In The Name of God The Most Compassionate, The Most Merciful

General Theory of Electric Machines

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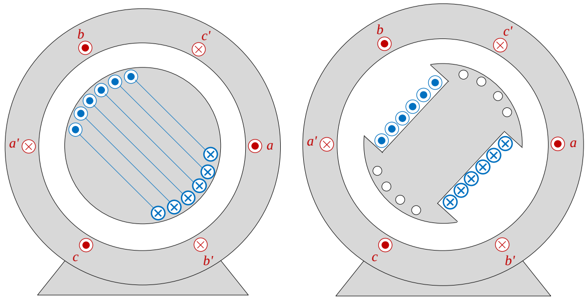
Chapter 5
Synchronous Machines

Introduction

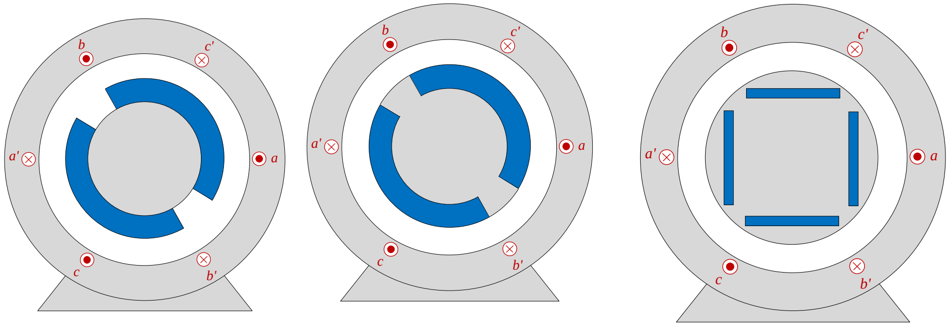
- The construction of 3-phase synchronous machines is relatively **more expensive** than that of induction machines.
- Synchronous machines are **more efficient** compared to the induction machines.
- The **stator** of synchronous machines is **similar** to that of induction machines.
- The synchronous machines are divided in terms of the rotor structures as **salient rotor** and **cylindrical rotor** machines.
- The synchronous machines are also categorized as:
  - **Wound rotor** (electrically excited rotor) synchronous machines,
  - **Permanent magnet** synchronous machines,
  - **Reluctance** synchronous machines.

Various Types of Synchronous Machines

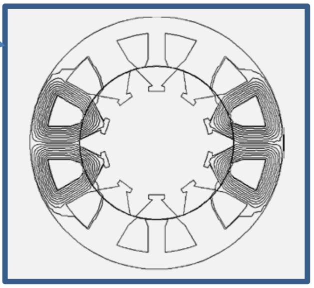
- **Wound rotor** (electrically excited rotor) synchronous machines
  - Cylindrical rotor
  - Salient rotor

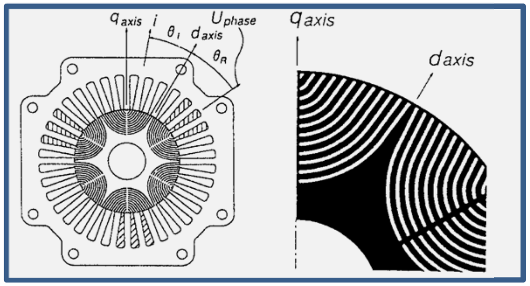


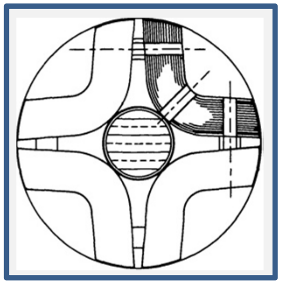
- **Permanent magnet** synchronous machines
  - Surface mounted PM (cylindrical rotor)
  - Surface inset PM (salient rotor)
  - Buried PM (Salient rotor)
    - Radially placed
    - Axially placed
    - inclined



- **Reluctance** synchronous machines
  - Segmential rotor
  - Flux barrier rotor
  - Axially laminated rotor

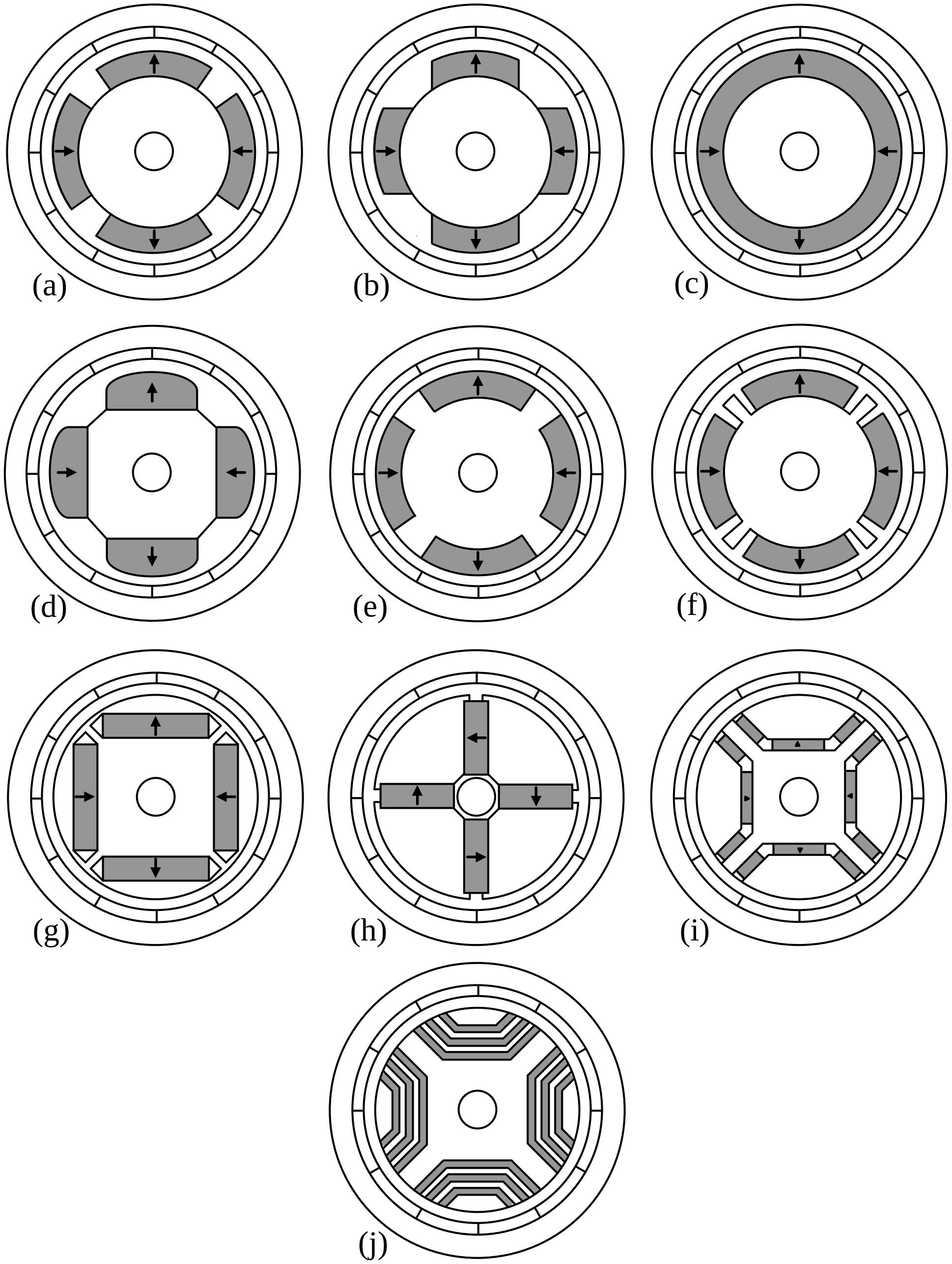






PM Synchronous Machines

Classification in terms of Magnet structure

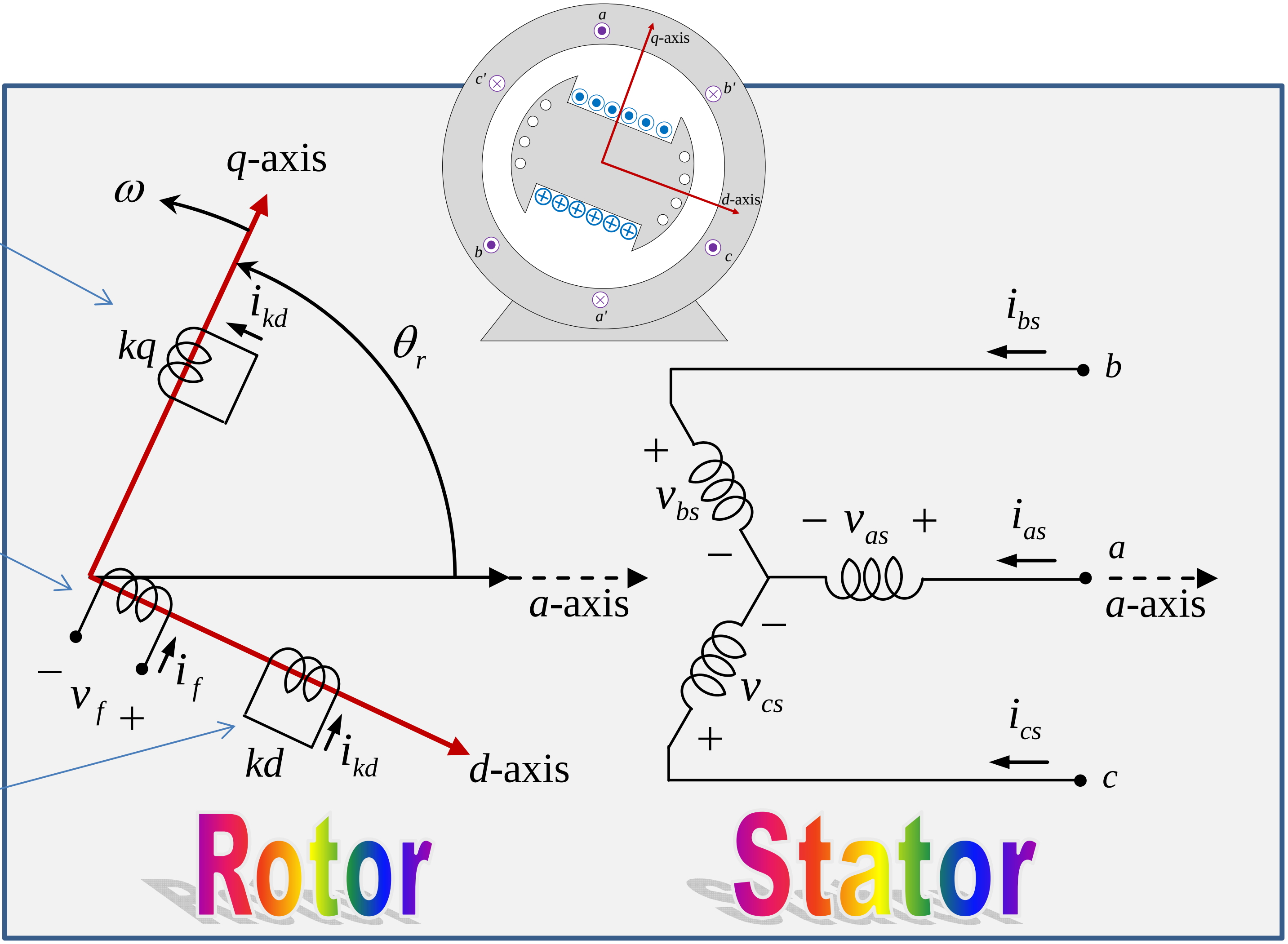


Different structures of PM in radial flux PM synchronous motors:

(a) surface mounted magnet
(b) surface mounted with parallel edges
(c) surface mounted ring magnet
(d) surface mounted bread-loaf magnet
(e) surface inset magnet
(f) surface inset magnet with airspace between magnets and iron inter-poles
(g) buried or interior magnet
(h) spoke magnet
(i) multi-segment interior magnet
(j) multilayer interior magnet

Wound Rotor Synchronous Machines

Circuit Representation



***q*-axis damper**

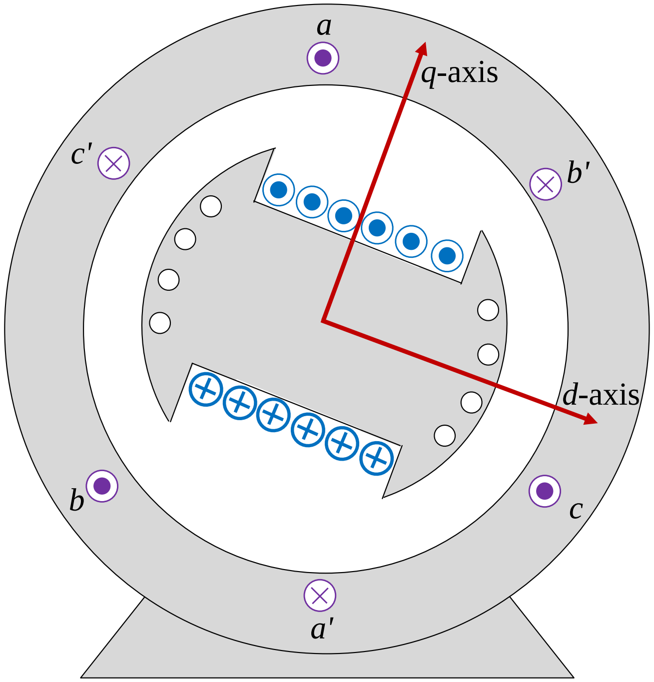
Field winding

***d*-axis damper**

Wound Rotor Synchronous Machines

Circuit Representation

- The stator has a set of **three sinusoidally distributed windings**.
- The rotor has normally one **field winding** which represented on the *d*-axis of the frame fixed to the rotor.
- For salient rotor machines, the **dampers** can be represented as two short-circuited damper windings along the *d*- and *q*-axis of the frame fixed to the rotor.
- Another winding is considered on the *q*-axis to represent the effects of **current flow in the rotor iron**.



Wound Rotor Synchronous Machines

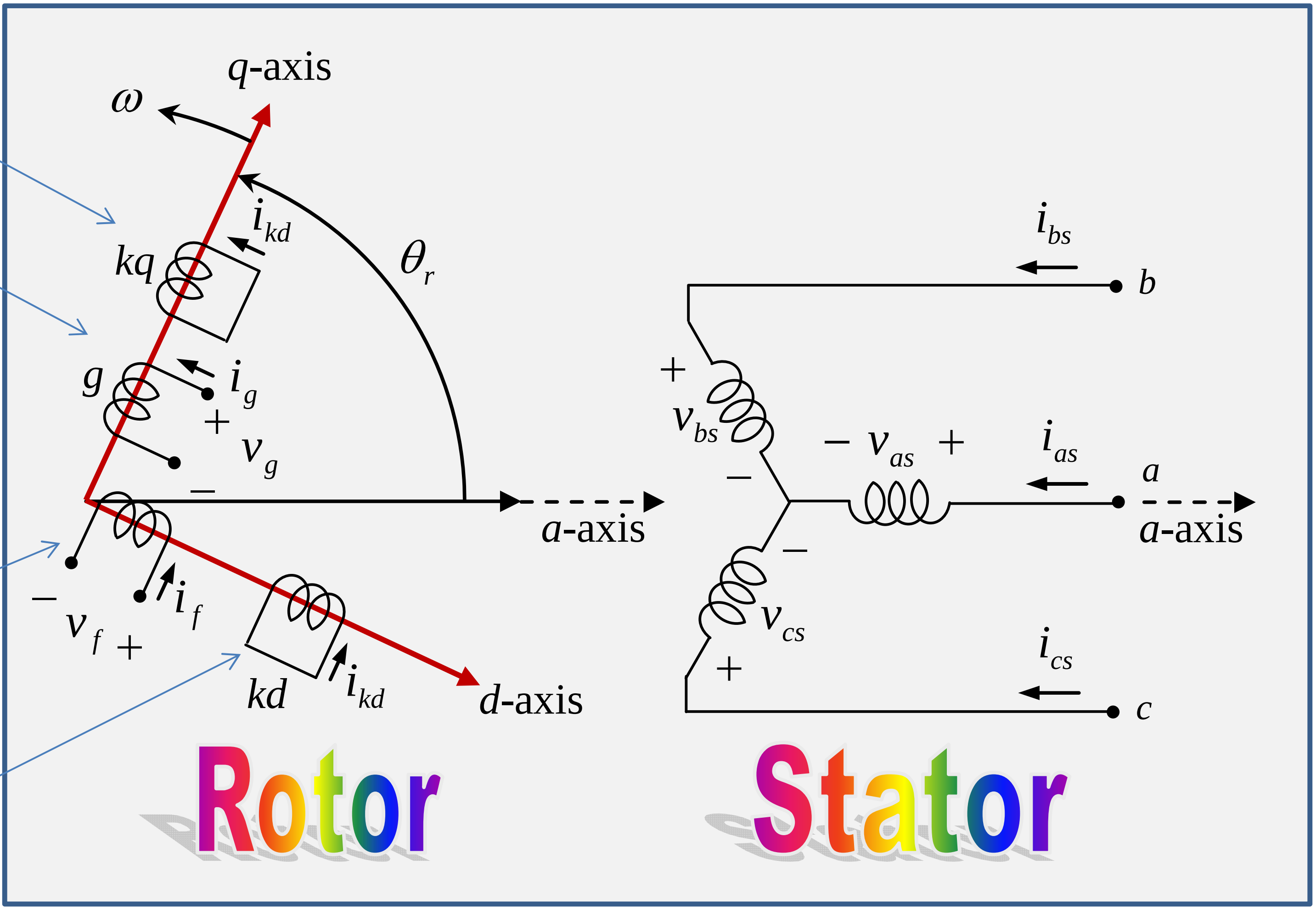
Circuit Representation

***q*-axis damper**

***q*-axis field**

***d*-axis field**

***d*-axis damper**



Wound Rotor Synchronous Machines

Mathematical model: Voltage equations

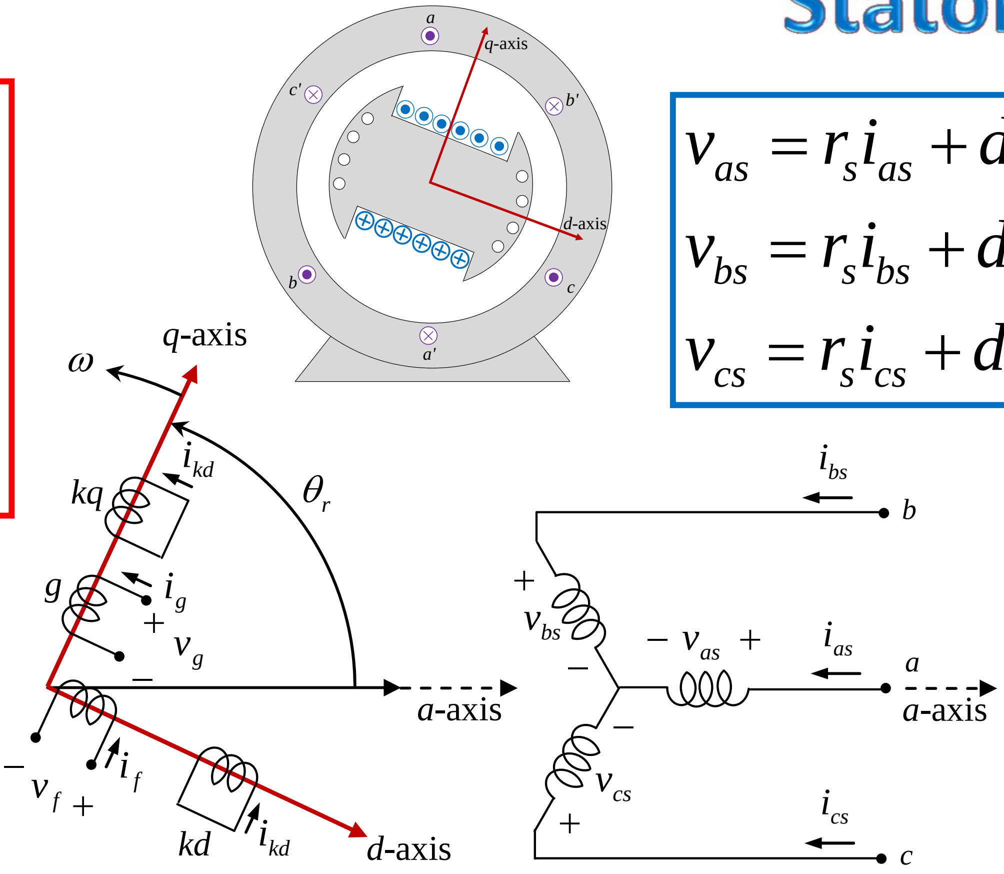
- The stator-rotor voltage equations can be expressed as:

