the turbulent flow of fluid, the load torque varies as the square of speed.
- It is the windage torque.
- The windage is a dominant component at high speeds for trains and car.

Traction excluding gravity

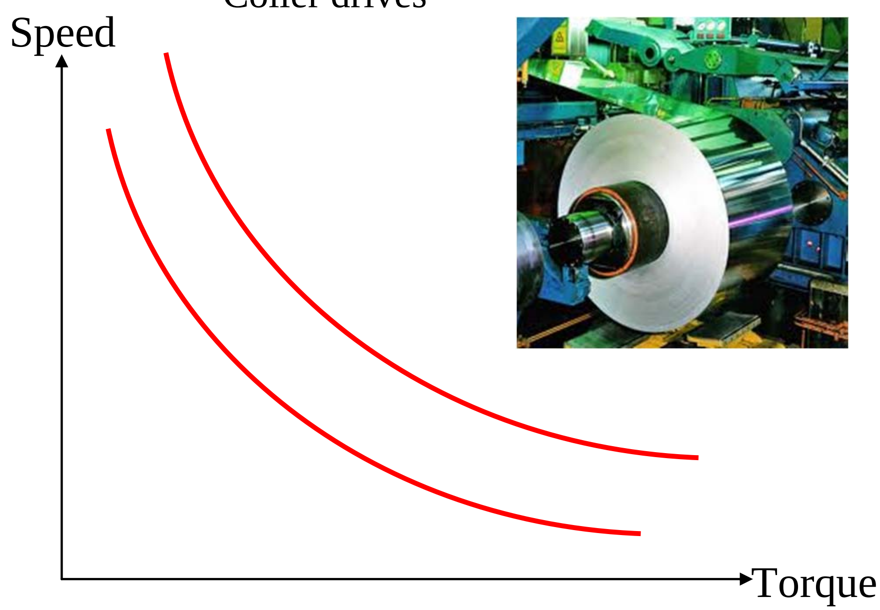


- It is applicable to electric trains and road vehicles.
- It is comprised of the windage, viscous friction, coulomb friction and stiction.

G. K. Dubey, Power Semiconductor Controlled Drives, 1989, Prentice-Hall, Inc.

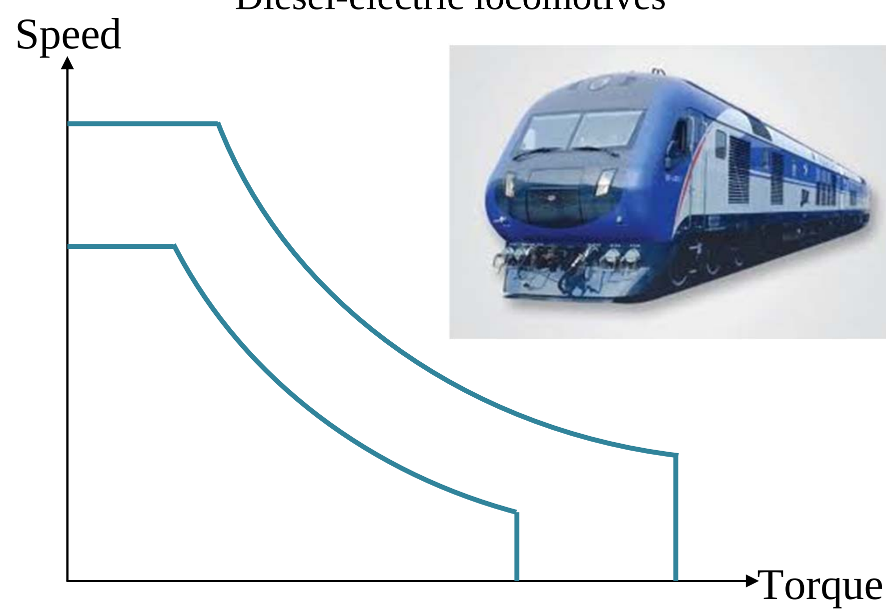
Examples of Mechanical Load

Coiler drives



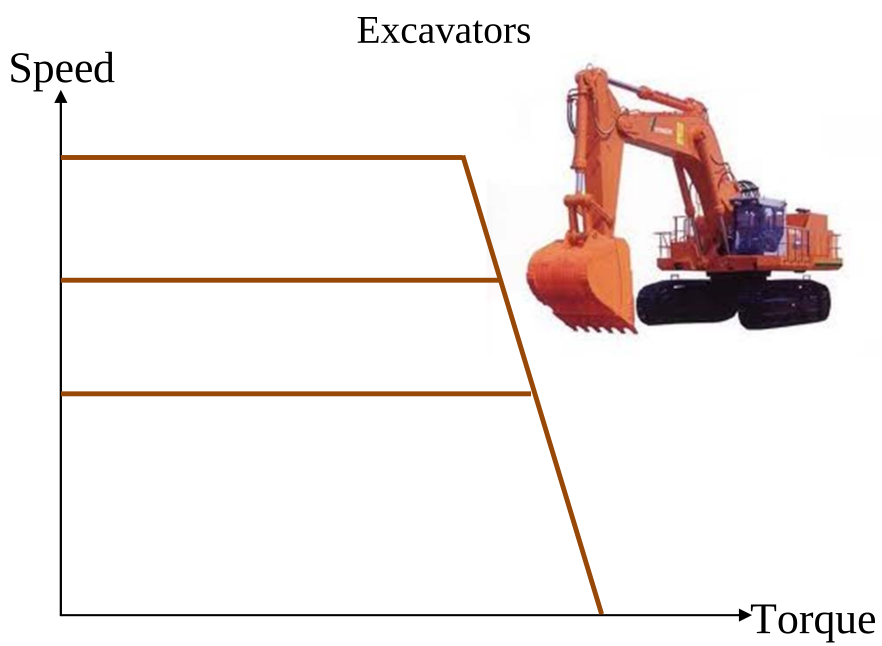
- It is for applications where the motor is required to operate at constant power.
- Coiler drives are used in steel strip, paper and plastic mills

Diesel-electric locomotives

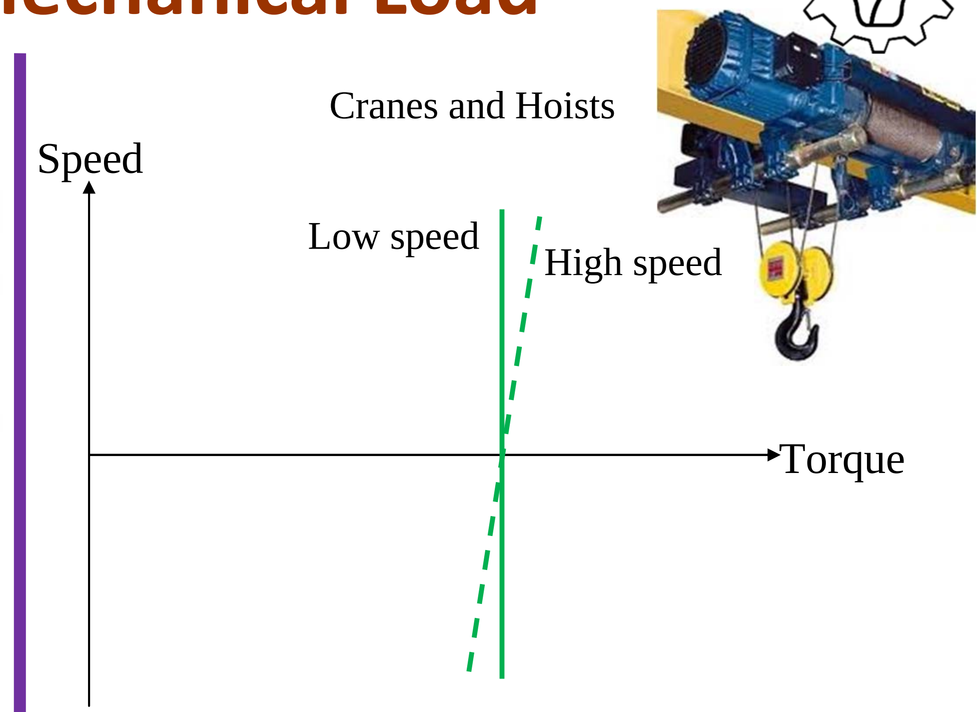


- A diesel-electric locomotive employs a dc motor fed by a dc generator driven by a diesel engine.
- Instead of d generator, ac generator followed by a rectifier may be used.

Examples of Mechanical Load



- The purpose of excavators is to dig earth.
- While digging, it may come across a rock, in this situation the motor should simply stop to prevent damage to the excavator.



- At low speeds, the torque is mainly due to gravity (constant and independent of speed).
- At high speeds, the viscous and windage will participate in the load torque.

Torque Development

The induced voltage in the armature winding having $\boldsymbol{Z}$ conductors and $\boldsymbol{a}$ parallel paths is obtained as:

$$E_a = \frac{pZ}{2\pi a}\phi\omega$$

$$E_a = k\phi\omega \qquad k = \frac{pZ}{2\pi a}$$

ω is the rotational velocity in rad/s
ϕ is the magnetic flux of each pole
p is the number of poles
Z is the total number of conductors
a is the number of parallel paths

The electromagnetic power which causes the rotation of rotor is

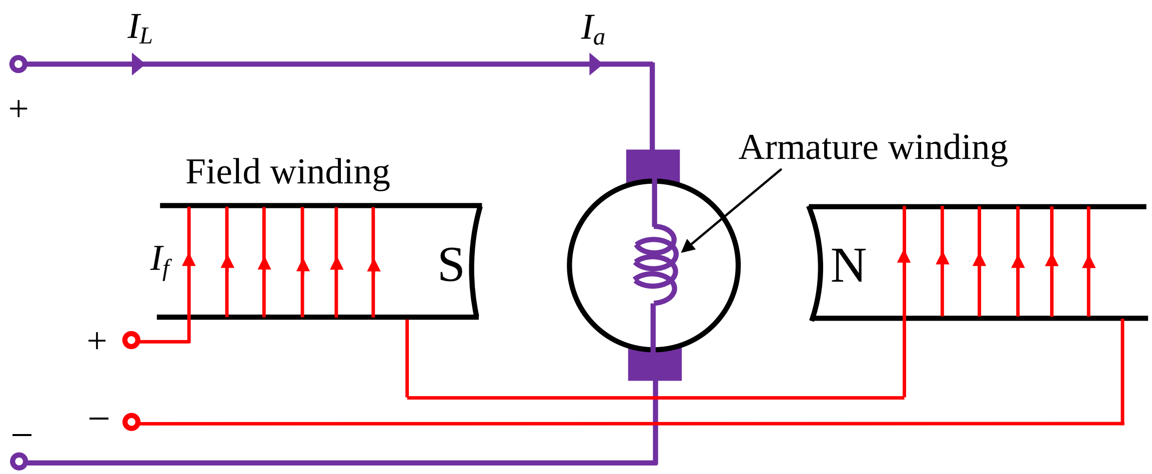
$$P_e = E_a I_a \qquad P_e = T_e\omega$$