Rotor

$$v_f = r_f i_f + d\lambda_f / dt$$
$$0 = r_{kd} i_{kd} + d\lambda_{kd} / dt$$
$$v_g = r_g i_g + d\lambda_g / dt$$
$$0 = r_{kq} i_{kq} + d\lambda_{kq} / dt$$

Stator

$$v_{as} = r_s i_{as} + d\lambda_{as} / dt$$
$$v_{bs} = r_s i_{bs} + d\lambda_{bs} / dt$$
$$v_{cs} = r_s i_{cs} + d\lambda_{cs} / dt$$



Wound Rotor Synchronous Machines

Mathematical model: Voltage equations

- The voltage equations can be rewritten in matrix form as follows:

$$\begin{bmatrix} \mathbf{v}_s^{abc} \\ \mathbf{v}_r^{dq} \end{bmatrix} = \begin{bmatrix} \mathbf{r}_s^{abc} & 0 \\ 0 & \mathbf{r}_r^{dq} \end{bmatrix} \begin{bmatrix} \mathbf{i}_s^{abc} \\ \mathbf{i}_r^{dq} \end{bmatrix} + \frac{d}{dt} \begin{bmatrix} \boldsymbol{\lambda}_s^{abc} \\ \boldsymbol{\lambda}_r^{dq} \end{bmatrix}$$

where

$$\mathbf{r}_s^{abc} = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \quad \mathbf{v}_s^{abc} = \begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} \quad \mathbf{i}_s^{abc} = \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} \quad \boldsymbol{\lambda}_s^{abc} = \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix}$$

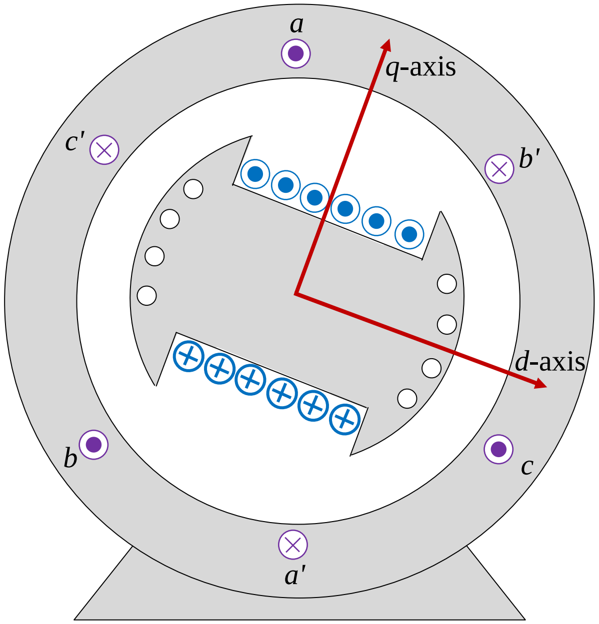
$$\mathbf{r}_r^{dq} = \begin{bmatrix} r_f & 0 & 0 & 0 \\ 0 & r_{kd} & 0 & 0 \\ 0 & 0 & r_g & 0 \\ 0 & 0 & 0 & r_{kq} \end{bmatrix} \quad \mathbf{v}_r^{dq} = \begin{bmatrix} v_f \\ 0 \\ v_g \\ 0 \end{bmatrix} \quad \mathbf{i}_r^{dq} = \begin{bmatrix} i_f \\ i_{kd} \\ i_g \\ i_{kq} \end{bmatrix} \quad \boldsymbol{\lambda}_r^{dq} = \begin{bmatrix} \lambda_f \\ \lambda_{kd} \\ \lambda_g \\ \lambda_{kq} \end{bmatrix}$$

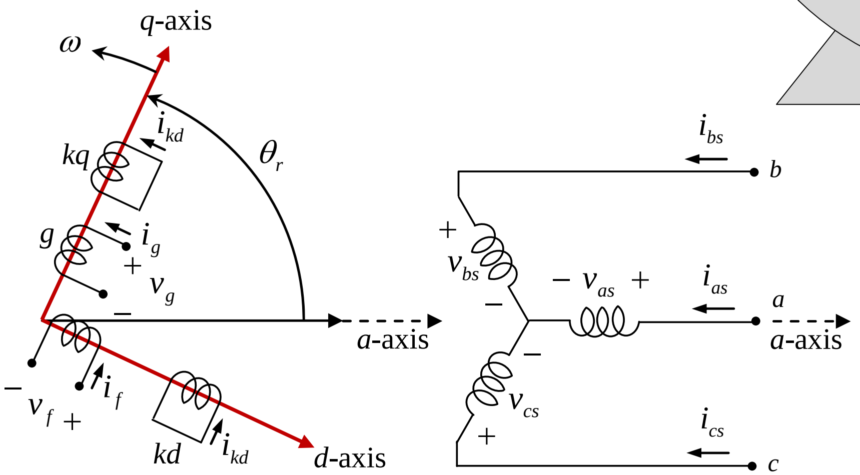
Wound Rotor Synchronous Machines

Mathematical model: Voltage equations

also

- r_s **stator** or armature winding resistance
- r_f ***d*-axis field** winding resistance
- r_g ***q*-axis field** winding resistance
- r_{kd} ***d*-axis damper** winding resistance
- r_{kq} ***q*-axis damper** winding resistance



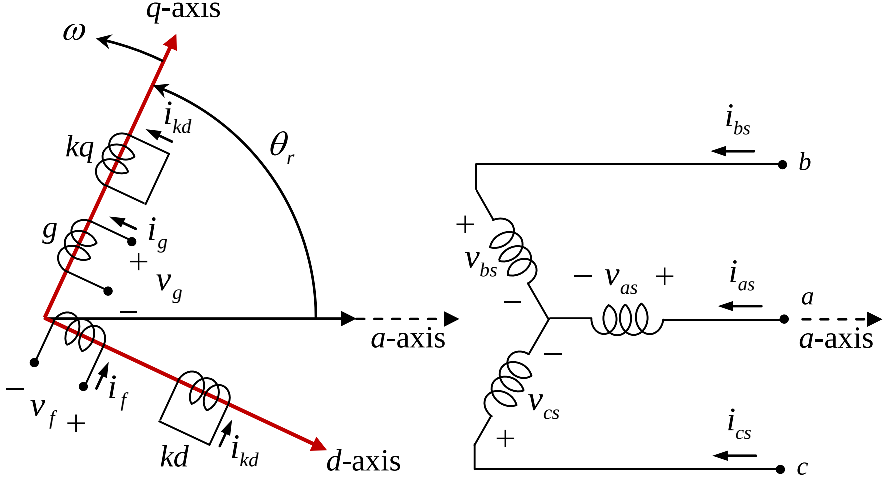


Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

- In matrix form, the **flux linkages** of the stator and rotor windings in terms of the winding **inductances** and **currents** are expressed as

$$\boxed{\begin{bmatrix} \boldsymbol{\lambda}_s^{abc} \\ \boldsymbol{\lambda}_r^{dq} \end{bmatrix} = \begin{bmatrix} \mathbf{L}_{ss}^{abc} & \mathbf{L}_{sr} \\ \mathbf{L}_{rs} & \mathbf{L}_{rr}^{dq} \end{bmatrix} \begin{bmatrix} \mathbf{i}_s^{abc} \\ \mathbf{i}_r^{dq} \end{bmatrix}}$$



where

- $\mathbf{L}_{ss}^{abc}$ stator windings inductance matrix
- $\mathbf{L}_{rr}^{dq}$ rotor windings inductance matrix
- $\mathbf{L}_{sr} = [\mathbf{L}_{rs}]^T$ the matrix of mutual inductances between stator and rotor windings

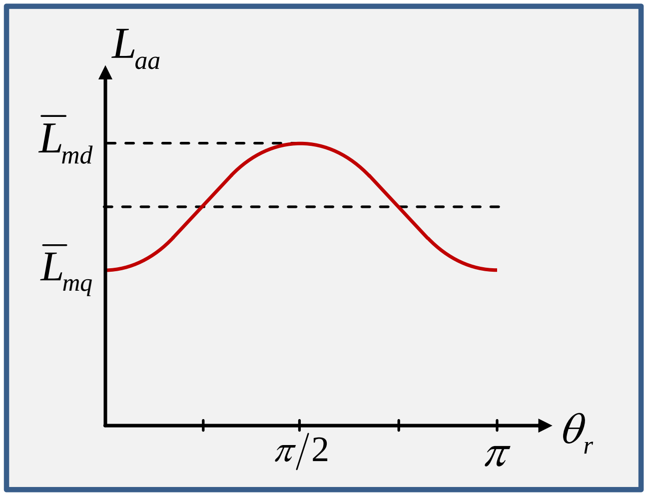
Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

- Now the **inductance matrices** have to be represented.
- For **salient rotor** machines, the stator winding inductances in *abc* system **vary with the rotor position**.
- So it is more convenient to obtain the stator winding inductances along the ***d*- and *q*-axis** of the frame fixed to the rotor.
- Also it is assumed that the variation of the stator winding inductances is **sinusoidally**.

$$\boxed{L_{aa} = \frac{\overline{L}_{md} + \overline{L}_{mq}}{2} - \frac{\overline{L}_{md} - \overline{L}_{mq}}{2} \cos\left(2\theta_r\right)}$$

$$\Rightarrow \quad \boxed{L_{aa} = L_0 - L_{ms} \cos\left(2\theta_r\right)}$$



Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

- Assume the **stator winding inductance** matrix in ***abc*** system is

$$\boxed{\mathbf{L}_{ss}^{abc} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}}$$

- In the following slides, the **stator phase-a self inductance** (L_{aa}) without the leakage inductance and the **mutual inductance between phase-a and phase-b** (L_{ab}) will be obtained.

- The other inductances can be determined in the **same manner**.

Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

Self inductance

- The **stator phase-*a* self inductance** (L_{aa}) can be calculated using the following expression

$$\boxed{L_{aa} = \frac{\lambda_{aa}}{i_a}}$$

where λ_{aa} is the magnetic flux produced by only the current of phase-a (i_a), linked with the winding of phase-a and obtained as

$$\boxed{\lambda_{aa} = N_s\,\phi_a}$$

where ϕ_a is the magnetic flux of phase-a and N_s is the number of turns of the winding of phase-a.

Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

Self inductance

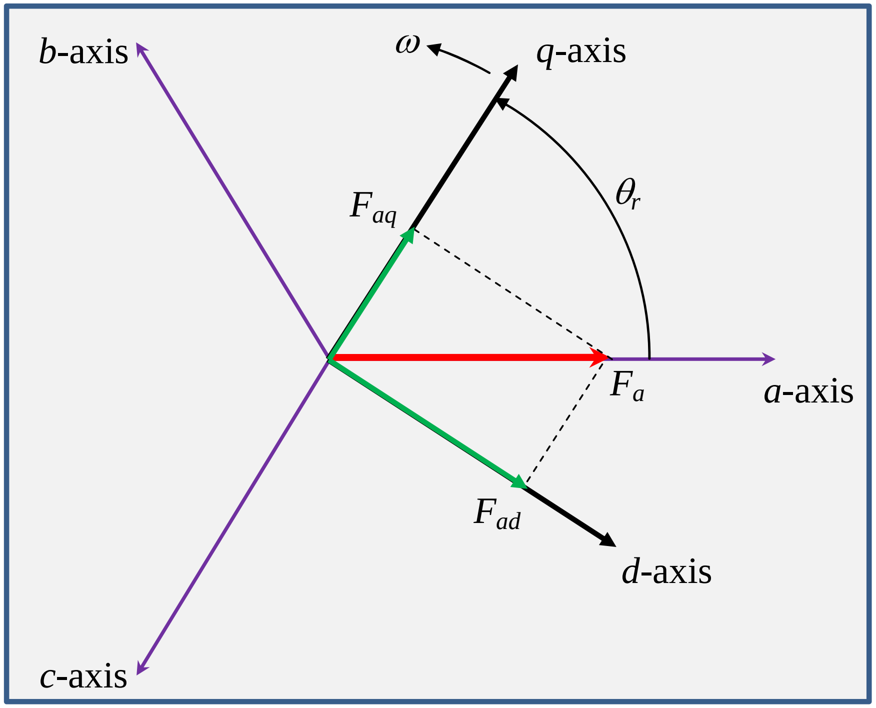
- ϕ_a is normally calculated by the multiplication of the MMF of phase-a ($F_a = N_s\, i_a$) and the permeance.

- But the permeance is a **function of the rotor position**.

- Therefore the MMF is resolved along the *d*- and *q*- axis as:

$$\boxed{F_{ad} = F_a \sin\theta_r = N_s\, i_a \sin\theta_r}$$

$$\boxed{F_{aq} = F_a \cos\,\theta_r = N_s i_a \cos\,\theta_r}$$



Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

Self inductance

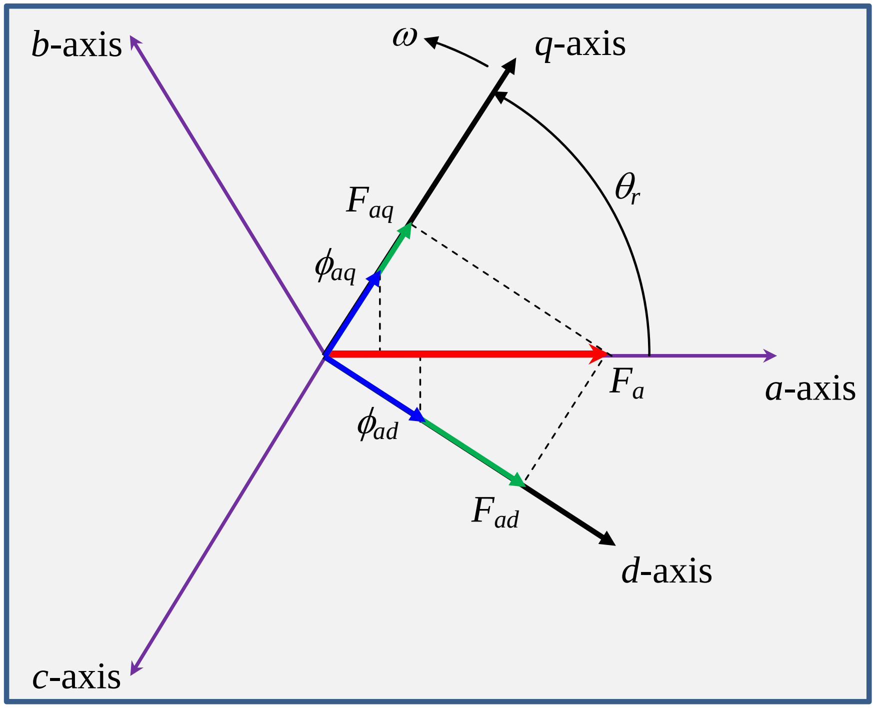
- Now ϕ_{ad} and ϕ_{aq} can be calculated by the multiplication of the corresponding **MMFs** (F_{ad} and F_{aq} respectively) and their related **permeances** (P_d and P_q respectively):

$$\boxed{\phi_{ad} = P_d F_{ad} = P_d\, F_a \sin\theta_r}$$

$$\boxed{\phi_{aq} = P_q F_{aq} = P_q\, F_a \cos\theta_r}$$

- ϕ_a is now calculated by the projection of ϕ_{ad} and ϕ_{aq} on *a*-axis:

$$\boxed{\phi_a = \phi_{ad} \sin\theta_r + \phi_{aq} \cos\theta_r}$$



Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

Self inductance

$$\boxed{\phi_{ad} = P_d\, F_a \sin\theta_r}$$

$$\boxed{\phi_{aq} = P_q\, F_a \cos\theta_r}$$

$$\boxed{\phi_a = \phi_{ad} \sin\theta_r + \phi_{aq} \cos\theta_r}$$

$$\boxed{\phi_a = F_a\left(P_d \sin^2\theta_r + P_q \cos^2\theta_r\right)}$$

where $\boxed{F_a = N_s\, i_a}$

$$\boxed{\lambda_{aa} = N_s\, \phi_a = N_s^2 i_a\left(P_d \sin^2\theta_r + P_q \cos^2\theta_r\right)}$$

$$\boxed{L_{aa} = \frac{\lambda_{aa}}{i_a} = N_s^2\left(P_d \sin^2\theta_r + P_q \cos^2\theta_r\right)}$$

Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

Self inductance

$$L_{aa} = N_s^2\left(P_d \sin^2\theta_r + P_q \cos^2\theta_r\right)$$

$$\sin^2\theta_r = \tfrac{1}{2}(1-\cos 2\theta_r)$$

$$\cos^2\theta_r = \tfrac{1}{2}(1+\cos 2\theta_r)$$

⇨ $$L_{aa} = N_s^2\left(\frac{P_d+P_q}{2} - \frac{P_d-P_q}{2}\cos 2\theta_r\right)$$

$$\overline{L}_{md} = N_s^2 P_d \qquad \overline{L}_{mq} = N_s^2 P_q$$

⇨ $$L_{aa} = \frac{\overline{L}_{md}+\overline{L}_{mq}}{2} - \frac{\overline{L}_{md}-\overline{L}_{mq}}{2}\cos\left(2\theta_r\right)$$

$$L_0 = \frac{\overline{L}_{md}+\overline{L}_{mq}}{2}$$

$$L_{ms} = \frac{\overline{L}_{md}-\overline{L}_{mq}}{2}$$

⇨ $$L_{aa} = L_0 - L_{ms}\cos\left(2\theta_r\right)$$

Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

Mutual inductance

$$\boxed{\phi_{ad} = P_d\, F_a \sin\theta_r}$$

$$\boxed{\phi_{aq} = P_q\, F_a \cos\theta_r}$$

- Similarly the linkage of the flux components (ϕ_{ad} and ϕ_{aq}) by phase-*b* winding that is 2π/3 electrical radian ahead can be written as:

$$\boxed{\lambda_{ab} = N_s\left[\phi_{ad}\sin\left(\theta_r - \tfrac{2\pi}{3}\right) + \phi_{aq}\cos\left(\theta_r - \tfrac{2\pi}{3}\right)\right]}$$

$$\boxed{\lambda_{ab} = N_s^2 i_a\left[P_d \sin\theta_r \sin\left(\theta_r - \tfrac{2\pi}{3}\right) + P_q \cos\theta_r \cos\left(\theta_r - \tfrac{2\pi}{3}\right)\right]}$$

$$\boxed{L_{ab} = \frac{\lambda_{ab}}{i_a} = N_s^2\left[P_d \sin\theta_r \sin\left(\theta_r - \tfrac{2\pi}{3}\right) + P_q \cos\theta_r \cos\left(\theta_r - \tfrac{2\pi}{3}\right)\right]}$$

Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

Mutual inductance

$$\boxed{L_{ab} = N_s^2\left[P_d \sin\theta_r \sin\left(\theta_r - \tfrac{2\pi}{3}\right) + P_q \cos\theta_r \cos\left(\theta_r - \tfrac{2\pi}{3}\right)\right]}$$

$$\boxed{L_{ab} = N_s^2\left[-\frac{P_d + P_q}{4} - \frac{P_d - P_q}{2}\cos 2\left(\theta_r - \tfrac{\pi}{3}\right)\right]}$$

$$\boxed{L_{ab} = \left[-\frac{\bar{L}_{md} + \bar{L}_{mq}}{4} - \frac{\bar{L}_{md} - \bar{L}_{mq}}{2}\cos 2\left(\theta_r - \tfrac{\pi}{3}\right)\right]}$$

$$\boxed{L_{ab} = -\tfrac{1}{2} L_0 - L_{ms}\cos 2\left(\theta_r - \tfrac{\pi}{3}\right)}$$

$$\boxed{\bar{L}_{md} = N_s^2 P_d}$$

$$\boxed{\bar{L}_{mq} = N_s^2 P_q}$$

$$\boxed{L_0 = \frac{\bar{L}_{md} + \bar{L}_{mq}}{2}}$$

$$\boxed{L_{ms} = \frac{\bar{L}_{md} - \bar{L}_{mq}}{2}}$$

Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

- Therefore the **stator inductance matrix** with considering the leakage inductance is written as:

$$\mathbf{L}_{ss}^{abc}=\begin{bmatrix} L_{ls}+L_0-L_{ms}\cos 2\theta_r & -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r-\frac{\pi}{3}) & -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r+\frac{\pi}{3}) \\ -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r-\frac{\pi}{3}) & L_{ls}+L_0-L_{ms}\cos 2(\theta_r-\frac{2\pi}{3}) & -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r+\pi) \\ -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r+\frac{\pi}{3}) & -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r+\pi) & L_{ls}+L_0-L_{ms}\cos 2(\theta_r+\frac{2\pi}{3}) \end{bmatrix}$$

- The **rotor inductance matrix** can be expressed as:

$$\mathbf{L}_{rr}^{dq}=\begin{bmatrix} L_{lf}+L_{mf} & L_{fkd} & 0 & 0 \\ L_{kdf} & L_{lkd}+L_{mkd} & 0 & 0 \\ 0 & 0 & L_{lg}+L_{mg} & L_{gkq} \\ 0 & 0 & L_{kqg} & L_{lkq}+L_{mkq} \end{bmatrix}$$

Why?

Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

- The mutual **inductance matrix** between the rotor and stator windings is:

$$\mathbf{L}_{sr} = \begin{bmatrix} L_{sf}\sin\theta_r & L_{skd}\sin\theta_r & L_{sg}\cos\theta_r & L_{skq}\cos\theta_r \\ L_{sf}\sin(\theta_r - \tfrac{2\pi}{3}) & L_{skd}\sin(\theta_r - \tfrac{2\pi}{3}) & L_{sg}\cos(\theta_r - \tfrac{2\pi}{3}) & L_{skq}\cos(\theta_r - \tfrac{2\pi}{3}) \\ L_{sf}\sin(\theta_r + \tfrac{2\pi}{3}) & L_{skd}\sin(\theta_r + \tfrac{2\pi}{3}) & L_{sg}\cos(\theta_r + \tfrac{2\pi}{3}) & L_{skq}\cos(\theta_r + \tfrac{2\pi}{3}) \end{bmatrix}$$

$$\mathbf{L}_{rs} = [\mathbf{L}_{sr}]^T$$

Why?

Since both $\mathbf{L}_{ss}^{abc}$ and $\mathbf{L}_{rs}$ are functions of the rotor position, the stator quantities are transformed to the rotor *qd*0 RF.

Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

- L_{ls} stator or armature winding **leakage** inductance
- L_{lf} *d*-axis field winding **leakage** inductance
- L_{lg} *q*-axis field winding **leakage** inductance
- L_{lkd} *d*-axis damper winding **leakage** inductance
- L_{lkq} *q*-axis damper winding **leakage** inductance
- L_{md} *d*-axis stator **magnetizing** inductance
- L_{mq} *q*-axis stator **magnetizing** inductance
- L_{mf} *d*-axis field winding **magnetizing** inductance
- L_{mg} *q*-axis field winding **magnetizing** inductance
- L_{mkd} *d*-axis damper winding **magnetizing** inductance
- L_{mkq} *q*-axis damper winding **magnetizing** inductance

Wound Rotor Synchronous Machines

Mathematical model: Flux linkage equations

- L_{fkd} **mutual** inductance between *d*-axis field winding and *d*-axis damper winding. $L_{kdf}=L_{fkd}$
- L_{gkq} **mutual** inductance between *q*-axis field winding and *q*-axis damper winding. $L_{kqg}=L_{gkq}$
- L_{sf} **peak value of mutual** inductance between stator windings and *d*-axis field winding.
- L_{skd} **peak value of mutual** inductance between stator windings and *d*-axis damper winding.
- L_{sg} **peak value of mutual** inductance between stator windings and *q*-axis field winding.
- L_{skq} **peak value of mutual** inductance between stator windings and *q*-axis damper winding.

Wound Rotor Synchronous Machines

*qd*0 stator voltage equations

- Note that **only the stator quantities** need to be **transformed** to rotor *qd*0 reference frame.
- The transformation is done using:

$$\left[\mathbf{f}_{qd0}\right]=\left[\mathbf{T}_{qd0}(\theta_r)\right]\left[\mathbf{f}_{abc}\right]$$

$$\left[\mathbf{f}_{abc}\right]=\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\left[\mathbf{f}_{qd0}\right]$$

where

$$\left[\mathbf{T}_{qd0}(\theta_r)\right]=\frac{2}{3}\begin{bmatrix}\cos\theta_r & \cos\left(\theta_r-\frac{2\pi}{3}\right) & \cos\left(\theta_r+\frac{2\pi}{3}\right)\\ \sin\theta_r & \sin\left(\theta_r-\frac{2\pi}{3}\right) & \sin\left(\theta_r+\frac{2\pi}{3}\right)\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{bmatrix}$$

$$\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}=\begin{bmatrix}\cos\theta_r & \sin\theta_r & 1\\ \cos\left(\theta_r-\frac{2\pi}{3}\right) & \sin\left(\theta_r-\frac{2\pi}{3}\right) & 1\\ \cos\left(\theta_r+\frac{2\pi}{3}\right) & \sin\left(\theta_r+\frac{2\pi}{3}\right) & 1\end{bmatrix}$$

Wound Rotor Synchronous Machines

*qd*0 stator voltage equations

- The stator voltage equations in *abc* system are

$$\boxed{\mathbf{v}_s^{abc} = \mathbf{r}_s^{abc}\,\mathbf{i}_s^{abc} + p\boldsymbol{\lambda}_s^{abc}}$$

where $p = d/dt$

$$\boxed{\left[\mathbf{f}_{abc}\right] = \left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\left[\mathbf{f}_{qd0}\right]}$$

- Applying the transformation $\mathbf{T}_{qd0}(\theta_r)$ to the voltage, current and flux yields

$$\boxed{\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\mathbf{v}_s^{qd0} = \mathbf{r}_s^{abc}\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\mathbf{i}_s^{qd0} + p\left(\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\boldsymbol{\lambda}_s^{qd0}\right)}$$

- Multiplying both sides by the transformation matrix yields

$$\boxed{\mathbf{v}_s^{qd0} = \left[\mathbf{T}_{qd0}(\theta_r)\right]\mathbf{r}_s^{abc}\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\mathbf{i}_s^{qd0} + \left[\mathbf{T}_{qd0}(\theta_r)\right]p\left(\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\boldsymbol{\lambda}_s^{qd0}\right)}$$

Wound Rotor Synchronous Machines

*qd*0 stator voltage equations

$$\boxed{\mathbf{v}_s^{qd0} = \underbrace{\left[\mathbf{T}_{qd0}(\theta_r)\right]\mathbf{r}_s^{abc}\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}}_{\mathbf{r}_s^{qd0}}\mathbf{i}_s^{qd0} + \left[\mathbf{T}_{qd0}(\theta_r)\right]p\left(\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\boldsymbol{\lambda}_s^{qd0}\right)}$$

- Substituting the following relations in the above expression

$$\boxed{\mathbf{r}_s^{qd0} = \mathbf{r}_s^{abc} = r_s\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\boxed{\left[\mathbf{T}_{qd0}(\theta_r)\right]p\left(\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\boldsymbol{\lambda}_s^{qd0}\right) = \omega_r\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\boldsymbol{\lambda}_s^{qd0} + p\boldsymbol{\lambda}_s^{qd0}}$$

yields

$$\boxed{\mathbf{v}_s^{qd0} = \mathbf{r}_s^{qd0}\mathbf{i}_s^{qd0} + \omega_r\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\boldsymbol{\lambda}_s^{qd0} + p\,\boldsymbol{\lambda}_s^{qd0}}$$

Wound Rotor Synchronous Machines

*qd*0 stator voltage equations

*qd*0 stator voltage equations

$$\mathbf{v}_s^{qd0} = \underbrace{\mathbf{r}_s^{qd0}\mathbf{i}_s^{qd0}}_{\text{Ohmic drop}} + \underbrace{\omega_r \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \boldsymbol{\lambda}_s^{qd0}}_{\text{Rotational EMF}} + \underbrace{p\,\boldsymbol{\lambda}_s^{qd0}}_{\text{Transformer EMF}}$$

$$\mathbf{v}_s^{qd0} = \begin{bmatrix} v_{qs} \\ v_{ds} \\ v_{0s} \end{bmatrix}$$

$$\mathbf{i}_s^{qd0} = \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \end{bmatrix}$$

$$\boldsymbol{\lambda}_s^{qd0} = \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \end{bmatrix}$$

$$\Rightarrow \left\{ \begin{aligned} v_{qs} &= r_s\, i_{qs} + p\lambda_{qs} + \omega_r\, \lambda_{ds} \\ v_{ds} &= r_s\, i_{ds} + p\lambda_{ds} - \omega_r\, \lambda_{qs} \\ v_{0s} &= r_s\, i_{0s} + p\lambda_{0s} \end{aligned} \right.$$

Wound Rotor Synchronous Machines

*qd*0 stator flux linkage equations

- The **stator flux linkage** equations in *abc* system is expressed as:

$$\boxed{\boldsymbol{\lambda}_s^{abc} = \mathbf{L}_{ss}^{abc}\,\mathbf{i}_s^{abc} + \mathbf{L}_{sr}\,\mathbf{i}_r^{dq}}$$

$$\boxed{\left[\mathbf{f}_{abc}\right] = \left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\left[\mathbf{f}_{qd0}\right]}$$

- Applying the transformation $\mathbf{T}_{qd0}(\theta_r)$ to the stator quantities yields

$$\boxed{\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\boldsymbol{\lambda}_s^{qd0} = \mathbf{L}_{ss}^{abc}\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\mathbf{i}_s^{qd0} + \mathbf{L}_{sr}\mathbf{i}_r^{dq}}$$

- Multiplying both sides by $\mathbf{T}_{qd0}(\theta_r)$ yields

$$\boxed{\boldsymbol{\lambda}_s^{qd0} = \left[\mathbf{T}_{qd0}(\theta_r)\right]\mathbf{L}_{ss}^{abc}\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\mathbf{i}_s^{qd0} + \left[\mathbf{T}_{qd0}(\theta_r)\right]\mathbf{L}_{sr}\mathbf{i}_r^{dq}}$$

Wound Rotor Synchronous Machines

*qd*0 stator flux linkage equations

$$\boldsymbol{\lambda}_s^{qd0} = \underbrace{\left[\mathbf{T}_{qd0}(\theta_r)\right]\mathbf{L}_{ss}^{abc}\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}}_{\mathbf{L}_{ss}^{qd0}}\ \mathbf{i}_s^{qd0} + \underbrace{\left[\mathbf{T}_{qd0}(\theta_r)\right]\ \mathbf{L}_{sr}}_{\mathbf{L}_{sr}^{qd0}}\ \mathbf{i}_r^{dq}$$

➡ $$\boxed{\boldsymbol{\lambda}_s^{qd0} = \mathbf{L}_{ss}^{qd0}\,\mathbf{i}_s^{qd0} + \mathbf{L}_{sr}^{qd0}\,\mathbf{i}_r^{dq}}$$

The inductance matrices L_{ss} and L_{sr} in *qd*0 reference frame need to be obtained.

Wound Rotor Synchronous Machines

*qd*0 stator flux linkage equations

$$\mathbf{L}_{ss}^{abc}=\begin{bmatrix} L_{ls}+L_0-L_{ms}\cos 2\theta_r & -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r-\frac{\pi}{3}) & -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r+\frac{\pi}{3}) \\ -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r-\frac{\pi}{3}) & L_{ls}+L_0-L_{ms}\cos 2(\theta_r-\frac{2\pi}{3}) & -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r+\pi) \\ -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r+\frac{\pi}{3}) & -\frac{1}{2}L_0-L_{ms}\cos 2(\theta_r+\pi) & L_{ls}+L_0-L_{ms}\cos 2(\theta_r+\frac{2\pi}{3}) \end{bmatrix}$$

where $\mathbf{L}_{ss}^{qd0}=\left[\mathbf{T}_{qd0}(\theta_r)\right]\mathbf{L}_{ss}^{abc}\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}$ is the **stator self-inductance** matrix in *qd*0 reference frame

$$\mathbf{L}_{ss}^{qd0}=\begin{bmatrix} L_{ls}+\frac{3}{2}(L_0-L_{ms}) & 0 & 0 \\ 0 & L_{ls}+\frac{3}{2}(L_0+L_{ms}) & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

Diagonal matrix and independent of θ_r.

Wound Rotor Synchronous Machines

*qd*0 stator flux linkage equations

$$\mathbf{L}_{sr} = \begin{bmatrix} L_{sf}\sin\theta_r & L_{skd}\sin\theta_r & L_{sg}\cos\theta_r & L_{skq}\cos\theta_r \\ L_{sf}\sin(\theta_r - \tfrac{2\pi}{3}) & L_{skd}\sin(\theta_r - \tfrac{2\pi}{3}) & L_{sg}\cos(\theta_r - \tfrac{2\pi}{3}) & L_{skq}\cos(\theta_r - \tfrac{2\pi}{3}) \\ L_{sf}\sin(\theta_r + \tfrac{2\pi}{3}) & L_{skd}\sin(\theta_r + \tfrac{2\pi}{3}) & L_{sg}\cos(\theta_r + \tfrac{2\pi}{3}) & L_{skq}\cos(\theta_r + \tfrac{2\pi}{3}) \end{bmatrix}$$

and $\mathbf{L}_{sr}^{qd0} = \left[\mathbf{T}_{qd0}(\theta_r)\right]\ \mathbf{L}_{sr}$ is the **stator-to-rotor mutual inductance** matrix in $qd0$ reference frame

$$\mathbf{L}_{sr}^{qd0} = \begin{bmatrix} 0 & 0 & L_{sg} & L_{skq} \\ L_{sf} & L_{skd} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix is independent of θ_r.

Wound Rotor Synchronous Machines

*qd*0 rotor flux linkage equations

- The **rotor flux linkage** equations in dq RF is expressed as:

$$\boxed{\boldsymbol{\lambda}_r^{dq} = \mathbf{L}_{rs}\,\mathbf{i}_s^{abc} + \mathbf{L}_{rr}^{dq}\,\mathbf{i}_r^{dq}}$$

$$\boxed{\left[\mathbf{f}_{abc}\right] = \left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}\left[\mathbf{f}_{qd0}\right]}$$

- Applying the transformation $\mathbf{T}_{qd0}(\theta_r)$ to the stator current yields

$$\boldsymbol{\lambda}_r^{dq} = \underbrace{\mathbf{L}_{rs}\left[\mathbf{T}_{qd0}(\theta_r)\right]^{-1}}_{\mathbf{L}_{rs}^{qd0}}\mathbf{i}_s^{qd0} + \mathbf{L}_{rr}^{dq}\mathbf{i}_r^{dq}$$

$$\mathbf{L}_{rs}^{qd0} = \begin{bmatrix} 0 & \frac{3}{2}L_{sf} & 0 \\ 0 & \frac{3}{2}L_{skd} & 0 \\ \frac{3}{2}L_{sg} & 0 & 0 \\ \frac{3}{2}L_{skq} & 0 & 0 \end{bmatrix} = \frac{3}{2}\left[\mathbf{L}_{sr}^{qd0}\right]^T$$

$$\Rightarrow \boxed{\boldsymbol{\lambda}_r^{dq} = \mathbf{L}_{rs}^{qd0}\,\mathbf{i}_s^{qd0} + \mathbf{L}_{rr}^{dq}\,\mathbf{i}_r^{dq}}$$

Wound Rotor Synchronous Machines

flux linkage equations

$$\boxed{\boldsymbol{\lambda}_s^{qd0}=\mathbf{L}_{ss}^{qd0}\,\mathbf{i}_s^{qd0}+\mathbf{L}_{sr}^{qd0}\,\mathbf{i}_r^{dq}} \quad \& \quad \boxed{\boldsymbol{\lambda}_r^{dq}=\mathbf{L}_{rs}^{qd0}\,\mathbf{i}_s^{qd0}+\mathbf{L}_{rr}^{dq}\,\mathbf{i}_r^{dq}} \Rightarrow$$

$$\boxed{\begin{bmatrix}\lambda_{qs}\\ \lambda_{ds}\\ \lambda_{0s}\\ \hline \lambda_f\\ \lambda_{kd}\\ \lambda_g\\ \lambda_{kq}\end{bmatrix}=\left[\begin{array}{ccc|cccc} L_{ls}+L_{mq} & 0 & 0 & 0 & 0 & L_{sg} & L_{skq}\\ 0 & L_{ls}+L_{md} & 0 & L_{sf} & L_{skd} & 0 & 0\\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0\\ \hline 0 & \frac{3}{2}L_{sf} & 0 & L_{lf}+L_{mf} & L_{fkd} & 0 & 0\\ 0 & \frac{3}{2}L_{skd} & 0 & L_{kdf} & L_{lkd}+L_{mkd} & 0 & 0\\ \frac{3}{2}L_{sg} & 0 & 0 & 0 & 0 & L_{lg}+L_{mg} & L_{gkq}\\ \frac{3}{2}L_{skq} & 0 & 0 & 0 & 0 & L_{kqg} & L_{lkq}+L_{mkq}\end{array}\right]\begin{bmatrix}i_{qs}\\ i_{ds}\\ i_{0s}\\ \hline i_f\\ i_{kd}\\ i_g\\ i_{kq}\end{bmatrix}}$$

where $L_{md}=\frac{3}{2}(L_0+L_{ms})=\frac{3}{2}N_s^2P_d$ and $L_{mq}=\frac{3}{2}(L_0-L_{ms})=\frac{3}{2}N_s^2P_q$ are the **equivalent magnetizing inductances** of the *d*- and *q*-axis stator winding respectively.

Wound Rotor Synchronous Machines

flux linkage equations

- As evident, the inductance matrix is **non-symmetric** due to the presence of 3/2 factor. Replacing the actual rotor currents with the following **equivalent currents**

$$\boxed{\hat{i}_f = \tfrac{2}{3} i_f} \qquad \boxed{\hat{i}_{kd} = \tfrac{2}{3} i_{kd}} \qquad \boxed{\hat{i}_g = \tfrac{2}{3} i_g} \qquad \boxed{\hat{i}_{kq} = \tfrac{2}{3} i_{kq}}$$

yields

$$\boxed{\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \\ \hline \lambda_f \\ \lambda_{kd} \\ \lambda_g \\ \lambda_{kq} \end{bmatrix} = \left[\begin{array}{ccc|cccc} L_{ls}+L_{mq} & 0 & 0 & 0 & 0 & \frac{3}{2}L_{sg} & \frac{3}{2}L_{skq} \\ 0 & L_{ls}+L_{md} & 0 & \frac{3}{2}L_{sf} & \frac{3}{2}L_{skd} & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 \\ \hline 0 & \frac{3}{2}L_{sf} & 0 & \frac{3}{2}(L_{lf}+L_{mf}) & \frac{3}{2}L_{fkd} & 0 & 0 \\ 0 & \frac{3}{2}L_{skd} & 0 & \frac{3}{2}L_{kdf} & \frac{3}{2}(L_{lkd}+L_{mkd}) & 0 & 0 \\ \frac{3}{2}L_{sg} & 0 & 0 & 0 & 0 & \frac{3}{2}(L_{lg}+L_{mg}) & \frac{3}{2}L_{gkq} \\ \frac{3}{2}L_{skq} & 0 & 0 & 0 & 0 & \frac{3}{2}L_{kqg} & \frac{3}{2}(L_{lkq}+L_{mkq}) \end{array}\right] \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ \hline \hat{i}_f \\ \hat{i}_{kd} \\ \hat{i}_g \\ \hat{i}_{kq} \end{bmatrix}}$$

Wound Rotor Synchronous Machines

Referring rotor quantities to the stator side

- Now all the rotor quantities and parameters are referred to the stator side:

$$\boxed{i'_f = \frac{N_f}{N_s}\hat{i}_f = \frac{2}{3}\frac{N_f}{N_s} i_f} \quad \boxed{i'_{kd} = \frac{N_{kd}}{N_s}\hat{i}_{kd} = \frac{2}{3}\frac{N_{kd}}{N_s} i_{kd}} \quad \boxed{v'_f = \frac{N_s}{N_f} v_f} \quad \boxed{v'_{kd} = \frac{N_s}{N_{kd}} v_{kd}}$$

$$\boxed{i'_g = \frac{N_g}{N_s}\hat{i}_g = \frac{2}{3}\frac{N_g}{N_s} i_g} \quad \boxed{i'_{kq} = \frac{N_{kq}}{N_s}\hat{i}_{kq} = \frac{2}{3}\frac{N_{kq}}{N_s} i_{kq}} \quad \boxed{v'_g = \frac{N_s}{N_g} v_g} \quad \boxed{v'_{kq} = \frac{N_s}{N_{kq}} v_{kq}}$$

$$\boxed{\lambda'_f = \frac{N_s}{N_f}\lambda_f} \quad \boxed{\lambda'_{kd} = \frac{N_s}{N_{kd}}\lambda_{kd}} \quad \boxed{r'_f = \frac{3}{2}\left(\frac{N_s}{N_f}\right)^2 r_f} \quad \boxed{r'_{kd} = \frac{3}{2}\left(\frac{N_s}{N_{kd}}\right)^2 r_{kd}}$$

$$\boxed{\lambda'_g = \frac{N_s}{N_g}\lambda_g} \quad \boxed{\lambda'_{kq} = \frac{N_s}{N_{kq}}\lambda_{kq}} \quad \boxed{r'_g = \frac{3}{2}\left(\frac{N_s}{N_g}\right)^2 r_g} \quad \boxed{r'_{kq} = \frac{3}{2}\left(\frac{N_s}{N_{kq}}\right)^2 r_{kq}}$$

Wound Rotor Synchronous Machines

Referring rotor quantities to the stator side

$$L_{md}=\tfrac{3}{2}N_s^2P_d \qquad L_{mq}=\tfrac{3}{2}N_s^2P_q$$

- Using the above relations, the inductances are expressed as:

Mutual inductances between rotor and stator windings:

$$L_{sf}=N_sN_fP_d=\frac{2}{3}\frac{N_f}{N_s}L_{md} \qquad L_{skd}=N_sN_{kd}P_d=\frac{2}{3}\frac{N_{kd}}{N_s}L_{md}$$

$$L_{sg}=N_sN_gP_q=\frac{2}{3}\frac{N_g}{N_s}L_{mq} \qquad L_{skq}=N_sN_{kq}P_q=\frac{2}{3}\frac{N_{kq}}{N_s}L_{mq}$$

Magnetizing inductances of the rotor windings:

$$L_{mf}=N_f^2P_d=\frac{2}{3}\left(\frac{N_f}{N_s}\right)^2L_{md} \qquad L_{mkd}=N_{kd}^2P_d=\frac{2}{3}\left(\frac{N_{kd}}{N_s}\right)^2L_{md}$$

$$L_{mg}=N_g^2P_q=\frac{2}{3}\left(\frac{N_g}{N_s}\right)^2L_{mq} \qquad L_{mkq}=N_{kq}^2P_q=\frac{2}{3}\left(\frac{N_{kq}}{N_s}\right)^2L_{mq}$$

Wound Rotor Synchronous Machines

Referring rotor quantities to the stator side

$$\boxed{L_{md} = \tfrac{3}{2} N_s^2 P_d} \qquad \boxed{L_{mq} = \tfrac{3}{2} N_s^2 P_q}$$

- Using the above relations, the inductances are expressed as:

Mutual inductances between *d*-axis rotor windings (between *f* and *kd*):

$$\boxed{L_{fkd} = L_{kdf} = N_f N_{kd} P_d = \tfrac{2}{3} \frac{N_f N_{kd}}{N_s^2} L_{md}}$$

Mutual inductances between *q*-axis rotor windings (between *g* and *kq*):

$$\boxed{L_{gkq} = L_{kqg} = N_g N_{kq} P_d = \tfrac{2}{3} \frac{N_g N_{kq}}{N_s^2} L_{mq}}$$

Wound Rotor Synchronous Machines

Referring rotor quantities to the stator side

- The remaining rotor parameters are referred to the stator side:

$$\boxed{L'_{ff} = \frac{3}{2}\left(\frac{N_s}{N_f}\right)^2 L_{lf} + L_{md}}$$

$$\boxed{L'_{kdkd} = \frac{3}{2}\left(\frac{N_s}{N_{kd}}\right)^2 L_{lkd} + L_{md}}$$

$$\boxed{L'_{gg} = \frac{3}{2}\left(\frac{N_s}{N_g}\right)^2 L_{lg} + L_{mq}}$$

$$\boxed{L'_{kqkq} = \frac{3}{2}\left(\frac{N_s}{N_{kq}}\right)^2 L_{lkq} + L_{mq}}$$

- The *d*- and *q*-axis **synchronous inductances** are defined as:

$$\boxed{L_d = L_{ls} + L_{md}}$$

$$\boxed{L_q = L_{ls} + L_{mq}}$$

Wound Rotor Synchronous Machines

*qd*0 flux linkage equations referred to the stator side

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \\ \hline \lambda'_f \\ \lambda'_{kd} \\ \lambda'_g \\ \lambda'_{kq} \end{bmatrix} = \left[\begin{array}{ccc|cccc} L_q & 0 & 0 & 0 & 0 & L_{mq} & L_{mq} \\ 0 & L_d & 0 & L_{md} & L_{md} & 0 & 0 \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 \\ \hline 0 & L_{md} & 0 & L'_{ff} & L_{md} & 0 & 0 \\ 0 & L_{md} & 0 & L_{md} & L'_{kdkd} & 0 & 0 \\ L_{mq} & 0 & 0 & 0 & 0 & L'_{gg} & L_{mq} \\ L_{mq} & 0 & 0 & 0 & 0 & L_{mq} & L'_{kqkq} \end{array}\right] \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ \hline i'_f \\ i'_{kd} \\ i'_g \\ i'_{kq} \end{bmatrix}$$

$$L'_{ff} = \tfrac{3}{2}\left(\frac{N_s}{N_f}\right)^2 L_{lf} + L_{md}$$

$$L'_{gg} = \tfrac{3}{2}\left(\frac{N_s}{N_g}\right)^2 L_{lg} + L_{mq}$$

$$L'_{kdkd} = \tfrac{3}{2}\left(\frac{N_s}{N_{kd}}\right)^2 L_{lkd} + L_{md}$$

$$L'_{kqkq} = \tfrac{3}{2}\left(\frac{N_s}{N_{kq}}\right)^2 L_{lkq} + L_{mq}$$

$$L_d = L_{ls} + L_{md}$$

$$L_{sf} = \frac{2}{3}\frac{N_f}{N_s} L_{md}$$

$$L_{sg} = \frac{2}{3}\frac{N_g}{N_s} L_{mq}$$

$$L_{gkq} = L_{kqg} = \frac{2}{3}\frac{N_g N_{kq}}{N_s^2} L_{mq}$$

$$L_q = L_{ls} + L_{mq}$$

$$L_{skd} = \frac{2}{3}\frac{N_{kd}}{N_s} L_{md}$$

$$L_{skq} = \frac{2}{3}\frac{N_{kq}}{N_s} L_{mq}$$

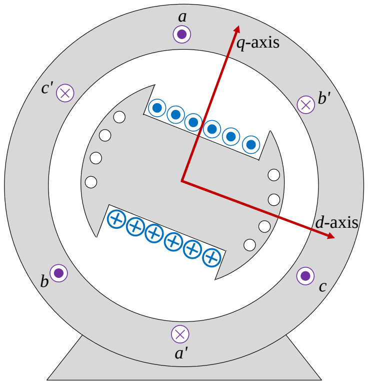
$$L_{fkd} = L_{kdf} = \frac{2}{3}\frac{N_f N_{kd}}{N_s^2} L_{md}$$

Wound Rotor Synchronous Machines

Summary: *qd*0 equations referred to the stator side

Stator

$$v_{qs} = r_s i_{qs} + d\lambda_{qs}/dt + \omega_r \lambda_{ds}$$
$$v_{ds} = r_s i_{ds} + d\lambda_{ds}/dt - \omega_r \lambda_{qs}$$
$$v_{0s} = r_s i_{0s} + d\lambda_{0s}/dt$$



Rotor

$$v'_f = r'_f i'_f + d\lambda'_f/dt$$
$$v'_{kd} = r'_{kd} i'_{kd} + d\lambda'_{kd}/dt$$
$$v'_g = r'_g i'_g + d\lambda'_g/dt$$
$$v'_{kq} = r'_{kq} i'_{kq} + d\lambda'_{kq}/dt$$

$$\lambda_{qs} = L_q i_{qs} + L_{mq} i'_g + L_{mq} i'_{kq}$$
$$\lambda_{ds} = L_d i_{ds} + L_{md} i'_f + L_{md} i'_{kd}$$
$$\lambda_{0s} = L_{ls} i_{0s}$$

$$\lambda'_f = L_{md} i_{ds} + L'_{ff} i'_f + L_{md} i'_{kd}$$
$$\lambda'_{kd} = L_{md} i_{ds} + L_{md} i'_f + L'_{kdkd} i'_{kd}$$
$$\lambda'_g = L_{mq} i_{qs} + L'_{gg} i'_g + L_{mq} i'_{kq}$$
$$\lambda'_{kq} = L_{mq} i_{qs} + L_{mq} i'_g + L'_{kqkq} i'_{kq}$$