I_a is the armature current
T_e is the electromagnetic torque

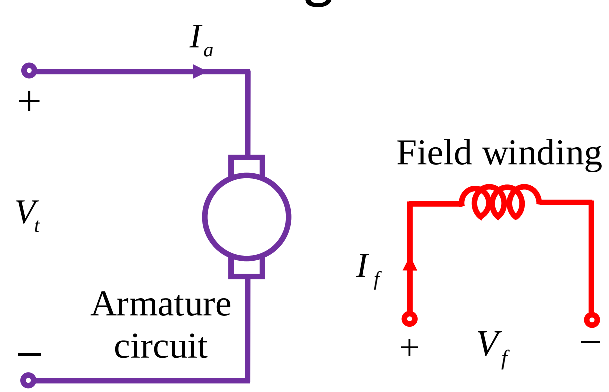
$$\Rightarrow \quad T_e = \frac{pZ}{2\pi a}\phi I_a \quad \Rightarrow \quad T_e = k\phi I_a$$

Separately Excited DC Motors

The schematic diagram of the separately excited dc motor:

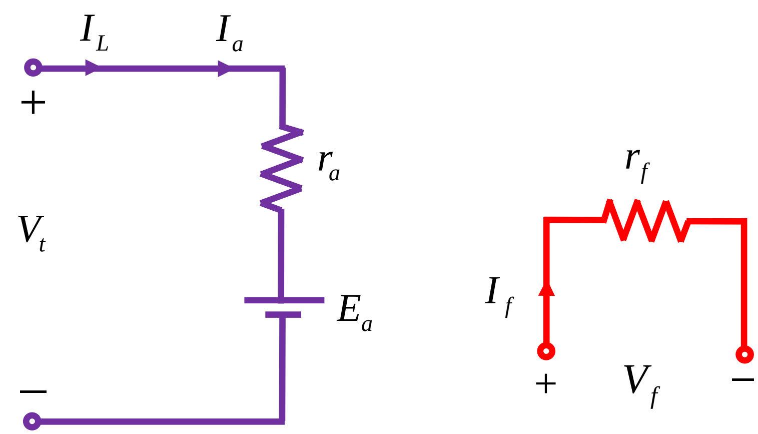


The simplified schematic diagram:



Separately Excited DC Motors

The equivalent circuit of the separately excited dc motor:



r_a is the armature winding resistance
r_f is the field winding resistance
I_a is the armature current
I_f is the field current
I_L is the line current
V_t is the motor terminal voltage (armature voltage)
V_f is the field winding voltage
E_a is the induced voltage in the armature winding

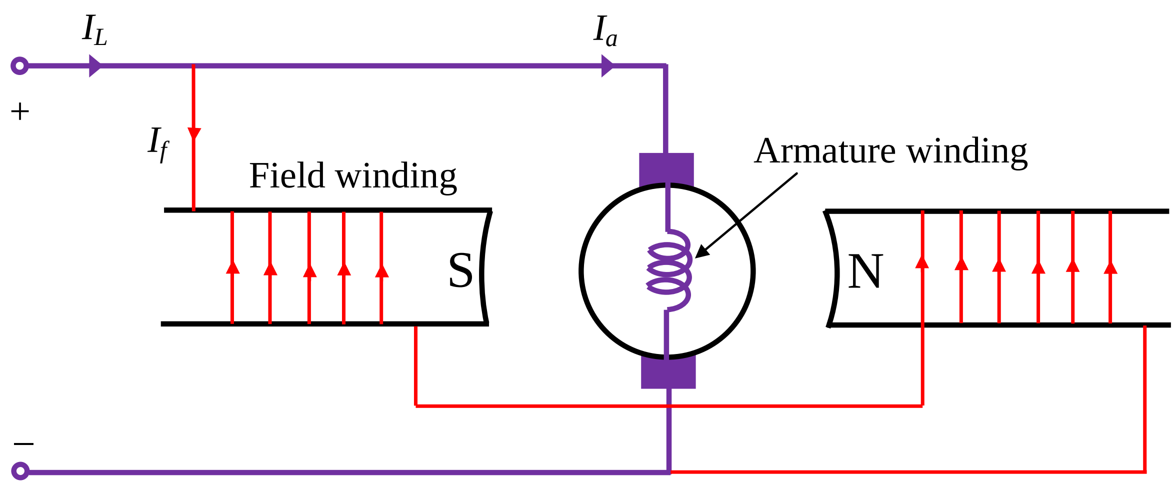
$$I_a = I_L$$

$$V_f = I_f r_f$$

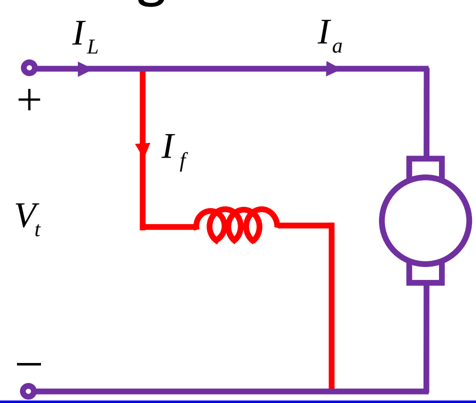
$$V_t = E_a + I_a r_a \qquad E_a = k\phi\omega \qquad T_e = k\phi I_a \qquad \phi \propto I_f$$

Shunt DC Motors

The schematic diagram of the shunt dc motor:

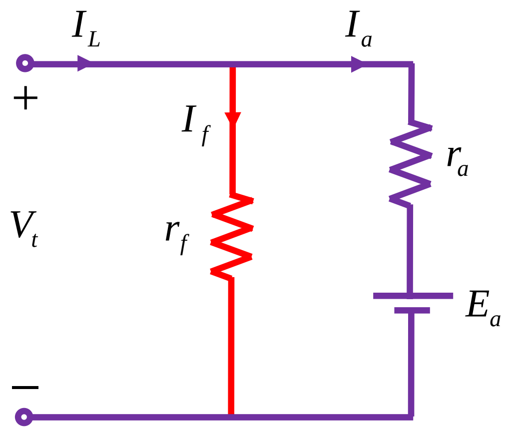


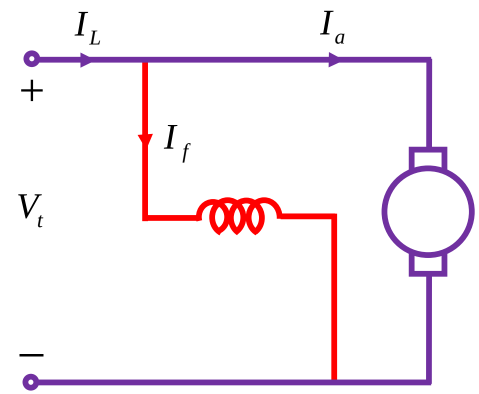
The simplified schematic diagram:



Shunt DC Motors

The equivalent circuit of the shunt dc motor:





$$I_L = I_a + I_f$$

$$I_f = \frac{V_t}{r_f}$$

$$\phi \propto I_f$$

$$V_t = E_a + I_a r_a$$

$$E_a = k \phi \omega$$

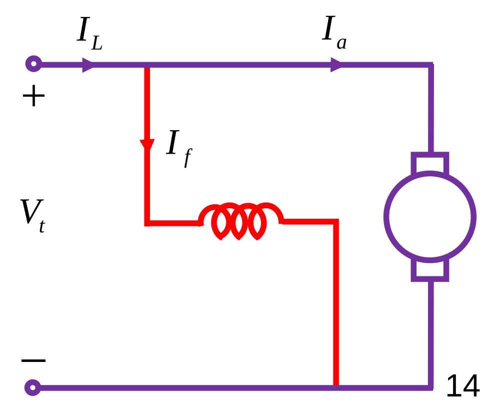
$$T_e = k \phi I_a$$

Effects of Field Winding Open-circuit

- Assume during the operation of a shunt or separately excited dc motor, the field circuit becomes open, then

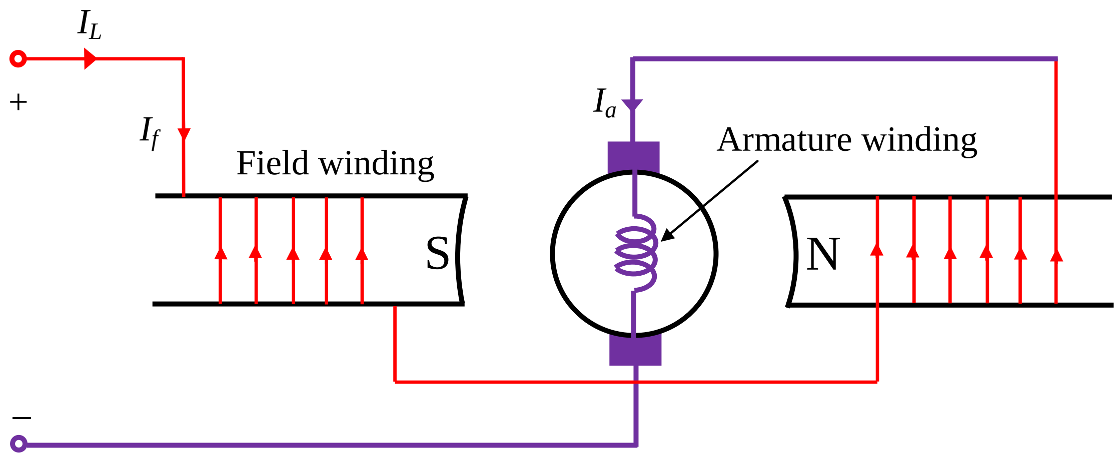
$$I_f = 0 \;\Rightarrow\; \phi = \phi_{res} \;\Rightarrow\; E_a = k\,\phi_{res}\,\omega$$

- Therefore back-emf decreases
- Since the armature voltage source is constant, the armature current increases significantly $V_t = E_a + I_a r_a$
- Despite the reduction of flux, the current increase is dominant and torque increases $T_e = k\,\phi_{res}\,I_a$
- Therefore the rotor accelerates $T_e = T_L + J\dfrac{d\omega}{dt}$
- In such a case, the motor should be disconnected from the voltage source.

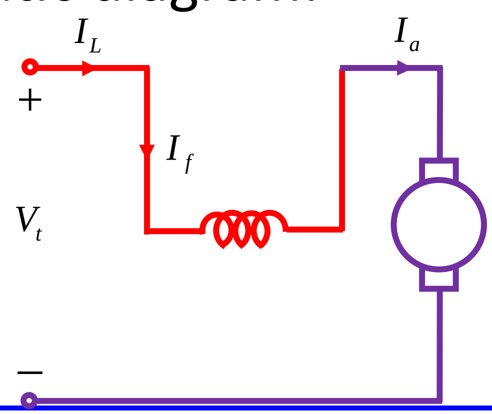


Series DC Motors

The schematic diagram of the series dc motor:

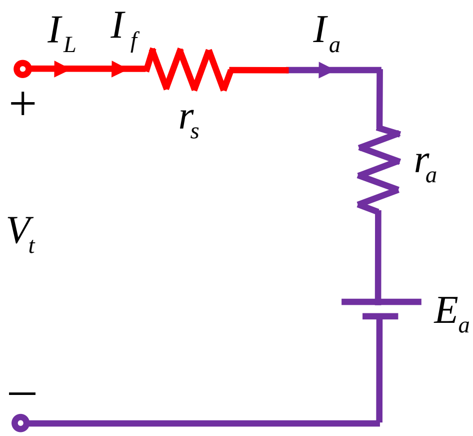


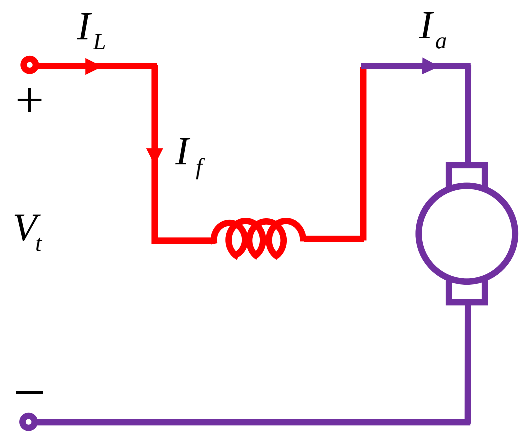
The simplified schematic diagram:



Series DC Motors

The equivalent circuit of the series dc motor:





$$I_a = I_f = I_L$$

$$E_a = k\phi\omega$$

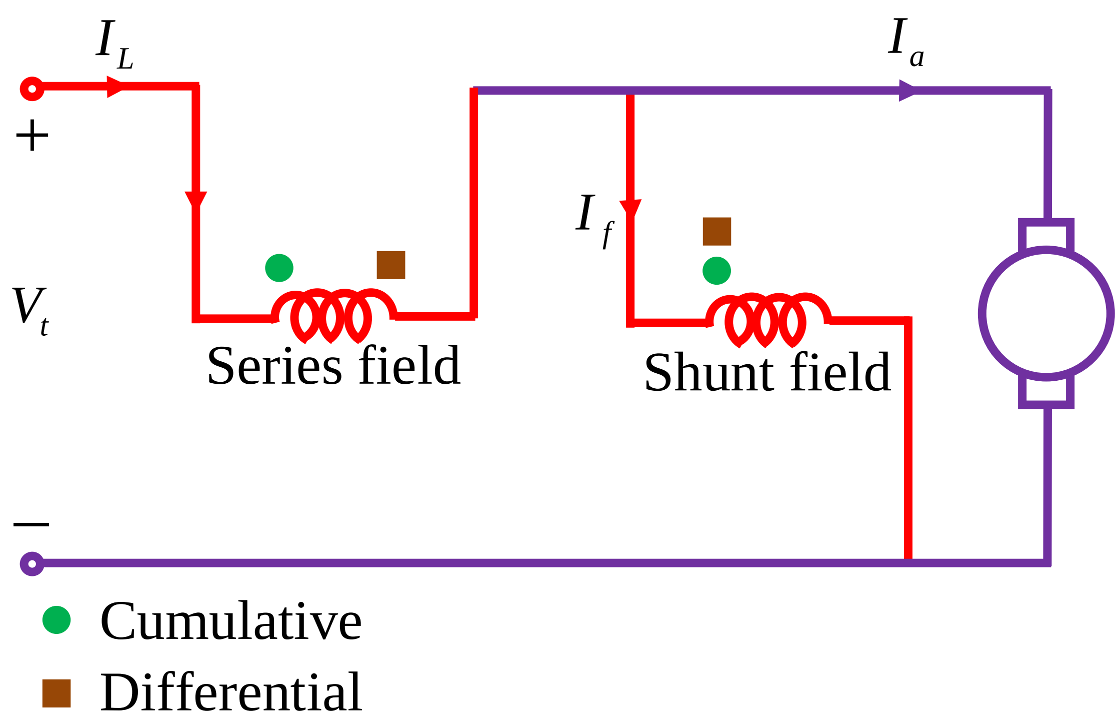
$$T_e = k\phi I_a$$

$$V_t = E_a + I_a\left(r_a + r_s\right)$$

$$\phi \propto I_f = I_a$$

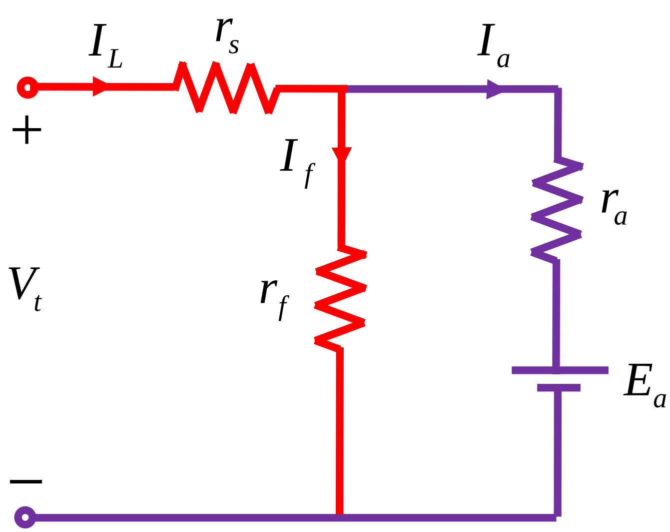
Compound DC Motors
Short-shunt

The simplified schematic diagram of the short-shunt compound dc motor:



Compound DC Motors
Short-shunt

The equivalent circuit of the short-shunt compound dc motor:



$$I_L = I_a + I_f$$

$$E_a = k\phi\omega$$

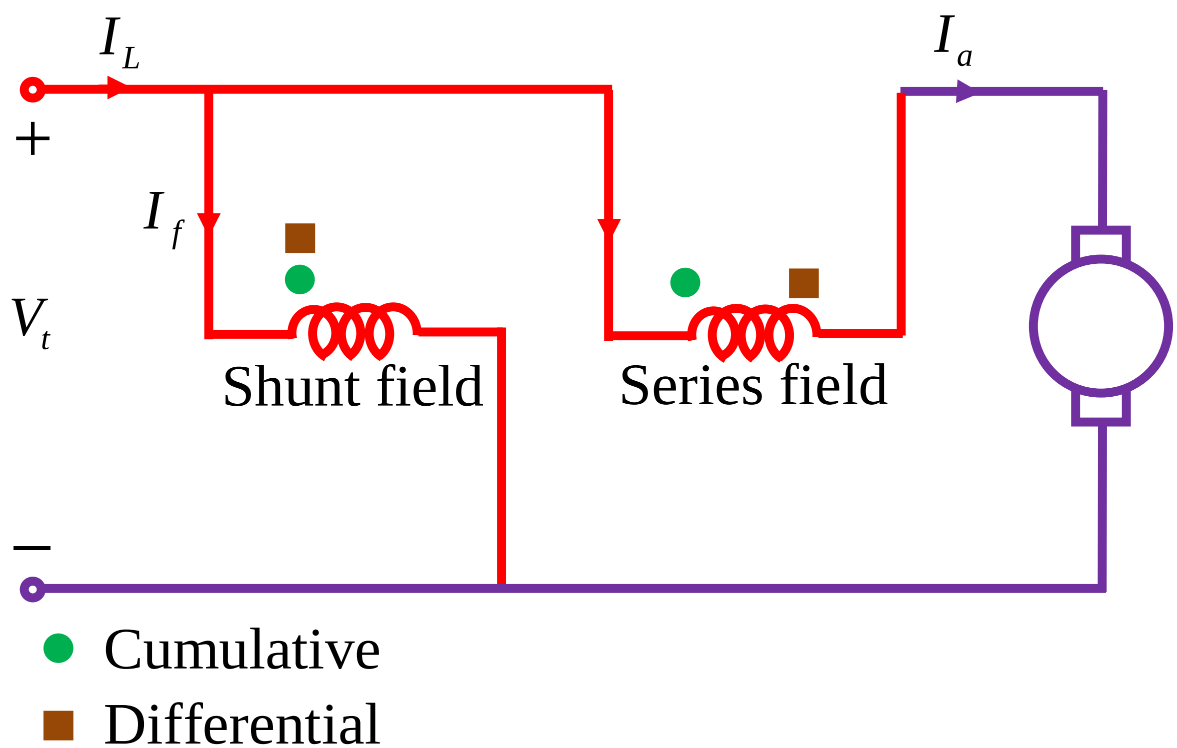
$$I_f = \frac{E_a + r_a I_a}{r_f}$$

$$V_t = E_a + r_a I_a + r_s I_L$$

$$\phi \propto k_{sh} I_f \pm k_s I_L \quad \begin{cases} + & \text{Cumulative} \\ - & \text{Differential} \end{cases}$$