Wound Rotor Synchronous Machines

q-axis equivalent circuit

Stator

$$v_{qs} = r_s i_{qs} + d\lambda_{qs}/dt + \omega\lambda_{ds}$$

$$\lambda_{qs} = L_{ls} i_{qs} + L_{mq}(i_{qs} + i'_g + i'_{kq})$$

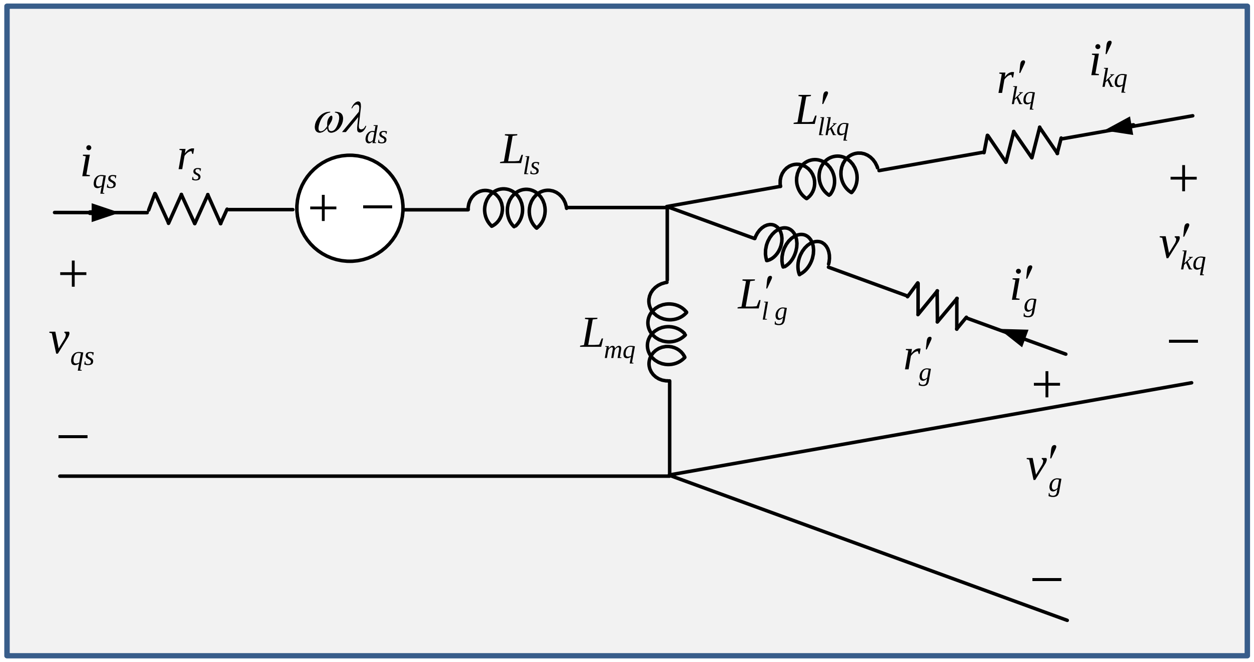
Rotor

$$\lambda'_g = L'_{lg} i'_g + L_{mq}(i_{qs} + i'_g + i'_{kq})$$

$$\lambda'_{kq} = L'_{lkq} i'_{kq} + L_{mq}(i_{qs} + i'_g + i'_{kq})$$

$$v'_g = r'_g i'_g + d\lambda'_g/dt$$

$$v'_{kq} = r'_{kq} i'_{kq} + d\lambda'_{kq}/dt$$



Wound Rotor Synchronous Machines

d-axis equivalent circuit

Stator

$$v_{ds} = r_s i_{ds} + d\lambda_{ds}/dt - \omega\lambda_{qs}$$

$$\lambda_{ds} = L_{ls} i_{ds} + L_{md}(i_{ds} + i'_f + i'_{kd})$$

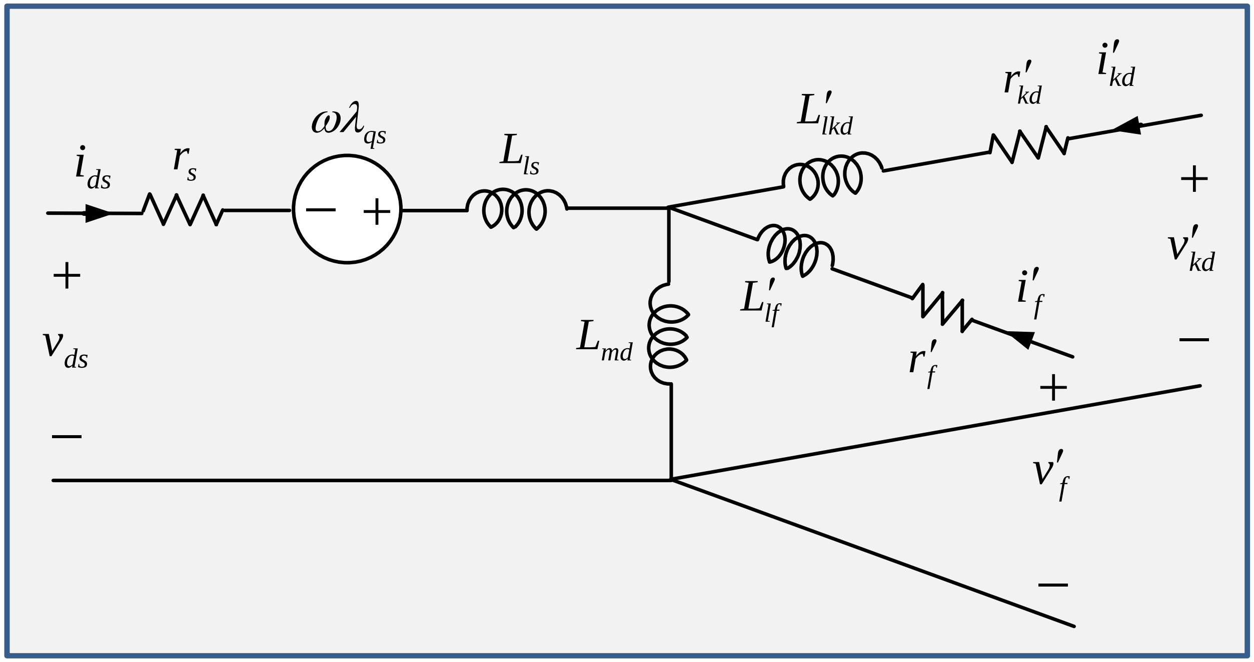
Rotor

$$\lambda'_f = L'_{lf} i'_f + L_{md}(i_{ds} + i'_f + i'_{kd})$$

$$\lambda'_{kd} = L'_{lkd} i'_{kd} + L_{md}(i_{ds} + i'_f + i'_{kd})$$

$$v'_f = r'_f i'_f + d\lambda'_f/dt$$

$$v'_{kd} = r'_{kd} i'_{kd} + d\lambda'_{kd}/dt$$



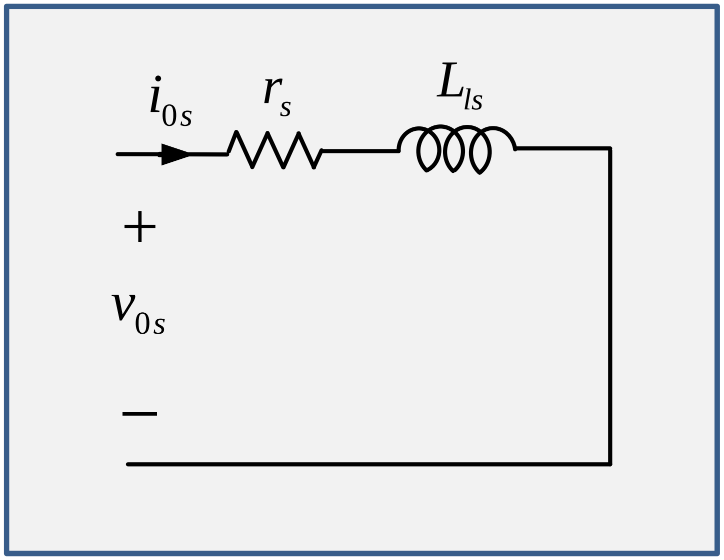
Wound Rotor Synchronous Machines

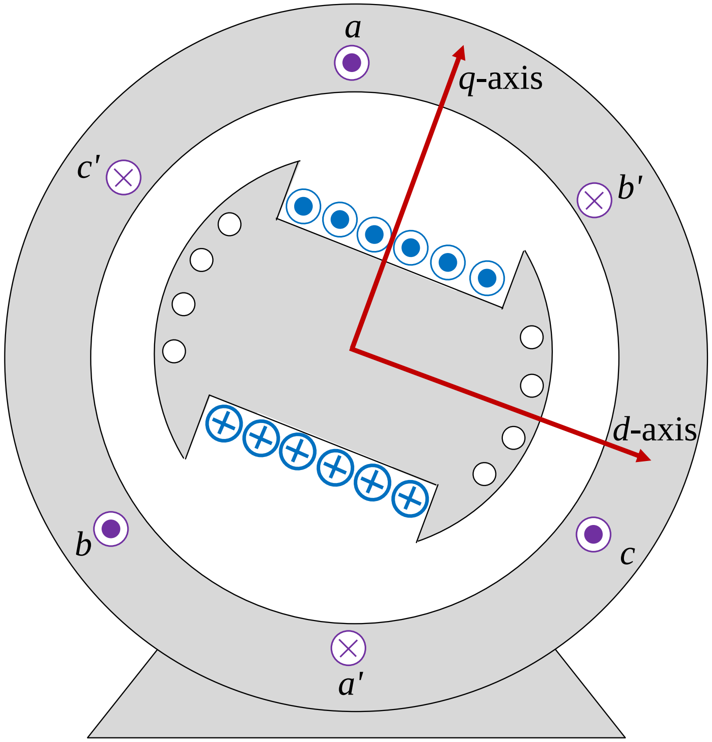
Zero-component equivalent circuit

Stator

$$\boxed{v_{0s} = r_s i_{0s} + d\lambda_{0s}/dt}$$

$$\boxed{\lambda_{0s} = L_{ls} i_{0s}}$$





Wound Rotor Synchronous Machines

Torque Equation

- The instantaneous input power is obtained as

$$\boxed{P_{in} = v_{as}\, i_{as} + v_{bs}\, i_{bs} + v_{cs}\, i_{cs} + v_{f}\, i_{f} + v_{g}\, i_{g}}$$

- In terms of qd0 quantities, the instantaneous input power is

$$\boxed{P_{in} = \tfrac{3}{2}\left(v_{qs}\, i_{qs} + v_{ds}\, i_{ds}\right) + 3v_{0s}\, i_{0s} + v_{f}\, i_{f} + v_{g}\, i_{g}}$$

- Substituting the voltage equations in the above expression yields

$$\boxed{\begin{aligned} P_{in} = \tfrac{3}{2}&\left(r_s\, i_{qs}^2 + i_{qs}\, p\lambda_{qs} + \omega_r \lambda_{ds} i_{qs} + r_s\, i_{ds}^2 + i_{ds}\, p\lambda_{ds} - \omega_r \lambda_{qs} i_{ds}\right) \\ &+ 3r_s\, i_{0s}^2 + 3i_{0s}\, p\lambda_{0s} + r_f\, i_f^2 + i_f\, p\lambda_f + r_g\, i_g^2 + i_g\, p\lambda_g \end{aligned}}$$

Wound Rotor Synchronous Machines

Torque Equation

$$\boxed{\begin{aligned}P_{in} = \tfrac{3}{2}\left(r_s\, i_{qs}^2 + i_{qs}\, p\lambda_{qs} + \omega_r \lambda_{ds} i_{qs} + r_s\, i_{ds}^2 + i_{ds}\, p\lambda_{ds} - \omega_r \lambda_{qs} i_{ds}\right)\\ + 3r_s\, i_{0s}^2 + 3i_{0s}\, p\lambda_{0s} + r_f\, i_f^2 + i_f\, p\lambda_f + r_g\, i_g^2 + i_g\, p\lambda_g\end{aligned}}$$

- The **ohmic losses** (ri^2 terms) and the **rate of exchange of magnetic field energy** between windings ($ip\lambda$ terms) **do not contribute** to electromagnetic (EM) torque development.

- Hence the **power terms contributed** to EM torque generation are:

$$\boxed{P_T = \tfrac{3}{2}\omega_r\left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}\right)}$$

Wound Rotor Synchronous Machines

Torque Equation

$$\boxed{P_T = \tfrac{3}{2}\omega_r\left(\lambda_{ds}i_{qs} - \lambda_{qs}i_{ds}\right)}$$

- The electromagnetic torque is obtained by dividing the above power by the **mechanical speed**.

$$\boxed{T_{em} = \tfrac{3}{2}\tfrac{P}{2}\left(\lambda_{ds}i_{qs} - \lambda_{qs}i_{ds}\right)}$$

where P is the number of poles.

- The relation between the mechanical speed and the electrical speed is as follows:

$$\boxed{\omega_{rm} = \tfrac{2}{P}\omega_r}$$

Wound Rotor Synchronous Machines

Mechanical Dynamic Equation

- Based on Newton's 2nd law for rotational movement, $\sum T = J\dfrac{d\omega_{rm}}{dt}$ the **mechanical dynamic equation** is obtained:

Motoring Mode

$$\boxed{T_{em} - T_{mech} - T_{damp} = J\frac{d\omega_{rm}}{dt}}$$

Generating Mode

$$\boxed{T_{em} + T_{mech} - T_{damp} = J\frac{d\omega_{rm}}{dt}}$$

where T_{mech} is the externally-applied mechanical torque, T_{damp} is the damping torque, J is the moment of inertia and ω_{rm} is the mechanical rotational velocity.

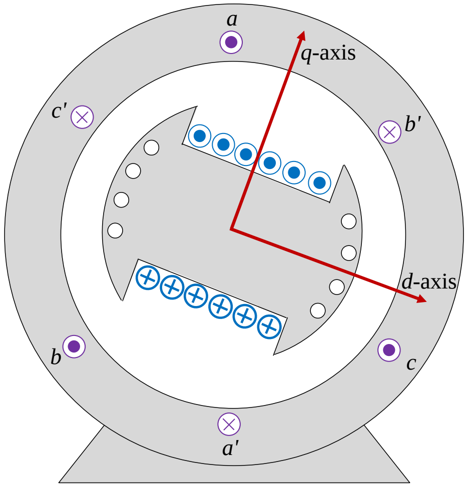
Wound Rotor Synchronous Machines

Currents in terms of flux linkages

- Synchronous machines can be simulated using the **flux linkages as the state variables**.
- Hence it is required to obtain the **currents in terms of flux linkages**.
- Let’s define the following **mutual flux linkages** in the *q*- and *d*-axis:

$$\boxed{\lambda_{mq} = L_{mq}(i_{qs} + i'_g + i'_{kq})}$$

$$\boxed{\lambda_{md} = L_{md}(i_{ds} + i'_f + i'_{kd})}$$



Wound Rotor Synchronous Machines

Currents in terms of flux linkages

- The following expressions for ***q*-axis currents** can be obtained:

$$\left.\begin{array}{r}\boxed{\lambda_{qs} = L_{ls} i_{qs} + L_{mq}(i_{qs} + i'_g + i'_{kq})} \\ \boxed{\lambda_{mq} = L_{mq}(i_{qs} + i'_g + i'_{kq})}\end{array}\right\} \Rightarrow \boxed{i_{qs} = \frac{1}{L_{ls}}(\lambda_{qs} - \lambda_{mq})}$$

$$\left.\begin{array}{r}\boxed{\lambda'_g = L'_{lg} i'_g + L_{mq}(i_{qs} + i'_g + i'_{kq})} \\ \boxed{\lambda_{mq} = L_{mq}(i_{qs} + i'_g + i'_{kq})}\end{array}\right\} \Rightarrow \boxed{i'_g = \frac{1}{L'_{lg}}(\lambda'_g - \lambda_{mq})}$$

$$\left.\begin{array}{r}\boxed{\lambda'_{kq} = L'_{lkq} i'_{kq} + L_{mq}(i_{qs} + i'_g + i'_{kq})} \\ \boxed{\lambda_{mq} = L_{mq}(i_{qs} + i'_g + i'_{kq})}\end{array}\right\} \Rightarrow \boxed{i'_{kq} = \frac{1}{L'_{lkq}}(\lambda'_{kq} - \lambda_{mq})}$$

Wound Rotor Synchronous Machines

Currents in terms of flux linkages

- The following expressions for ***d*-axis currents** can be obtained:

$$\left.\begin{array}{r}\boxed{\lambda_{ds} = L_{ls} i_{ds} + L_{md}(i_{ds} + i'_f + i'_{kd})} \\ \boxed{\lambda_{md} = L_{md}(i_{ds} + i'_f + i'_{kd})}\end{array}\right\} \Rightarrow \boxed{i_{ds} = \frac{1}{L_{ls}}(\lambda_{ds} - \lambda_{md})}$$

$$\left.\begin{array}{r}\boxed{\lambda'_f = L'_{lf} i'_f + L_{md}(i_{ds} + i'_f + i'_{kd})} \\ \boxed{\lambda_{md} = L_{md}(i_{ds} + i'_f + i'_{kd})}\end{array}\right\} \Rightarrow \boxed{i'_f = \frac{1}{L'_{lf}}(\lambda'_f - \lambda_{md})}$$

$$\left.\begin{array}{r}\boxed{\lambda'_{kd} = L'_{lkd} i'_{kd} + L_{md}(i_{ds} + i'_f + i'_{kd})} \\ \boxed{\lambda_{md} = L_{md}(i_{ds} + i'_f + i'_{kd})}\end{array}\right\} \Rightarrow \boxed{i'_{kd} = \frac{1}{L'_{lkd}}(\lambda'_{kd} - \lambda_{md})}$$

Wound Rotor Synchronous Machines

Currents in terms of flux linkages

- Substituting the expressions of the *q*-axis winding currents in the *q*-axis mutual flux linkages yields:

$$\left.\begin{array}{l}\boxed{\lambda_{mq}=L_{mq}(i_{qs}+i'_{g}+i'_{kq})}\\ \boxed{i_{qs}=\frac{1}{L_{ls}}(\lambda_{qs}-\lambda_{mq})}\\ \boxed{i'_{g}=\frac{1}{L'_{l\,g}}(\lambda'_{g}-\lambda_{mq})}\\ \boxed{i'_{kq}=\frac{1}{L'_{lkq}}(\lambda'_{kq}-\lambda_{mq})}\end{array}\right\}\Rightarrow\boxed{\lambda_{mq}=L_{MQ}\left(\frac{\lambda_{qs}}{L_{ls}}+\frac{\lambda'_{g}}{L'_{l\,g}}+\frac{\lambda'_{kq}}{L'_{lkq}}\right)}$$

where $$\boxed{\frac{1}{L_{MQ}}=\frac{1}{L_{ls}}+\frac{1}{L'_{l\,g}}+\frac{1}{L'_{lkq}}+\frac{1}{L_{mq}}}$$

Wound Rotor Synchronous Machines

Currents in terms of flux linkages

- Substituting the expressions of the *d*-axis winding currents in the *d*-axis mutual flux linkages yields:

$$\boxed{\lambda_{md} = L_{md}(i_{ds} + i'_f + i'_{kd})}$$

$$\boxed{i_{ds} = \frac{1}{L_{ls}}(\lambda_{ds} - \lambda_{md})}$$

$$\boxed{i'_f = \frac{1}{L'_{lf}}(\lambda'_f - \lambda_{md})}$$

$$\boxed{i'_{kd} = \frac{1}{L'_{lkd}}(\lambda'_{kd} - \lambda_{md})}$$

$$\Rightarrow \quad \boxed{\lambda_{md} = L_{MD}\left(\frac{\lambda_{ds}}{L_{ls}} + \frac{\lambda'_f}{L'_{lf}} + \frac{\lambda'_{kd}}{L'_{lkd}}\right)}$$

where

$$\boxed{\frac{1}{L_{MD}} = \frac{1}{L_{ls}} + \frac{1}{L'_{lf}} + \frac{1}{L'_{lkd}} + \frac{1}{L_{md}}}$$

Wound Rotor Synchronous Machines

Currents in terms of flux linkages

$$\boxed{\lambda_{mq} = L_{MQ}\left(\frac{\lambda_{qs}}{L_{ls}} + \frac{\lambda'_g}{L'_{l\,g}} + \frac{\lambda'_{kq}}{L'_{lkq}}\right)}$$