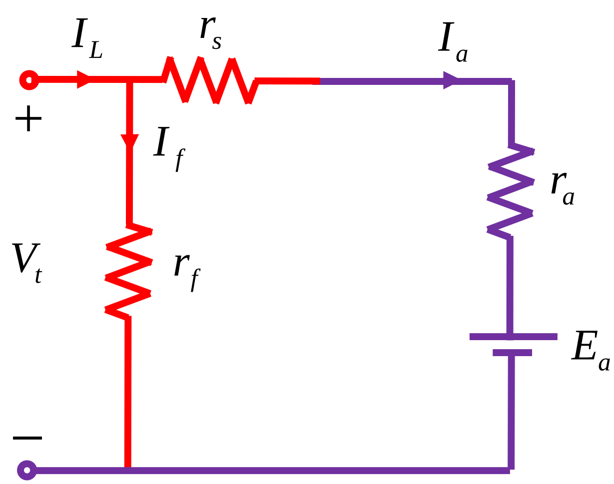
Compound DC Motors
Long-shunt

The simplified schematic diagram of the long-shunt compound dc motor:



Compound DC Motors
Long-shunt

The equivalent circuit of the long-shunt compound dc motor:



$$I_L = I_a + I_f$$

$$E_a = k\phi\omega$$

$$I_f = \frac{V_t}{r_f}$$

$$V_t = E_a + I_a(r_a + r_s)$$

$$\phi \propto k_{sh} I_f \pm k_s I_a \quad \begin{cases} + & \text{Cumulative} \\ - & \text{Differential} \end{cases}$$

Cumulative vs. Differential Compound DC Motors

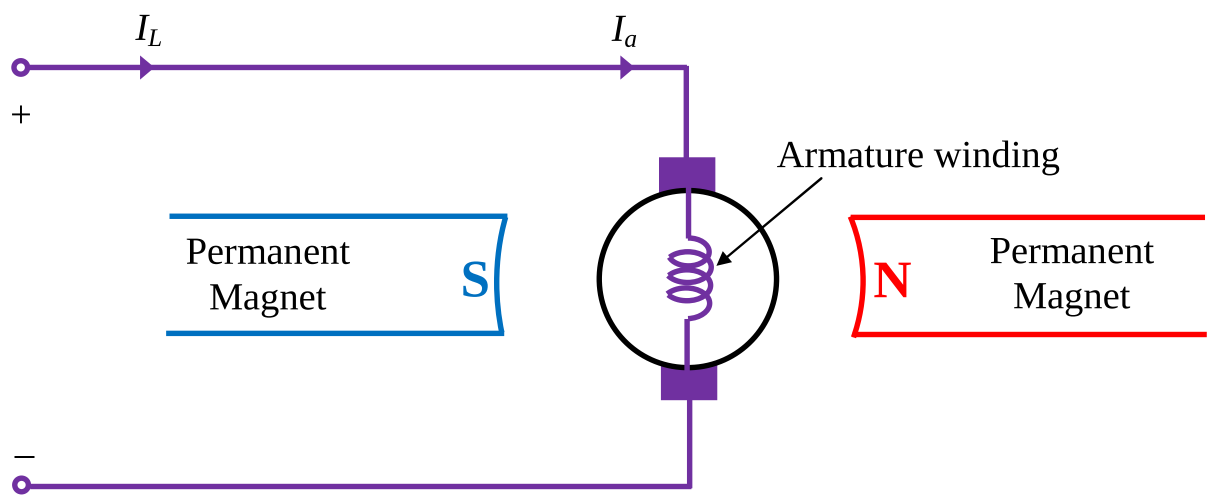
- If the direction of the fields produced by the series and shunt field windings are the same, the motor is cumulative;

- Otherwise it is differential.

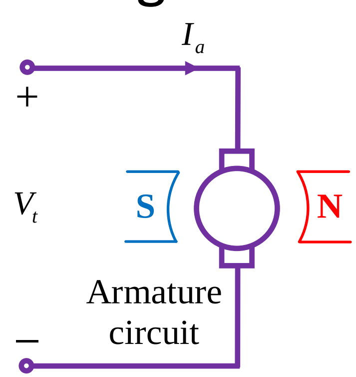
- A **cumulative compound dc motor** is a **differential compound dc generator** and vice versa.

Permanent Magnet DC Motors

The schematic diagram of the PM dc motor:

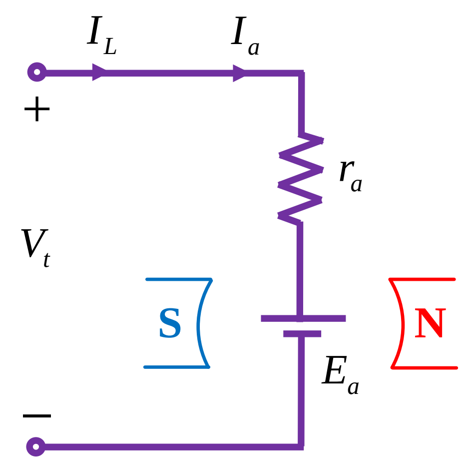


The simplified schematic diagram:



Permanent Magnet DC Motors

The equivalent circuit of the PM dc motor:



r_a is the armature winding resistance

I_a is the armature current

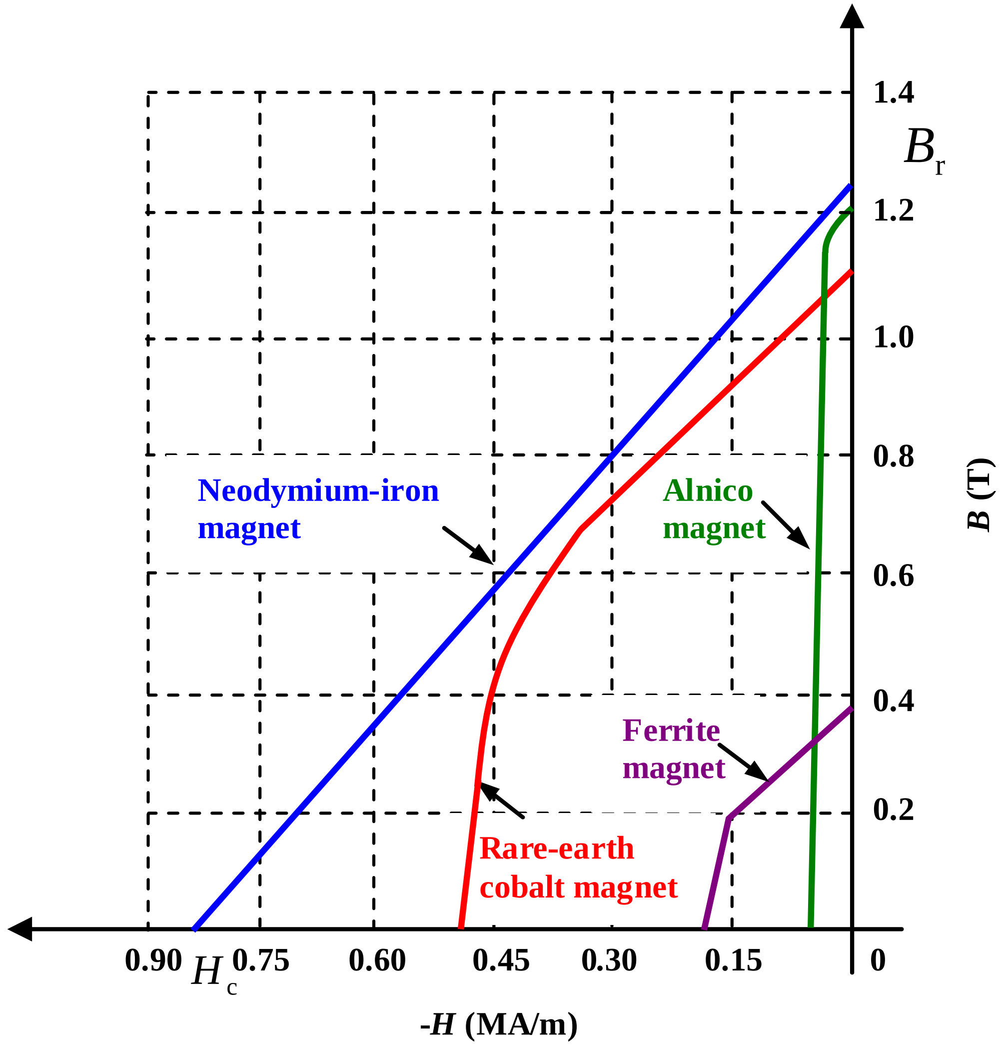
I_L is the line current

V_t is the motor terminal voltage (armature voltage)

E_a is the induced voltage in the armature winding

$V_t = E_a + I_a r_a$ $\quad$ $E_a = k\phi\omega$ $\quad$ $T_e = k\phi I_a$ $\quad$ $I_a = I_L$

Common types of Permanent Magnets



Demagnetization curve of four types of permanent magnets

Power Flow in DC Motors

- In dc motors the input power is electrical (Voltage multiplied by current) $P_{in} = P_{elec} = V_t I_L + (V_f I_f)$

$$P_{\text{elec}} - P_{\text{armature-losses}} - P_{\text{field-losses}} - P_{\text{brush-losses}} - P_{\text{core-losses}} = E_a I_a$$

- P_{elec}: Input electrical power
- $P_{\text{armature-losses}}$: Ohmic losses due to armature winding
- $P_{\text{field-losses}}$: Ohmic losses due to field windings
- $P_{\text{brush-losses}}$: Brushes losses
- $P_{\text{core-losses}}$: Core losses

- In dc motors the output power is mechanical (load torque multiplied by rotational velocity) $P_{out} = P_{mech} = P_L = T_L \omega$

$$E_a I_a - P_{\text{mech-losses}} = P_{\text{mech}}$$

- $P_{\text{mech-losses}}$: Mechanical losses
- P_{mech}: Output mechanical power