- Substituting the above expressions in the *q*-axis winding currents

$$\boxed{i_{qs} = \tfrac{1}{L_{ls}}(\lambda_{qs} - \lambda_{mq})} \qquad \boxed{i'_g = \tfrac{1}{L'_{l\,g}}(\lambda'_g - \lambda_{mq})} \qquad \boxed{i'_{kq} = \tfrac{1}{L'_{lkq}}(\lambda'_{kq} - \lambda_{mq})}$$

yields

$$\begin{bmatrix} i_{qs} \\ i'_g \\ i'_{kq} \end{bmatrix} = \begin{bmatrix} (1-\frac{L_{MQ}}{L_{ls}})\frac{1}{L_{ls}} & -\frac{L_{MQ}}{L_{ls}\,L'_{l\,g}} & -\frac{L_{MQ}}{L_{ls}\,L'_{lkq}} \\ -\frac{L_{MQ}}{L_{ls}\,L'_{l\,g}} & (1-\frac{L_{MQ}}{L'_{l\,g}})\frac{1}{L'_{l\,g}} & -\frac{L_{MQ}}{L'_{l\,g}\,L'_{lkq}} \\ -\frac{L_{MQ}}{L_{ls}\,L'_{lkq}} & -\frac{L_{MQ}}{L'_{l\,g}\,L'_{lkq}} & (1-\frac{L_{MQ}}{L'_{lkq}})\frac{1}{L'_{lkq}} \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda'_g \\ \lambda'_{kq} \end{bmatrix}$$

Wound Rotor Synchronous Machines

Currents in terms of flux linkages

$$\boxed{\lambda_{md} = L_{MD}\left(\frac{\lambda_{ds}}{L_{ls}} + \frac{\lambda'_f}{L'_{lf}} + \frac{\lambda'_{kd}}{L'_{lkd}}\right)}$$

- Substituting the above expressions in the *d*-axis winding currents

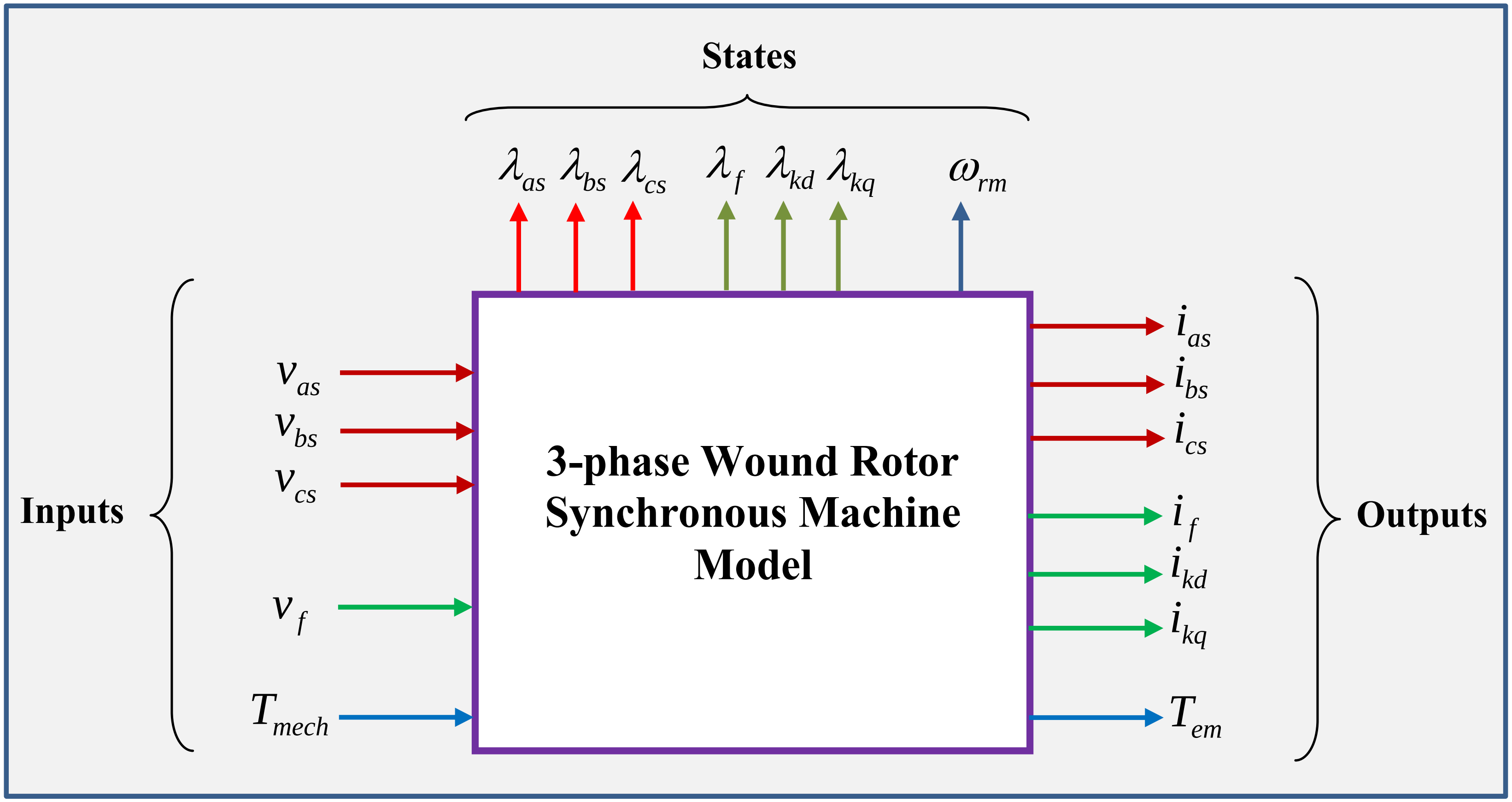
$$\boxed{i_{ds} = \frac{1}{L_{ls}}(\lambda_{ds} - \lambda_{md})} \quad \boxed{i'_f = \frac{1}{L'_{lf}}(\lambda'_f - \lambda_{md})} \quad \boxed{i'_{kd} = \frac{1}{L'_{lkd}}(\lambda'_{kd} - \lambda_{md})}$$

yields

$$\boxed{\begin{bmatrix} i_{ds} \\ i'_f \\ i'_{kd} \end{bmatrix} = \begin{bmatrix} (1-\frac{L_{MD}}{L_{ls}})\frac{1}{L_{ls}} & -\frac{L_{MD}}{L_{ls}\,L'_{lf}} & -\frac{L_{MD}}{L_{ls}\,L'_{lkd}} \\ -\frac{L_{MD}}{L_{ls}\,L'_{lf}} & (1-\frac{L_{MD}}{L'_{lf}})\frac{1}{L'_{lf}} & -\frac{L_{MD}}{L'_{lf}\,L'_{lkd}} \\ -\frac{L_{MD}}{L_{ls}\,L'_{lkd}} & -\frac{L_{MD}}{L'_{lf}\,L'_{lkd}} & (1-\frac{L_{MD}}{L'_{lkd}})\frac{1}{L'_{lkd}} \end{bmatrix} \begin{bmatrix} \lambda_{ds} \\ \lambda'_f \\ \lambda'_{kd} \end{bmatrix}}$$

Simulation of Wound Rotor Synchronous Machines

- The winding equations of the derived model is implemented in a simulation that uses **voltages as inputs** and **currents as outputs**.

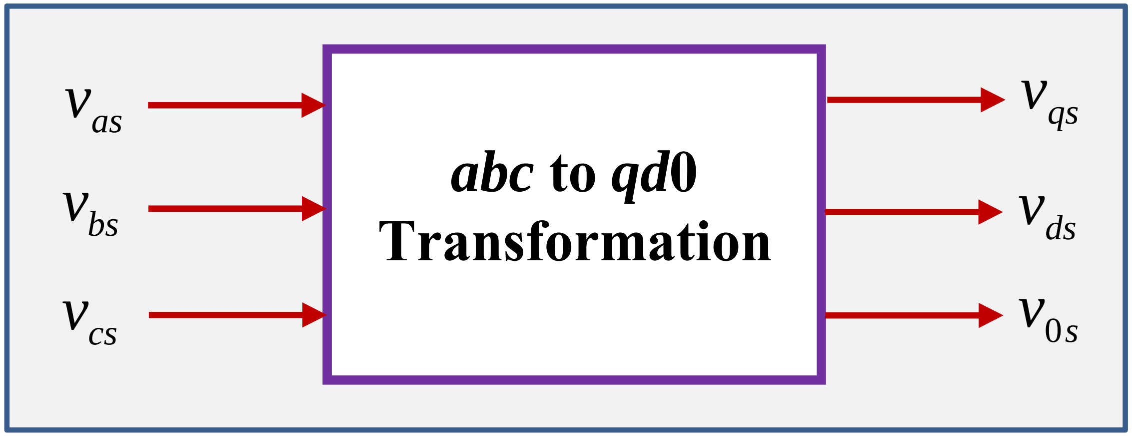


Simulation of Wound Rotor Synchronous Machines

Sub-systems: **Transformation of stator voltages from *abc* system to *qd*0 RF fixed to the rotor**

Stator voltages

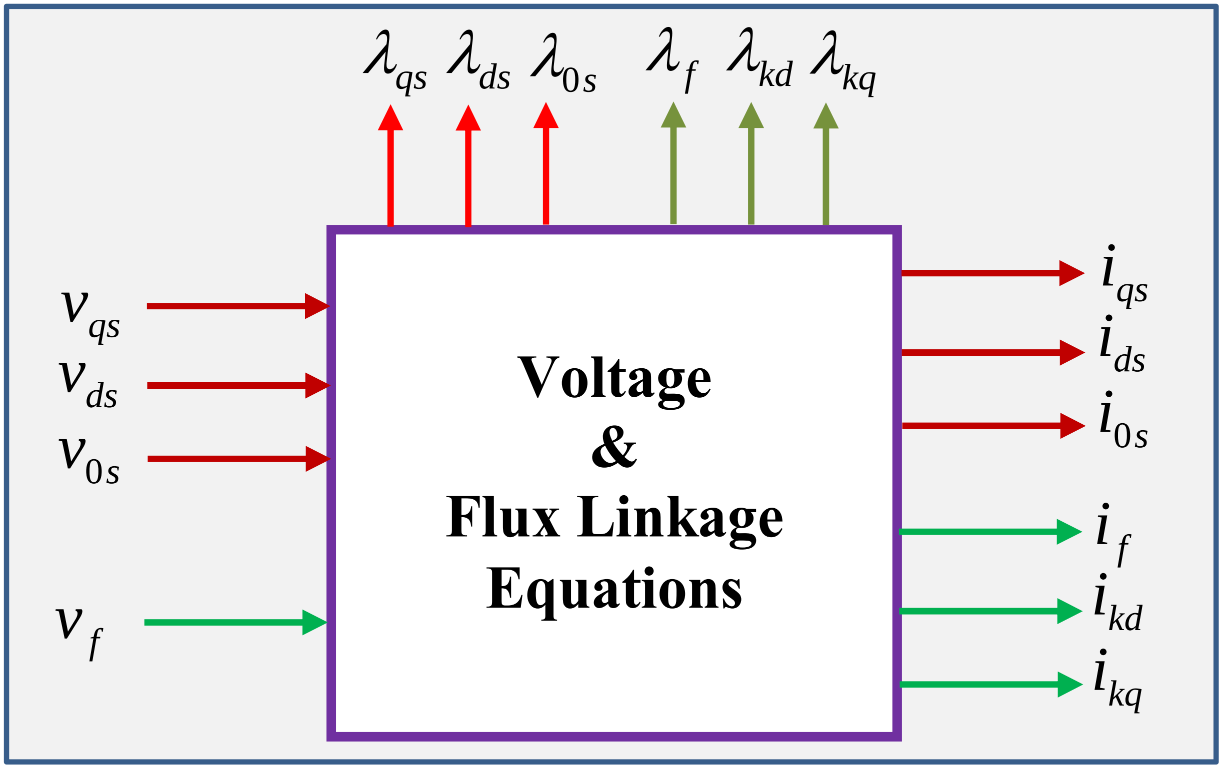
$$[\mathbf{f}_{qd0}]=[\mathbf{T}_{qd0}]\;[\mathbf{f}_{abc}]$$



- It can be done in **two steps** (1: *abc system* to stationary *qd*0 RF; 2: stationary *qd*0 RF to rotor *qd*0 RF) or in **single step**.

Simulation of Wound Rotor Synchronous Machines

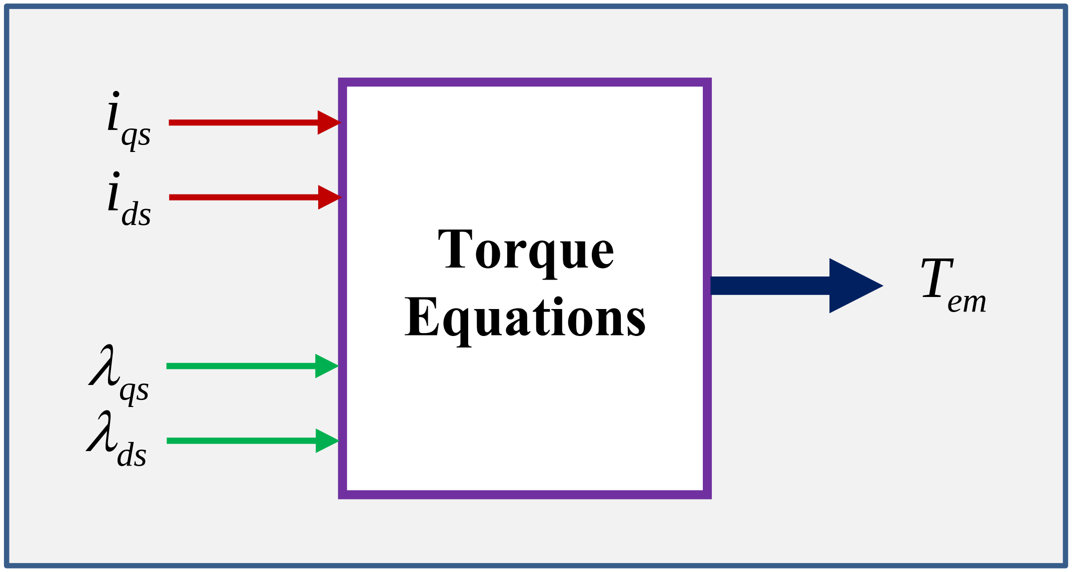
Sub-systems: Voltage & flux linkage equations



The stator and rotor equations are coupled.

Simulation of Wound Rotor Synchronous Machines

Sub-systems: Torque equation



$$\boxed{T_{em} = \frac{3}{2}\frac{P}{2}\left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}\right)}$$

or

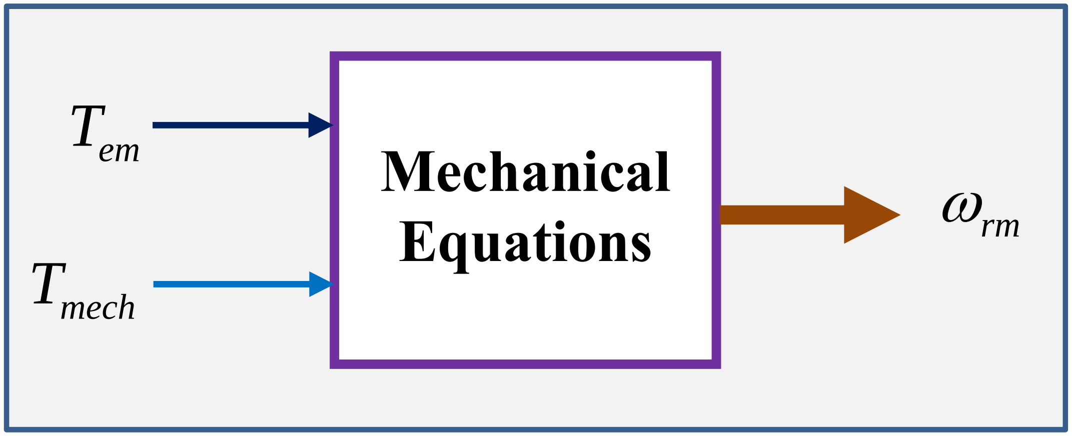
$$\boxed{T_{em} = \frac{3}{2}\frac{P}{2\omega_b}\left(\psi_{ds} i_{qs} - \psi_{qs} i_{ds}\right)}$$

where

$$\boxed{\psi_{ds} = \omega_b \lambda_{ds}} \quad \boxed{\psi_{qs} = \omega_b \lambda_{qs}}$$

Simulation of Wound Rotor Synchronous Machines

Sub-systems: Mechanical equation



Motoring Mode

$$\boxed{T_{em} - T_{mech} - T_{damp} = J\frac{d\omega_{rm}}{dt}}$$

Generating Mode

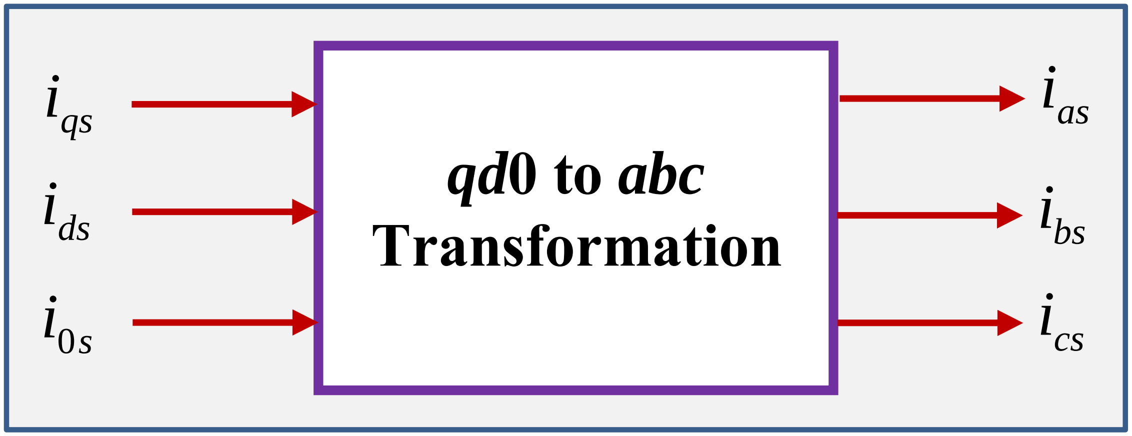
$$\boxed{T_{em} + T_{mech} - T_{damp} = J\frac{d\omega_{rm}}{dt}}$$

Simulation of Wound Rotor Synchronous Machines

Sub-systems: Transformation of stator current from rotor *qd*0 RF to *abc* system

Stator Currents

$$\boxed{\left[\mathbf{f}_{abc}\right]=\left[\mathbf{T}_{qd0}\right]^{-1}\left[\mathbf{f}_{qd0}\right]}$$



- It can be done in **two steps** (1: rotor *qd*0 RF to stationary *qd*0 RF; 2: stationary *qd*0 RF to abc system) or in **single step**.

Simulation of Wound Rotor Synchronous Machines

Transformation (Stator voltage)

- Transformation of the **stator voltages** from stator *abc* system to the **rotor** *qd*0 RF.

$$\left[\mathbf{T}_{qd0}\right]=\frac{2}{3}\begin{bmatrix}\cos\theta_r & \cos\left(\theta_r-\frac{2\pi}{3}\right) & \cos\left(\theta_r+\frac{2\pi}{3}\right)\\ \sin\theta_r & \sin\left(\theta_r-\frac{2\pi}{3}\right) & \sin\left(\theta_r+\frac{2\pi}{3}\right)\\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\end{bmatrix}$$

$$\left[\mathbf{v}_{qd0}\right]=\left[\mathbf{T}_{qd0}\right]\left[\mathbf{v}_{abc}\right]$$

$$\begin{cases} v_{qs}=\frac{2}{3}\left\{v_{as}\cos\theta_r+v_{bs}\cos(\theta_r-\frac{2\pi}{3})+v_{cs}\cos(\theta_r-\frac{4\pi}{3})\right\}\\ v_{ds}=\frac{2}{3}\left\{v_{as}\sin\theta_r+v_{bs}\sin(\theta_r-\frac{2\pi}{3})+v_{cs}\sin(\theta_r-\frac{4\pi}{3})\right\}\\ v_{0s}=\frac{1}{3}(v_{as}+v_{bs}+v_{cs})\end{cases}$$

Simulation of Wound Rotor Synchronous Machines

Flux linkage and voltage equations

- The **reactance** is used instead of the **inductance**

$$\boxed{x_y = \omega_b L_y} \Rightarrow \boxed{\psi_y = \omega_b \lambda_y}$$

1. *d*-axis stator equations

$$\boxed{v_{ds} = r_s i_{ds} + \tfrac{1}{\omega_b} d\psi_{ds}/dt - \tfrac{\omega_r}{\omega_b}\psi_{qs}}$$

$$\boxed{\psi_{ds} = x_{ls} i_{ds} + \psi_{md}} \Rightarrow \boxed{i_{ds} = \tfrac{1}{x_{ls}}(\psi_{ds} - \psi_{md})}$$

1

$$\Rightarrow \boxed{\psi_{ds} = \omega_b \int \left\{ v_{ds} + \tfrac{\omega_r}{\omega_b}\psi_{qs} + \tfrac{r_s}{x_{ls}}(\psi_{md} - \psi_{ds}) \right\} dt}$$

2

where $\boxed{\psi_{md} = x_{md}(i_{ds} + i'_f + i'_{kd})}$

Simulation of Wound Rotor Synchronous Machines

Flux linkage and voltage equations

2. q-axis stator equations

$$v_{qs} = r_s i_{qs} + \tfrac{1}{\omega_b} d\psi_{qs} / dt + \tfrac{\omega_r}{\omega_b} \psi_{ds}$$

$$\psi_{qs} = x_{ls} i_{qs} + \psi_{mq} \Rightarrow i_{qs} = \tfrac{1}{x_{ls}} (\psi_{qs} - \psi_{mq})$$

3

$$\Rightarrow \psi_{qs} = \omega_b \int \left\{ v_{qs} - \tfrac{\omega_r}{\omega_b} \psi_{ds} + \tfrac{r_s}{x_{ls}} (\psi_{mq} - \psi_{qs}) \right\} dt$$

4

where $\psi_{mq} = x_{mq}(i_{qs} + i'_{kq})$

3. Zero-component stator equations

$$v_{0s} = r_s i_{0s} + \tfrac{1}{\omega_b} d\psi_{0s} / dt$$

$$\psi_{0s} = x_{ls} i_{0s}$$

$$\Rightarrow \psi_{0s} = \omega_b \int \left\{ v_{0s} - \tfrac{r_s}{x_{ls}} \psi_{0s} \right\} dt$$

5

Simulation of Wound Rotor Synchronous Machines

Flux linkage and voltage equations

4. *d*-axis rotor field winding equations

$$\boxed{v'_f = r'_f i'_f + \tfrac{1}{\omega_b} d\psi'_f / dt}$$

$$\boxed{\psi'_f = x'_{lf} i'_f + \psi_{md}} \Rightarrow \boxed{i'_f = \tfrac{1}{x'_{lf}}(\psi'_f - \psi_{md})} \quad 6$$

$$\Rightarrow \boxed{\psi'_f = \omega_b \int \left\{ v'_f + \tfrac{r'_f}{x'_{lf}}(\psi_{md} - \psi'_f) \right\} dt} \quad 7$$

5. *d*-axis rotor damper equations

$$\boxed{0 = r'_{kd} i'_{kd} + \tfrac{1}{\omega_b} d\psi'_{kd} / dt}$$

$$\boxed{\psi'_{kd} = x'_{lkd} i'_{kd} + \psi_{md}} \Rightarrow \boxed{i'_{kd} = \tfrac{1}{x'_{lkd}}(\psi'_{kd} - \psi_{md})} \quad 8$$

$$\Rightarrow \boxed{\psi'_{kd} = \frac{\omega_b r'_{kd}}{x'_{lkd}} \int \left\{ (\psi_{md} - \psi'_{kd}) \right\} dt} \quad 9$$

Simulation of Wound Rotor Synchronous Machines

Flux linkage and voltage equations

6. ***q*-axis rotor damper equations**

$$\boxed{0 = r'_{kq} i'_{kq} + \tfrac{1}{\omega_b} d\psi'_{kq} / dt}$$

$$\boxed{\psi'_{kq} = x'_{lkq} i'_{kq} + \psi_{mq}} \Rightarrow \boxed{i'_{kq} = \tfrac{1}{x'_{lkq}} (\psi'_{kq} - \psi_{mq})} \quad \mathbf{10}$$

$$\Rightarrow \boxed{\psi'_{kq} = \frac{\omega_b r'_{kq}}{x'_{lkq}} \int \left\{ (\psi_{mq} - \psi'_{kq}) \right\} dt} \quad \mathbf{11}$$

Simulation of Wound Rotor Synchronous Machines

Flux linkage and voltage equations

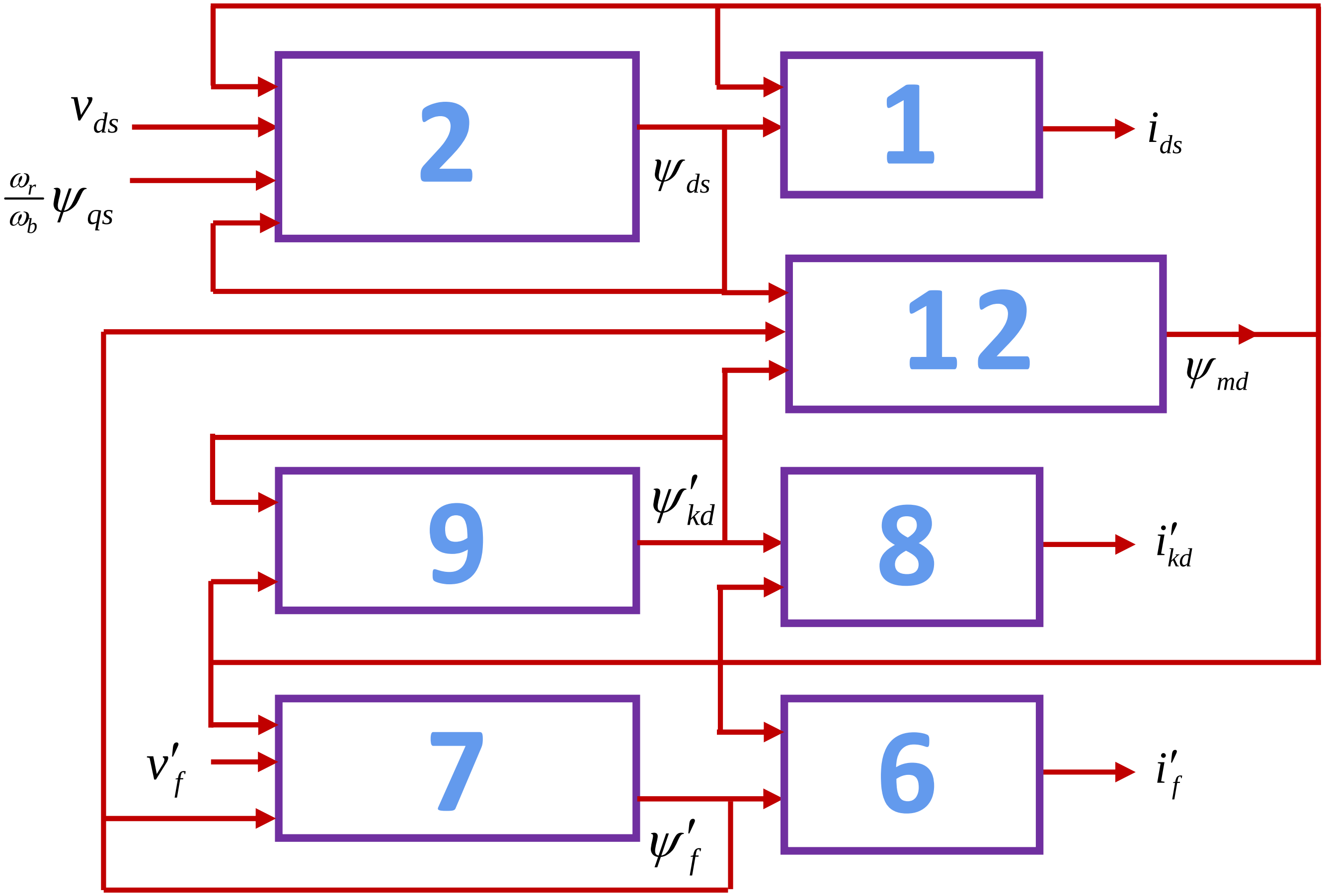
- As before, to handle the **cut-set of inductors**, the following expressions are used:

$$\boxed{\psi_{md} = x_{MD}\left(\frac{\psi_{ds}}{x_{ls}} + \frac{\psi'_{f}}{x'_{lf}} + \frac{\psi'_{kd}}{x'_{lkd}}\right)} \quad 12 \qquad \boxed{\frac{1}{x_{MD}} = \frac{1}{x_{ls}} + \frac{1}{x'_{lf}} + \frac{1}{x'_{lkd}} + \frac{1}{x_{md}}}$$

$$\boxed{\psi_{mq} = x_{MQ}\left(\frac{\psi_{qs}}{x_{ls}} + \frac{\psi'_{kq}}{x'_{lkq}}\right)} \quad 13 \qquad \boxed{\frac{1}{x_{MQ}} = \frac{1}{x_{ls}} + \frac{1}{x'_{lkq}} + \frac{1}{x_{mq}}}$$

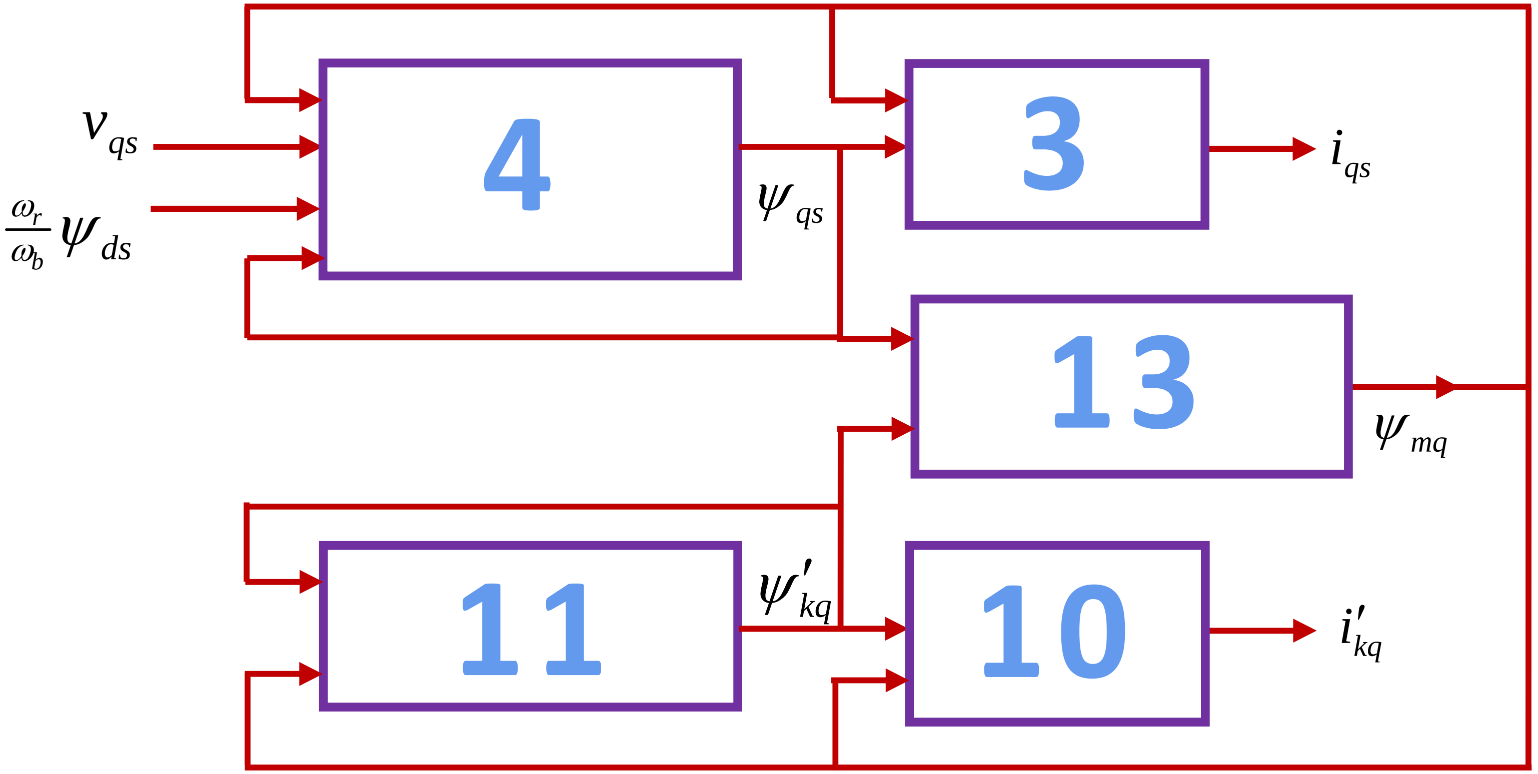
Simulation of Wound Rotor Synchronous Machines

Flux linkage and voltage equations: *d*-axis block diagram



Simulation of Wound Rotor Synchronous Machines

Flux linkage and voltage equations: *q*-axis block diagram



Simulation of Wound Rotor Synchronous Machines

2-Step Transformation of the Stator Currents

- Transformation from the **rotor** $qd0$ RF to the **stator** $qd0$ RF:

$$\begin{cases} \boxed{i_{qs}^{s} = i_{qs}\cos\theta_r + i_{ds}\sin\theta_r} \\ \boxed{i_{ds}^{s} = -i_{qs}\sin\theta_r + i_{ds}\cos\theta_r} \end{cases}$$

- Transformation from the **stator** $qd0$ RF to ***abc* system**:

$$\begin{cases} \boxed{i_{as} = i_{qs}^{s} + i_{0s}} \\ \boxed{i_{bs} = -\tfrac{1}{2} i_{qs}^{s} - \tfrac{1}{\sqrt{3}} i_{ds}^{s} + i_{0s}} \\ \boxed{i_{cs} = -\tfrac{1}{2} i_{qs}^{s} + \tfrac{1}{\sqrt{3}} i_{ds}^{s} + i_{0s}} \end{cases}$$

Per-Unit System

Base quantities

- The base quantities with **peak** rather than rms value of a P-pole, three-phase synchronous machine with **rated line-to-line rms voltage**, V_{rated}, and **rated volt-ampere**, S_{rated}, are as follows:

Base Voltage $\boxed{V_b = \sqrt{\frac{2}{3}} V_{rated}}$

Base Volt-ampere $\boxed{S_b = S_{rated}}$

Base Peak Current $\boxed{I_b = 2S_b/(3V_b)}$

Base Impedance $\boxed{Z_b = V_b / I_b}$

Base Torque $\boxed{T_b = S_b / \omega_{bm}}$ where $\boxed{\omega_{bm} = 2\omega_b / P}$

Per-Unit System

Torque Equation

$$\boxed{T_{em} = \frac{3}{2}\frac{P}{2\omega_b}\left(\psi_{ds} i_{qs} - \psi_{qs} i_{ds}\right)}$$

$$\boxed{T_b = \frac{S_b}{\omega_{bm}}} \xrightarrow{\omega_{bm} = 2\omega_b / P} \boxed{T_b = \frac{S_b}{\frac{2}{P}\omega_b}} \xrightarrow{I_b = 2S_b /(3V_b)} \boxed{T_b = \frac{3}{2}\frac{P}{2}\frac{V_b I_b}{\omega_b}}$$

$$\boxed{\frac{T_{em}}{T_b} = \frac{\frac{3}{2}\frac{P}{2\omega_b}\left(\psi_{ds} i_{qs} - \psi_{qs} i_{ds}\right)}{\frac{3}{2}\frac{P}{2\omega_b}V_b I_b}} \Rightarrow \boxed{T_{em}(pu) = \psi_{ds}(pu) i_{qs}(pu) - \psi_{qs}(pu) i_{ds}(pu)}$$

where

$$\boxed{\psi_{ds}(pu) = \frac{\psi_{ds}}{V_b}} \quad \boxed{\psi_{qs}(pu) = \frac{\psi_{qs}}{V_b}} \quad \boxed{i_{qs}(pu) = \frac{i_{qs}}{I_b}} \quad \boxed{i_{ds}(pu) = \frac{i_{ds}}{I_b}}$$

Per-Unit System

Mechanical Equation

$$\left.\begin{array}{l} \boxed{J\frac{d\omega_{rm}}{dt}=T_{em}+T_{mech}-T_{damp}} \\[2ex] \boxed{\omega_{rm}=2\omega_r/P} \end{array}\right\} \quad \boxed{\frac{2J\omega_b}{P}\frac{d(\omega_r/\omega_b)}{dt}=T_{em}+T_{mech}-T_{damp}} \quad *$$

$$\left.\begin{array}{l} * \\[2ex] \boxed{T_b=S_b/\omega_{bm}} \\[2ex] \boxed{\omega_{bm}=2\omega_b/P} \end{array}\right\} \quad \boxed{2H\frac{d(\omega_r/\omega_b)}{dt}=T_{em}(pu)+T_{mech}(pu)-T_{damp}(pu)}$$

where $H=J\omega_{bm}^2/(2S_b)$ is the inertia constant.