Power Flow in DC Motors

Important note

$$E_a I_a - P_{\text{mech}-\text{losses}} = P_{\text{mech}}$$

- At no-load condition, there is no load torque $T_L = 0$
- Therefore output power is zero $P_{\text{mech}} = T_L \omega = 0$

$$\left(E_a I_a\right)_{no-load} = P_{\text{mech}-\text{losses}}$$

Operating Characteristics of DC Motors

1. **Speed vs. armature current** $\omega - I_a$

2. **Torque vs. armature current** $T_e - I_a$

3. **Speed vs. Torque:** $\omega - T_e$

Operating Characteristics of Separately Excited and Shunt DC Motors

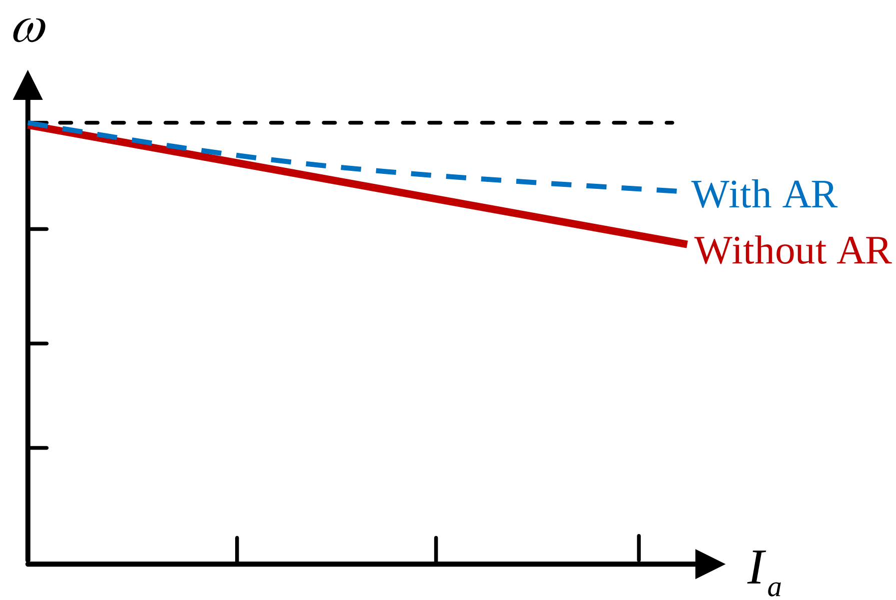
1. Speed vs. armature current $\omega - I_a$

$$V_t = E_a + I_a r_a$$

$$E_a = k\phi\omega$$

$$\Rightarrow V_t - I_a r_a = k\phi\omega \Rightarrow \omega = \frac{V_t - I_a r_a}{k\phi}$$

$$k = \frac{pZ}{2\pi a}$$

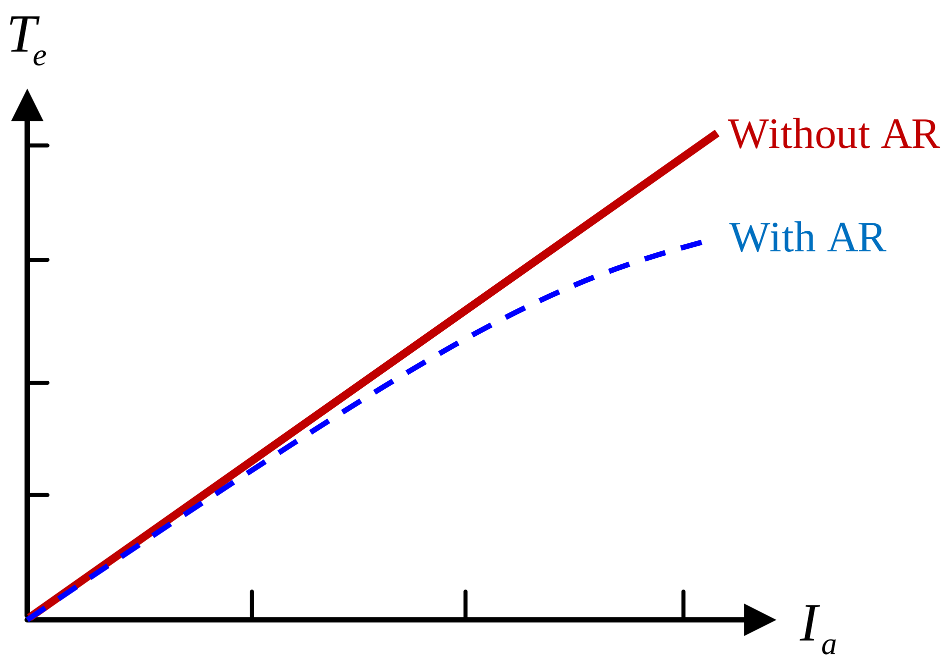


Operating Characteristics of Separately Excited and Shunt DC Motors

2. Torque vs. armature current $T - I_a$

$$T_e = k\phi I_a$$

$$k = \frac{pZ}{2\pi a}$$



Operating Characteristics of Separately Excited and Shunt DC Motors

3. Speed vs. torque

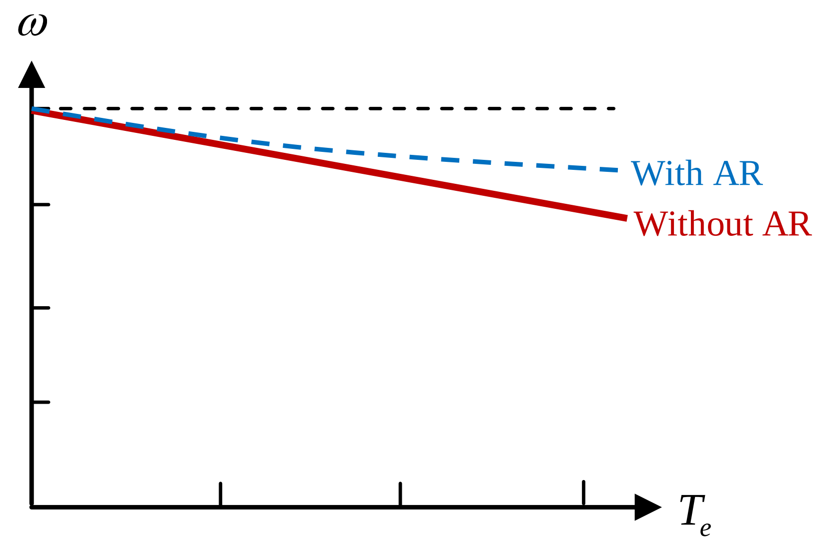
$\omega - T_e$

$$T_e = k\phi I_a \quad \Rightarrow \quad I_a = \frac{T_e}{k\phi}$$

$$\omega = \frac{V_t - I_a r_a}{k\phi}$$

$$k = \frac{pZ}{2\pi a}$$

$$\Rightarrow \quad \omega = \frac{V_t}{k\phi} - \frac{r_a}{(k\phi)^2} T_e$$



Operating Characteristics of Series DC Motors

1. **Speed vs. armature current** $\omega - I_a$

$$V_t = E_a + I_a(r_a + r_s)$$

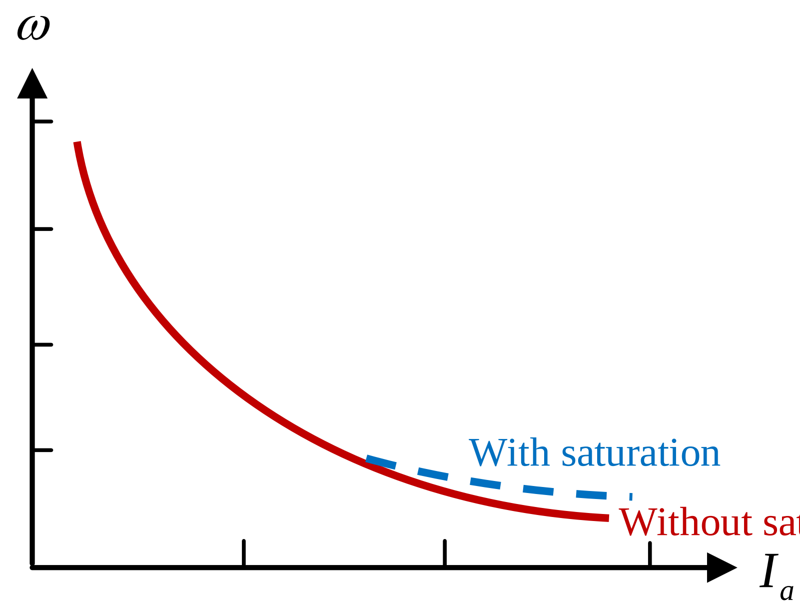
$$E_a = k\phi\omega$$

$$\Rightarrow V_t - I_a(r_a + r_s) = k\phi\omega$$

$$\Rightarrow \omega = \frac{V_t - I_a(r_a + r_s)}{k\phi}$$

$$\phi = cI_a$$

$$\Rightarrow \omega = \frac{V_t}{kcI_a} - \frac{(r_a + r_s)}{kc}$$



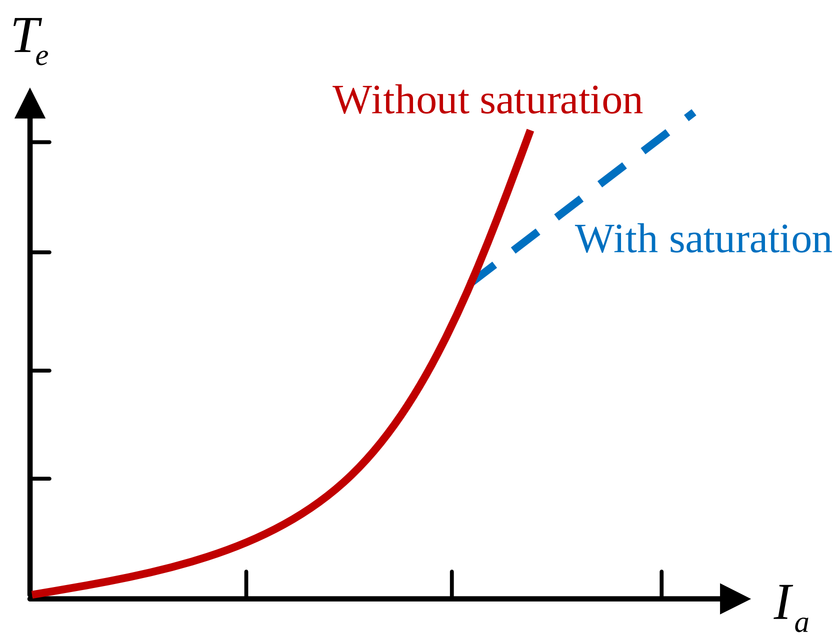
Operating Characteristics of Series DC Motors

2. Torque vs. armature current $T - I_a$

$$T_e = k\,\phi\, I_a$$

$$\phi = c I_a$$

$$\Rightarrow \quad T_e = k\,c\,I_a^2$$



Operating Characteristics of Series DC Motors

3. Speed vs. torque

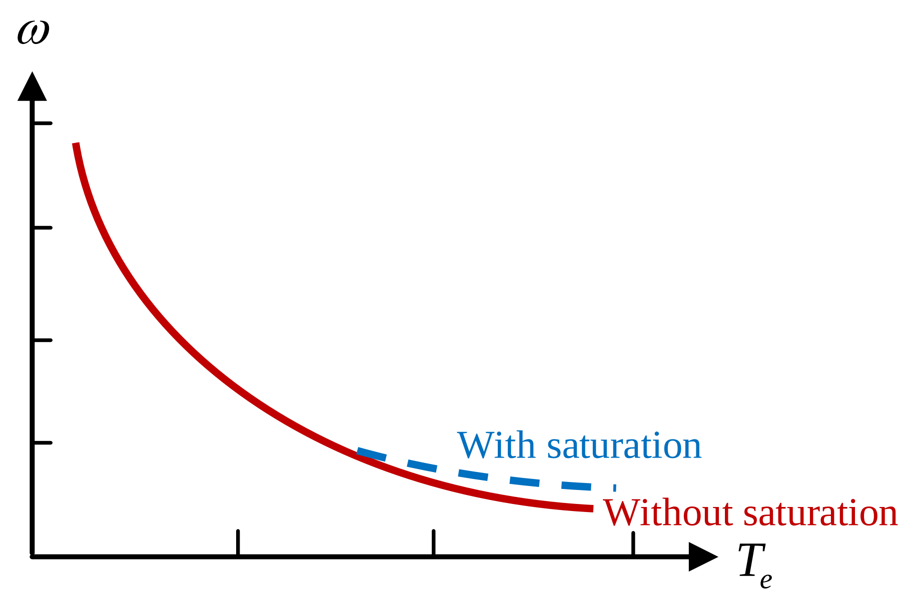
$\omega - T_e$

$$T_e = k\,c\,I_a^2 \quad \Rightarrow \quad I_a = \sqrt{\frac{T_e}{k\,c}}$$

$$\omega = \frac{V_t}{kcI_a} - \frac{(r_a + r_s)}{kc}$$

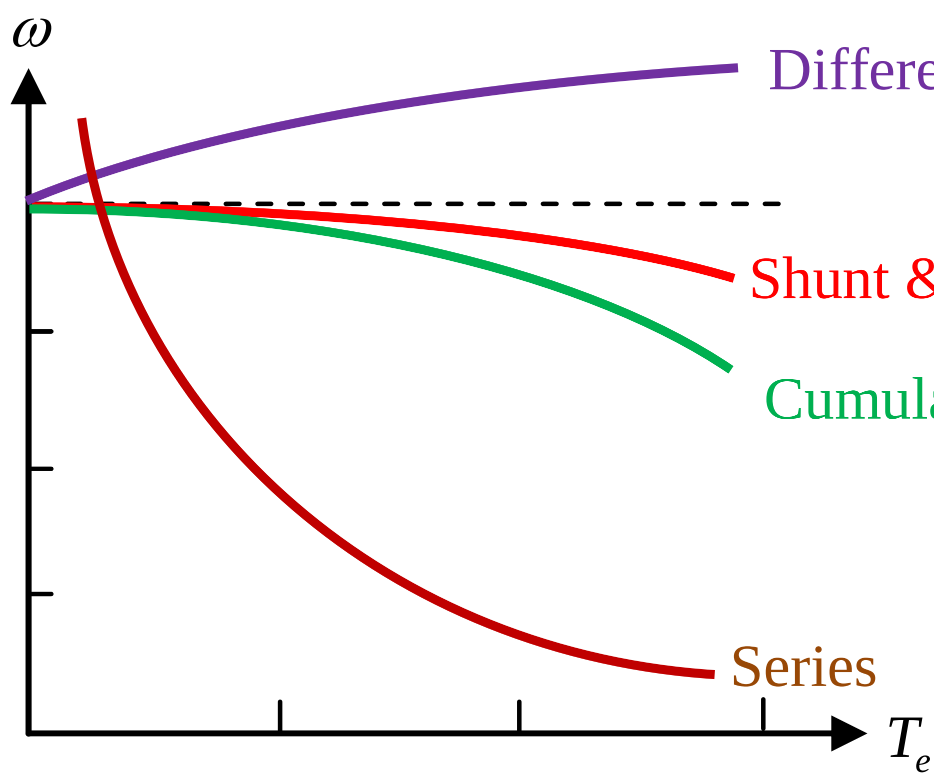
$$\Rightarrow \quad \omega = \frac{V_t}{\sqrt{kc}\sqrt{T_e}} - \frac{r_a + r_s}{kc}$$

Series DC motors should not run without load since it's speed increases severely.



Operating Characteristics of Compound DC Motors

3. Speed vs. torque $\omega - T_e$



Differential compound DC motors are not used due to their instability problem.

Changing the Direction of Rotation of DC Motors

In order to change the direction of rotation of DC motors,

1. **The direction of the field current should be changed while the direction of the armature current should be kept unchanged.**

   or

2. **The direction of the armature current should be changed while the direction of the field current should be kept unchanged.**

The second method is preferred since the field circuit have higher inductance.

DC Motors

Example 1: A **voltage of 230 V** is connected to the armature of a separately excited DC motor and under this condition the **nominal current of 205 A** flows in the armature. If the armature resistance is 0.2 ohms,

a) Find the **back-emf**.

b) Calculate the **output power and torque** if the **rotational losses are 1445 W** and the **rotational velocity is 1750 rpm**.

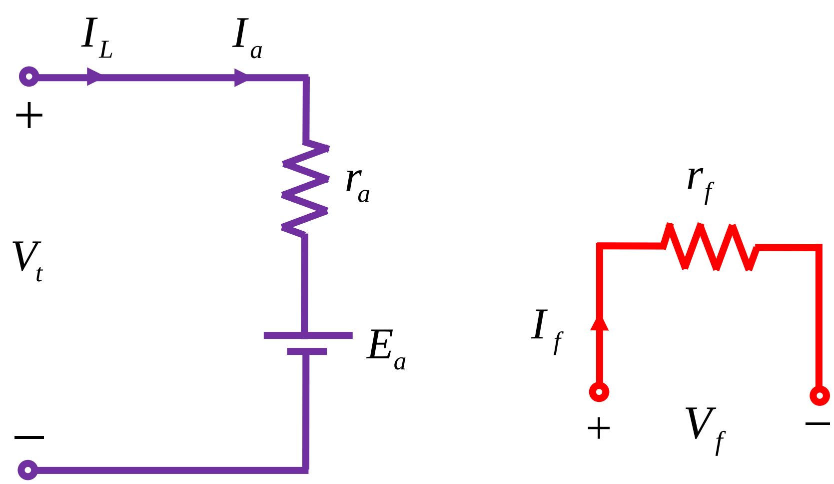
DC Motors

Solution 1: separately excited DC motor

$V_t = 230\ \text{V}$ $I_a = 205\ \text{A}$ $r_a = 0.2\ \Omega$

a) Calculate the **back-emf** (E_a).

$E_a = V_t - r_a I_a$ → $E_a = 230 - 0.2 \times 205$ → $E_a = 189\ \text{V}$



DC Motor

Solution 1: separately excited DC motor

$I_a = 205\ \text{A}$ $E_a = 189\ \text{V}$ $n = 1750\ \text{rpm}$ $P_{rot} = 1445\ \text{W}$

b) Calculate the **output power and torque**

$P_a = E_a I_a$ ⇒ $P_a = 189 \times 205$ ⇒ $P_a = 38745\ \text{W}$

$P_{out} = P_a - P_{rot}$ ⇒ $P_{out} = 38745 - 1445$ ⇒ $P_{out} = 37400\ \text{W}$

$T_{out} = \dfrac{P_{out}}{\omega}$ ⇒ $T_{out} = \dfrac{37400}{1750 \times \frac{2\pi}{60}}$ ⇒ $T_{out} = 203.5\ \text{Nm}$

DC Motors

Example 2: Consider a series DC motor with the following values for nominal voltage, nominal velocity, nominal terminal current, and series field and armature resistances:

$$V_{tn} = 600 \text{ V} \qquad n_n = 600 \text{ rpm} \qquad I_{tn} = 200 \text{ A} \qquad r_s = 0.04 \ \Omega \qquad r_a = 0.12 \ \Omega$$

a) At nominal condition, calculate the **back-emf** (E_a).

b) Calculate the **developed torque and developed power** at nominal condition.

c) If the load varies and the terminal current reduces to 150 A, calculate the **speed and developed torque**.

Assume the magnetic characteristics is linear.

DC Motors

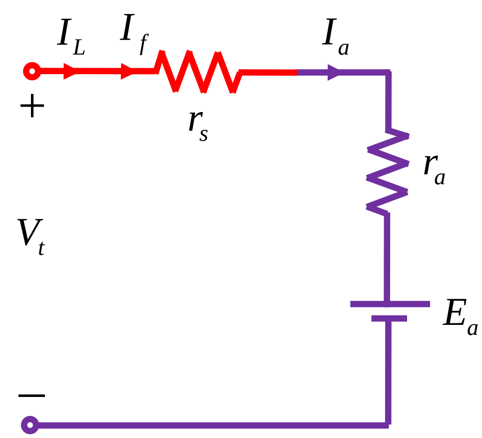
Solution 2: a series DC motor

$V_{tn} = 600 \text{ V}$ $n_n = 600 \text{ rpm}$ $I_{tn} = 200 \text{ A}$ $r_s = 0.04 \ \Omega$ $r_a = 0.12 \ \Omega$

a) At nominal condition, calculate the **back-emf** (E_a).

$$E_a = V_t - I_a(r_a + r_s) \Rightarrow E_a = 600 - 200 \times (0.12 + 0.04)$$

$$\Rightarrow E_a = 568 \text{ V}$$



DC Motors

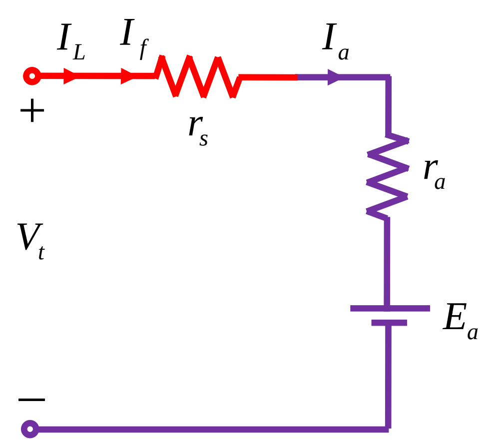
Solution 2: a series DC motor

$V_{tn} = 600\ \text{V} \qquad n_n = 600\ \text{rpm} \qquad I_{tn} = 200\ \text{A} \qquad r_s = 0.04\ \Omega \qquad r_a = 0.12\ \Omega$

b) Calculate the **developed torque and developed power** at nominal condition.

$$P_a = E_a I_a \Rightarrow P_a = 568 \times 200 \Rightarrow P_a = 113600\ \text{W}$$

$$T_a = \frac{P_a}{\omega} \Rightarrow T_a = \frac{113600}{600 \times \frac{2\pi}{60}} \Rightarrow T_a = 1808\ \text{Nm}$$



DC Motors

Solution 2: a series DC motor

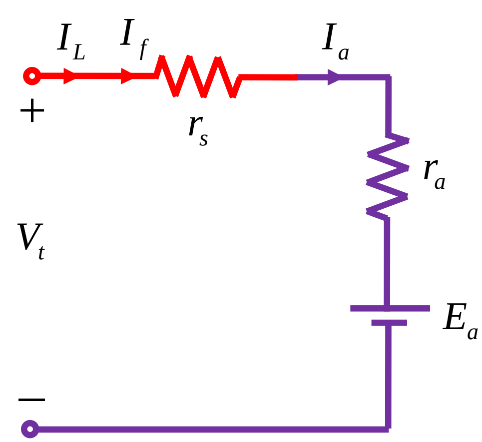
$V_{tn} = 600 \ \text{V} \qquad n_n = 600 \ \text{rpm} \qquad I_{tn} = 200 \ \text{A} \qquad r_s = 0.04 \ \Omega \qquad r_a = 0.12 \ \Omega$

c) If the load varies and the terminal current reduces to 150 A, calculate the **speed and developed torque**.

$$E_{a2} = V_t - I_{a2}(r_a + r_s) \Rightarrow E_{a2} = 600 - 150 \times (0.12 + 0.04)$$

$$\Rightarrow E_{a2} = 576 \quad \text{V}$$

$$\frac{E_{a2}}{E_{a1}} = \frac{k\phi_2\omega_2}{k\phi_1\omega_1} \Rightarrow \frac{E_{a2}}{E_{a1}} = \frac{\hat{k}I_{a2}n_2}{\hat{k}I_{a1}n_1}$$



DC Motors

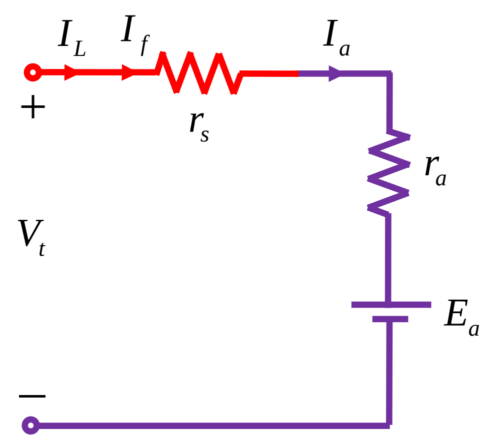
Solution 2: a series DC motor

$V_{tn} = 600 \text{ V} \quad n_n = 600 \text{ rpm} \quad I_{tn} = 200 \text{ A} \quad r_s = 0.04\ \Omega \quad r_a = 0.12\ \Omega$

c) If the load varies and the terminal current reduces to 150 A, calculate the **speed and developed torque**.

$E_{a2} = 576 \text{ V}$ $E_{a1} = 568 \text{ V}$

$I_{a2} = 150 \text{ A}$ $I_{a1} = 200 \text{ A}$



$$\frac{E_{a2}}{E_{a1}} = \frac{\hat{k} I_{a2} n_2}{\hat{k} I_{a1} n_1} \Rightarrow n_2 = n_1 \frac{E_{a2}}{E_{a1}} \frac{I_{a1}}{I_{a2}} \Rightarrow$$

$$n_2 = 600 \frac{576}{568} \frac{200}{150} \Rightarrow n_2 = 811 \text{ rpm}$$

DC Motors

Solution 2: a series DC motor

$V_{tn}=600\ \text{V}$ $n_n=600\ \text{rpm}$ $I_{tn}=200\ \text{A}$ $r_s=0.04\ \Omega$ $r_a=0.12\ \Omega$

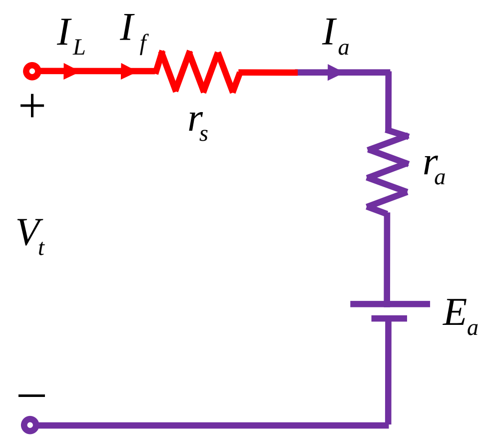
c) If the load varies and the terminal current reduces to 150 A, calculate the **speed and developed torque**.

$I_{a2}=150\ \text{A}$ $I_{a1}=200\ \text{A}$ $T_{a1}=1808\ \text{Nm}$

$$\frac{T_{a2}}{T_{a1}}=\frac{k\phi_2 I_{a2}}{k\phi_1 I_{a1}} \Rightarrow \frac{T_{a2}}{T_{a1}}=\frac{\hat{k}I_{a2}I_{a2}}{\hat{k}I_{a1}I_{a1}} \Rightarrow$$

$$T_{a2}=T_{a1}\left(\frac{I_{a2}}{I_{a1}}\right)^2 \Rightarrow T_{a2}=1808\left(\frac{150}{200}\right)^2$$

$$\Rightarrow T_{a2}=1017\ \text{Nm}$$



DC Motors

Example 3: Consider the series DC motor of the previous example

$V_{tn} = 600\ \text{V} \quad n_n = 600\ \text{rpm} \quad I_{tn} = 200\ \text{A} \quad r_s = 0.04\ \Omega \quad r_a = 0.12\ \Omega$

a) If the starting current needs to be 150% of nominal current, calculate the **external resistance** to be connected between the motor and the voltage source

b) With the external resistance calculate the **starting torque**.

c) If the external resistance remains in the circuit and the terminal current becomes 200 A, calculate the **back-emf and speed**.

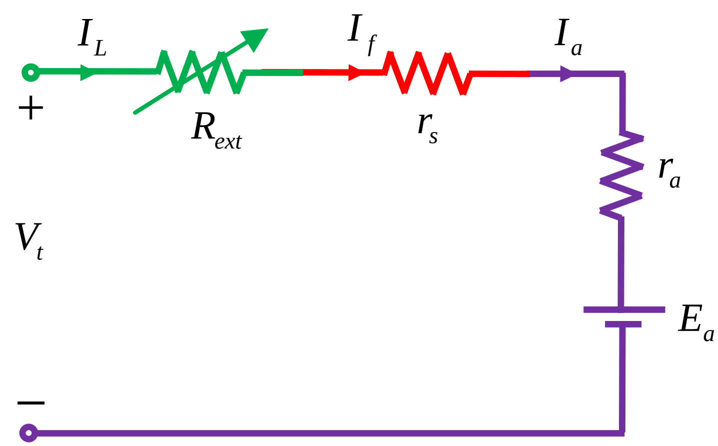
Assume the magnetic characteristics is linear.

DC Motors

Solution 3: series DC motor

$V_{tn} = 600 \ \text{V} \qquad n_n = 600 \ \text{rpm} \qquad I_{tn} = 200 \ \text{A} \qquad r_s = 0.04 \ \Omega \qquad r_a = 0.12 \ \Omega$

a) If the starting current needs to be 150% of nominal current, calculate the **external resistance** to be connected between the motor and the voltage source.



$$V_t = E_a + I_a(r_a + r_s + R_{ext})$$

At starting time, $E_a = 0$, therefore

$$V_t = I_a(r_a + r_s + R_{ext})$$

$$R_{ext} = \frac{V_t}{I_a} - r_a - r_s \Rightarrow R_{ext} = \frac{600}{1.5 \times 200} - 0.12 - 0.04 \Rightarrow R_{ext} = 1.84 \ \Omega$$

DC Motors

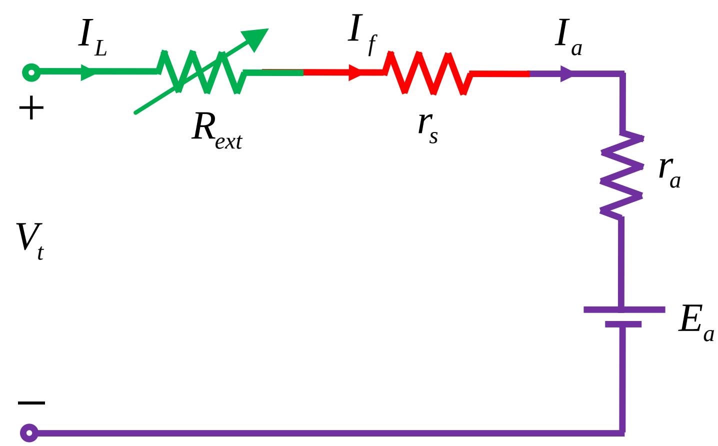
Solution 3: series DC motor

$V_{tn} = 600\ \text{V}$ $n_n = 600\ \text{rpm}$ $I_{tn} = 200\ \text{A}$ $r_s = 0.04\ \Omega$ $r_a = 0.12\ \Omega$

b) With the external resistance calculate the **starting torque**.

From previous example:

$I_a = 200\ \text{A}$ $T_a = 1808\ \text{Nm}$



$$T_{start} = T_a \left(\frac{I_{start}}{I_a} \right)^2 \Rightarrow T_{start} = 1808 \left(\frac{1.5 \times 200}{200} \right)^2$$

$$\Rightarrow T_{start} = 4068\ \text{Nm}$$

DC Motors

Solution 3: series DC motor

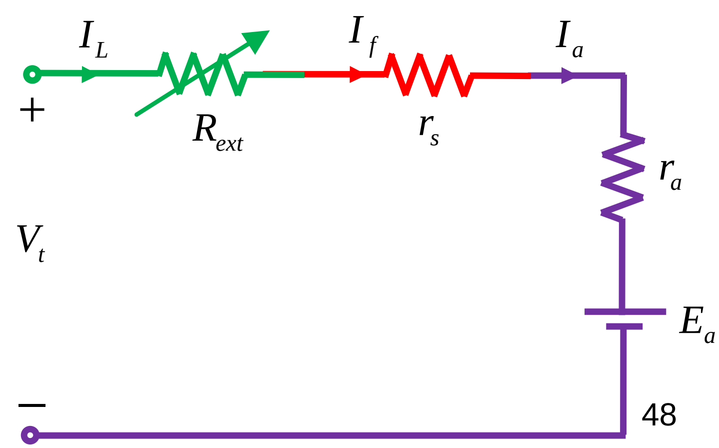
$V_{tn} = 600 \text{ V} \quad n_n = 600 \text{ rpm} \quad I_{tn} = 200 \text{ A} \quad r_s = 0.04 \ \Omega \quad r_a = 0.12 \ \Omega$

c) If the external resistance remains in the circuit and the terminal current becomes 200 A, calculate the **back-emf and speed**.

$$E_a = V_t - I_a(r_a + r_s + R_{ext}) \Rightarrow E_a = 600 - 200 \times 2 \Rightarrow E_a = 200 \text{ V}$$

$$n_2 = n_1 \frac{E_{a2}}{E_{a1}} \frac{I_{a1}}{I_{a2}} \Rightarrow n_2 = 600 \frac{200}{568} \frac{200}{200}$$

$$\Rightarrow n_2 = 211 \text{ rpm}$$



DC Motors

Example 4: Consider a shunt DC motor with the following values for nominal terminal voltage, armature resistance and field resistance:

$$V_{tn} = 230 \ \mathrm{V} \qquad r_a = 0.3 \ \Omega \qquad r_f = 160 \ \Omega$$

Assume the no-load rotational velocity is 1200 rpm, the terminal current at no-load condition is 3.938 A and the nominal armature current is 40 A.

a) Calculate the **armature current** at **no-load condition**.
b) Calculate the **developed power** at **no-load condition**.
c) Calculate the **efficiency** of the motor at **nominal condition**.
d) Calculate the **rotational velocity** at **nominal condition**.

DC Motors

Solution 4: shunt DC motor

$V_{tn} = 230\ \text{V}$ $r_a = 0.3\ \Omega$ $r_f = 160\ \Omega$

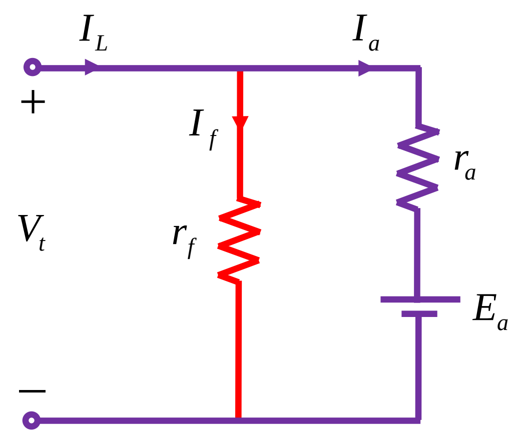
$n_{no-load} = 1200\ \text{rpm}$ $I_{L(no-load)} = 3.938\ \text{A}$ $I_{an} = 40\ \text{A}$

a) Calculate the **armature current** at **no-load condition**.

$$I_f = \frac{V_t}{r_f} \Rightarrow I_f = \frac{230}{160} \Rightarrow I_f = 1.438\ \text{A}$$

$$I_{a(no-load)} = I_{L(no-load)} - I_f \Rightarrow$$

$$I_{a(no-load)} = 3.938 - 1.438 \Rightarrow I_{a(no-load)} = 2.5\ \text{A}$$



DC Motors

Solution 4: shunt DC motor

$V_{tn} = 230 \text{ V}$ $r_a = 0.3 \ \Omega$ $r_f = 160 \ \Omega$

$n_{no-load} = 1200 \text{ rpm}$ $I_{L(no-load)} = 3.938 \text{ A}$ $I_{an} = 40 \text{ A}$

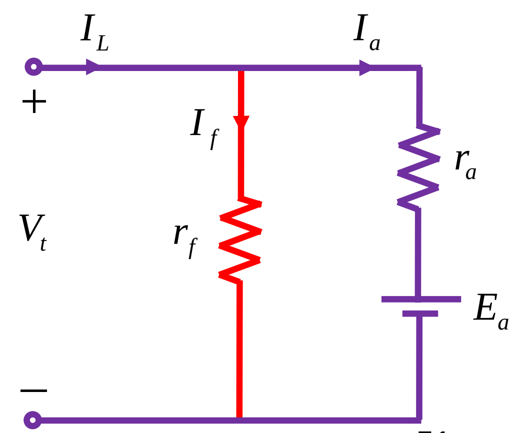
b) Calculate the **developed power** at **no-load condition**.

$$E_{a(no-load)} = V_t - r_a I_{a(no-load)} \Rightarrow E_{a(no-load)} = 230 - 0.3 \times 2.5$$

$$\Rightarrow E_{a(no-load)} = 229.25 \text{ V}$$

$$P_{a(no-load)} = E_{a(no-load)} I_{a(no-load)}$$

$$\Rightarrow P_{a(no-load)} = 229.25 \times 2.5 \Rightarrow P_{a(no-load)} = 573 \text{ W}$$



DC Motors

Solution 4: shunt DC motor

$V_{tn} = 230 \ \text{V}$ $r_a = 0.3 \ \Omega$ $r_f = 160 \ \Omega$

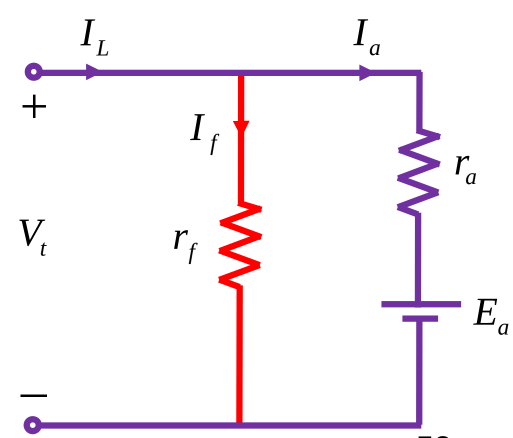
$n_{no-load} = 1200 \ \text{rpm}$ $I_{L(no-load)} = 3.938 \ \text{A}$ $I_{an} = 40 \ \text{A}$

c) Calculate the **efficiency** of the motor at **nominal condition**.

$E_{an} = V_t - r_a I_{an}$ ⇒ $E_{an} = 230 - 0.3 \times 40$ ⇒ $E_{an} = 218 \ \text{V}$

$P_{an} = E_{an} I_{an}$ ⇒ $P_{an} = 218 \times 40$

⇒ $P_{an} = 8720 \ \text{W}$



DC Motors

Solution 4: shunt DC motor

$V_{tn} = 230 \text{ V}$ $r_a = 0.3 \ \Omega$ $r_f = 160 \ \Omega$

$n_{no-load} = 1200 \text{ rpm}$ $I_{L(no-load)} = 3.938 \text{ A}$ $I_{an} = 40 \text{ A}$

c) Calculate the **efficiency** of the motor at **nominal condition**.

$$P_{an} = 8720 \text{ W}$$

Note that no-load developed power is the rotational losses

$$P_{rot} = P_{a(no-load)} = 573 \text{ W}$$

$$P_{out} = P_{an} - P_{rot} \Rightarrow P_{out} = 8720 - 573 \Rightarrow P_{out} = 8147 \text{ W}$$

DC Motors

Solution 4: shunt DC motor

$V_{tn} = 230 \text{ V}$ $\quad r_a = 0.3 \ \Omega$ $\quad r_f = 160 \ \Omega$

$n_{no-load} = 1200 \text{ rpm}$ $\quad I_{L(no-load)} = 3.938 \text{ A}$ $\quad I_{an} = 40 \text{ A}$

c) Calculate the **efficiency** of the motor at **nominal condition**.

$P_{out} = 8147 \text{ W}$

$P_{in} = V_t I_L \Rightarrow P_{in} = 230 \times (40 + 1.438) \Rightarrow P_{in} = 9530.6 \text{ W}$

$\eta = \dfrac{P_{out}}{P_{in}} \Rightarrow \eta = \dfrac{8147}{9530.6} \Rightarrow \eta = 0.855$

DC Motors

Solution 4: shunt DC motor

$V_{tn} = 230\ \text{V}$ $r_a = 0.3\ \Omega$ $r_f = 160\ \Omega$

$n_{no-load} = 1200\ \text{rpm}$ $I_{L(no-load)} = 3.938\ \text{A}$ $I_{an} = 40\ \text{A}$

d) Calculate the **rotational velocity** at **nominal condition**.

$E_{a(no-load)} = 229.25\ \text{V}$ $E_{an} = 218\ \text{V}$

$$n_n = n_{no-load} \frac{E_{an}}{E_{a(no-load)}} \Rightarrow n_n = 1200 \frac{218}{229.25} \Rightarrow n_n = 1141\ \text{rpm}$$