Machine Parameters

The following machine parameters are required for the simulation

- x_{ls} stator or armature winding **leakage** reactance
- x'_{lf} *d*-axis field winding **leakage** reactance referred to stator
- x'_{lkd} *d*-axis damper winding **leakage** reactance referred to stator
- x'_{lkq} *q*-axis damper winding **leakage** reactance referred to stator
- x_{md} *d*-axis **magnetizing** reactance
- x_{mq} *q*-axis **magnetizing** reactance
- r_s stator or armature winding **resistance**
- r'_f *d*-axis field winding **resistance** referred to stator
- r'_{kd} *d*-axis damper winding **resistance** referred to stator
- r'_{kq} *q*-axis damper winding **resistance** referred to stator

Machine Parameters

Other parameters/quantities required for simulation are:

- J rotor **moment of inertia**
- P **number of poles**
- ω_b rotor **base speed**
- v_{rated} rated line-to-line rms **voltage of stator**
- S_{rared} rated **volt-ampere**
- T_{mech} mechanical **load torque** profile

Machine Parameters

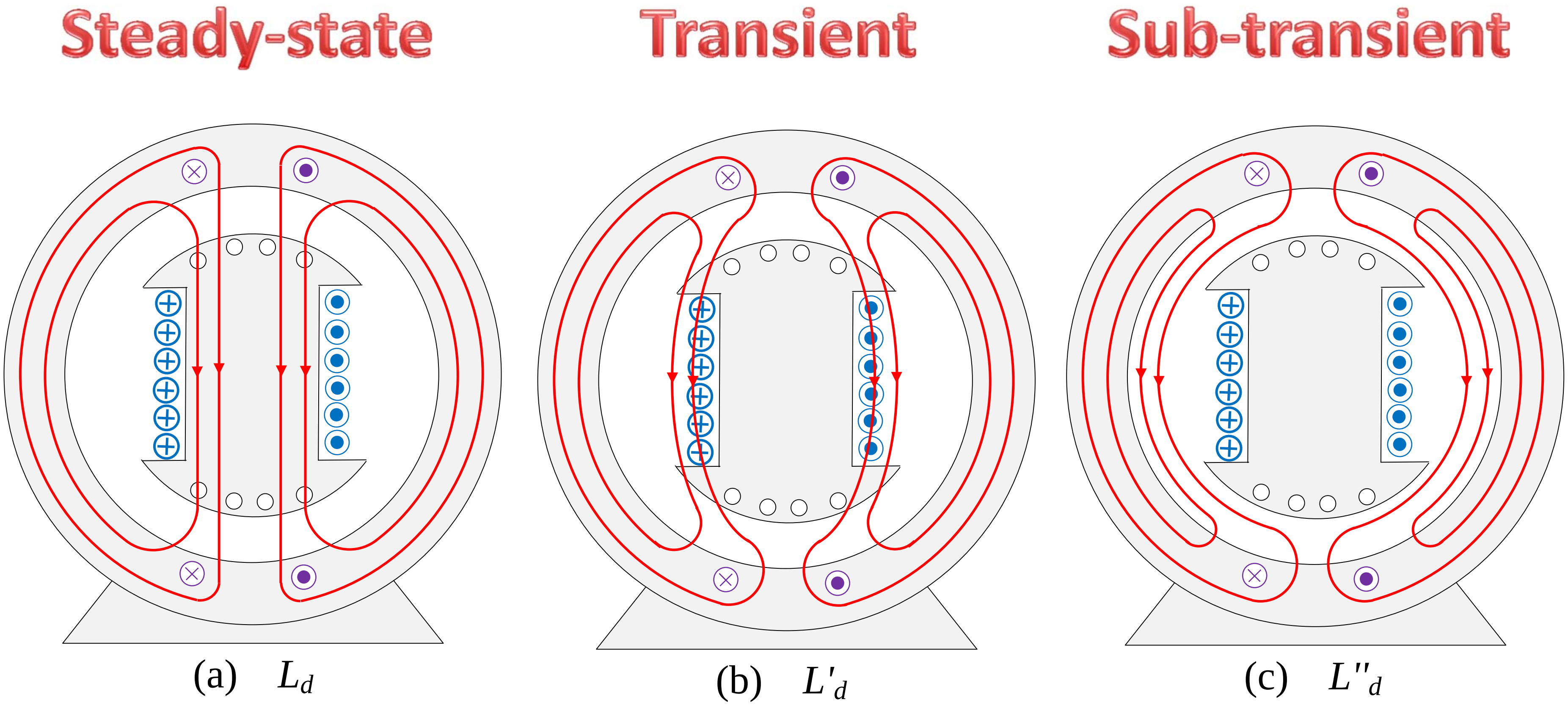
- The machine data from manufacturers are usually in the form of
  - **reactances**,
  - **time constants**, and
  - **resistances**.
- Most of them are derived from **measurements** taken from the **stator** windings.
- Rotor winding parameters can be obtained from stator measurements, because the effective **time constants** of various rotor currents are significantly **different**.
- A common approach is the **short-circuit oscillogram** of the stator currents, when a 3-phase short-circuit is applied to the machine whose stator is initially open-circuit and its field held constant.

Machine Parameters

- The **short-circuit current** consists of:
  - **DC offset**
  - **Sub-transient** period
  - **Transient** period
  - **Steady-state** period
- During the **sub-transient** period the current decay is very **rapid,** due to the changes in the currents of **damper** windings.
- The current decay is **slower** during the **transient** period and is attributed to changes in the currents of the **field** windings.

Machine Parameters

The flux path of *d*-axis during different periods

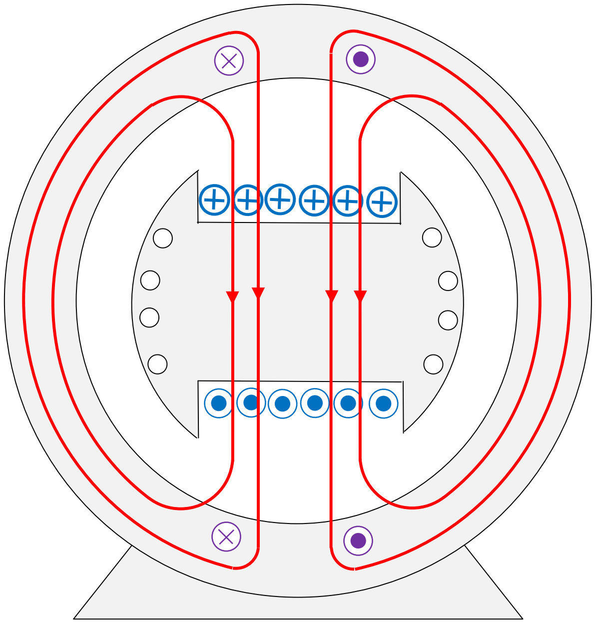


(a) L_d (b) L'_d (c) L''_d

Machine Parameters

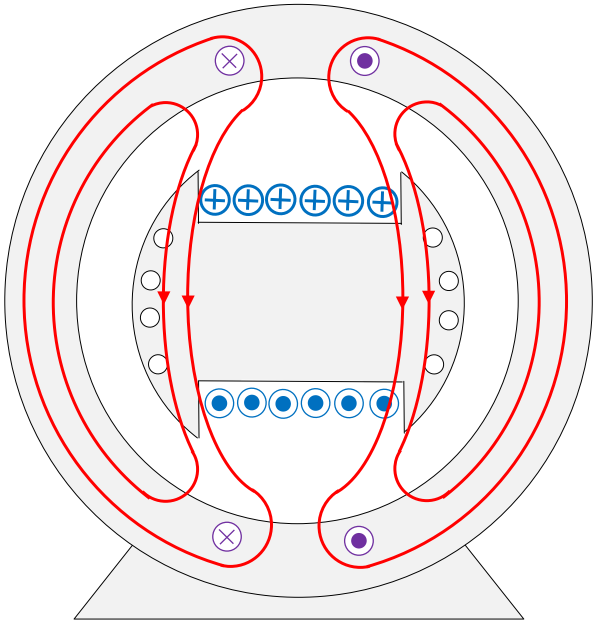
The flux path of *q*-axis during different periods

Steady-state



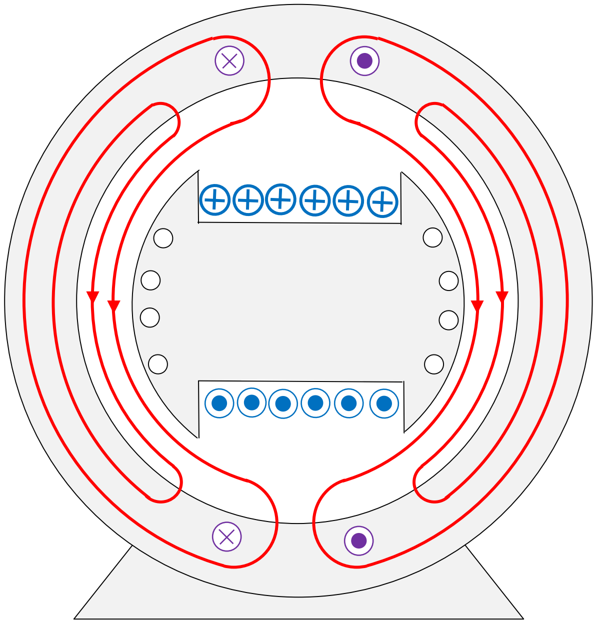
(a) L_q

Transient



(b) L'_q

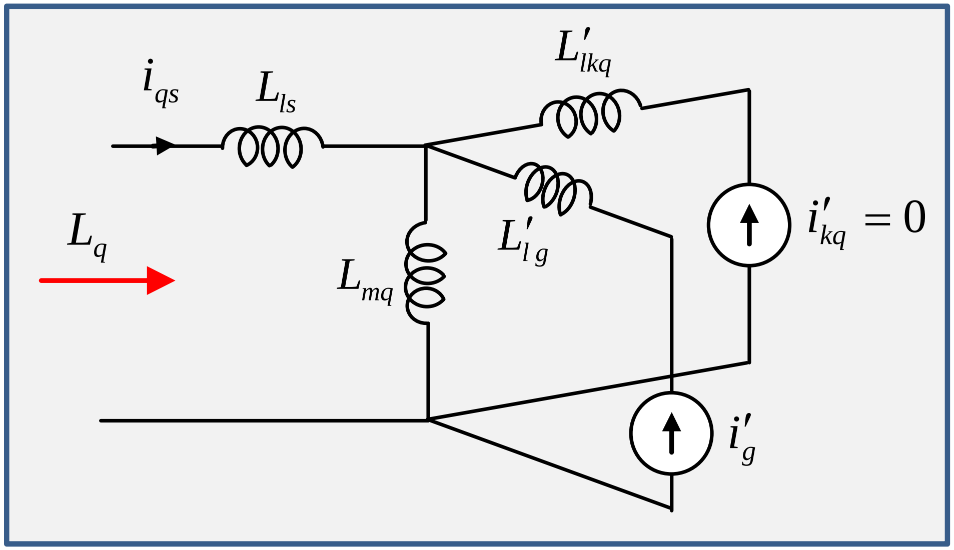
Sub-transient



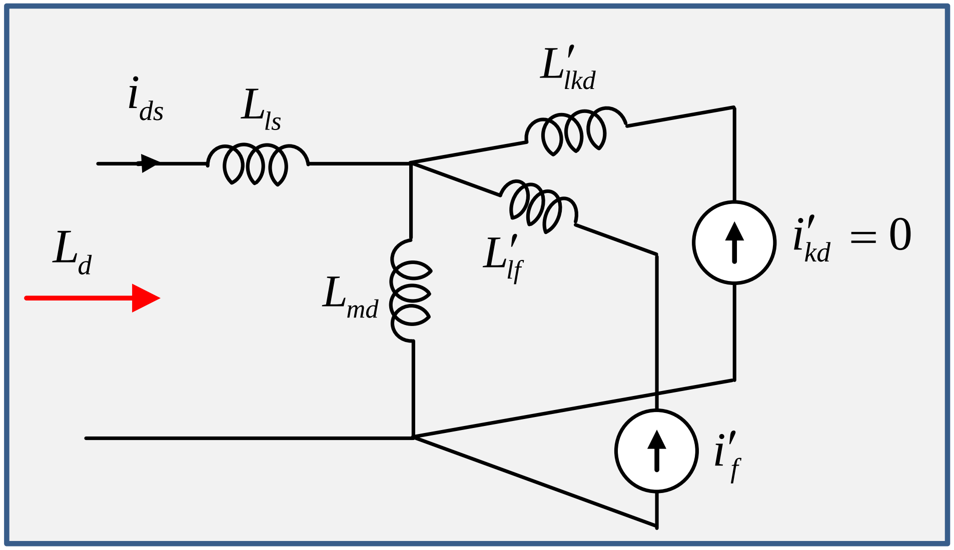
(c) L''_q

Machine Parameters

Synchronous Inductances



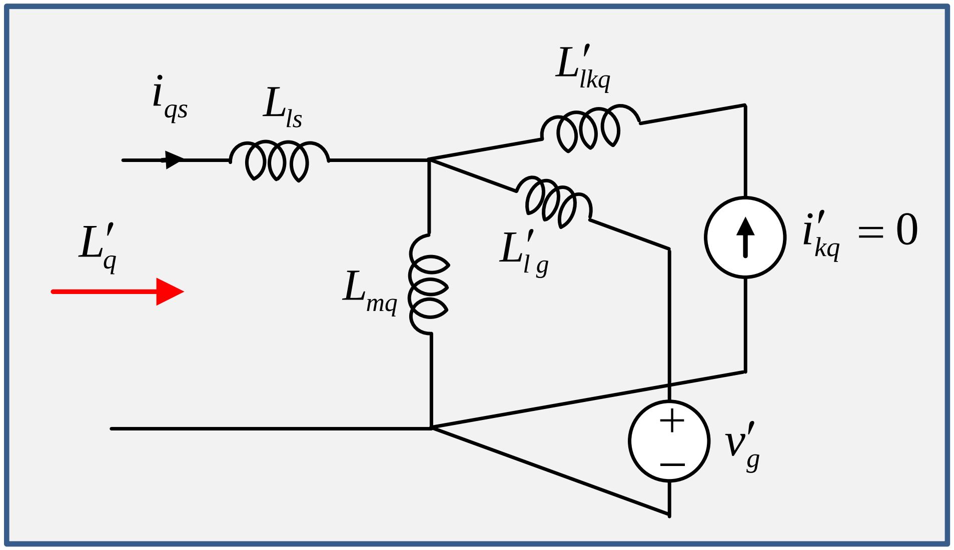
$$\boxed{L_q = L_{ls} + L_{mq}}$$



$$\boxed{L_d = L_{ls} + L_{md}}$$

Machine Parameters

Transient Inductances

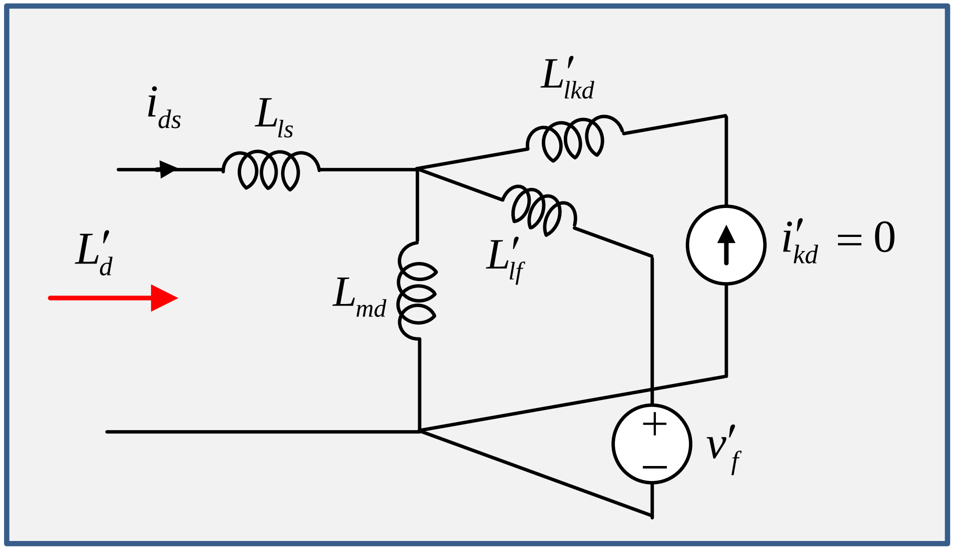


$$L'_q = L_{ls} + L_{mq} \,||\, L'_{l\,g}$$

$$L'_q = L_{ls} + \frac{L_{mq} L'_{l\,g}}{L_{mq} + L'_{l\,g}}$$

$$L'_d = L_{ls} + L_{md} \,||\, L'_{lf}$$

$$L'_d = L_{ls} + \frac{L_{md} L'_{lf}}{L_{md} + L'_{lf}}$$



Machine Parameters

Transient Inductances

- During the transient period $\Delta\lambda_f = 0$ and $\Delta i'_{kd} = 0$

$$\left.\begin{array}{l} \boxed{\Delta\lambda_f = L_{md}\Delta i_d + L'_{ff}\Delta i'_f = 0} \\ \\ \boxed{\Delta\lambda_d = L_d\Delta i_d + L_{md}\Delta i'_f} \end{array}\right\} \Rightarrow \boxed{\Delta\lambda_d = \left(L_d - \frac{L_{md}^2}{L'_{ff}}\right)\Delta i_d}$$

$$\boxed{L'_d = \frac{\Delta\lambda_d}{\Delta i_d} = L_d - \frac{L_{md}^2}{L'_{ff}}}$$

which is the same as

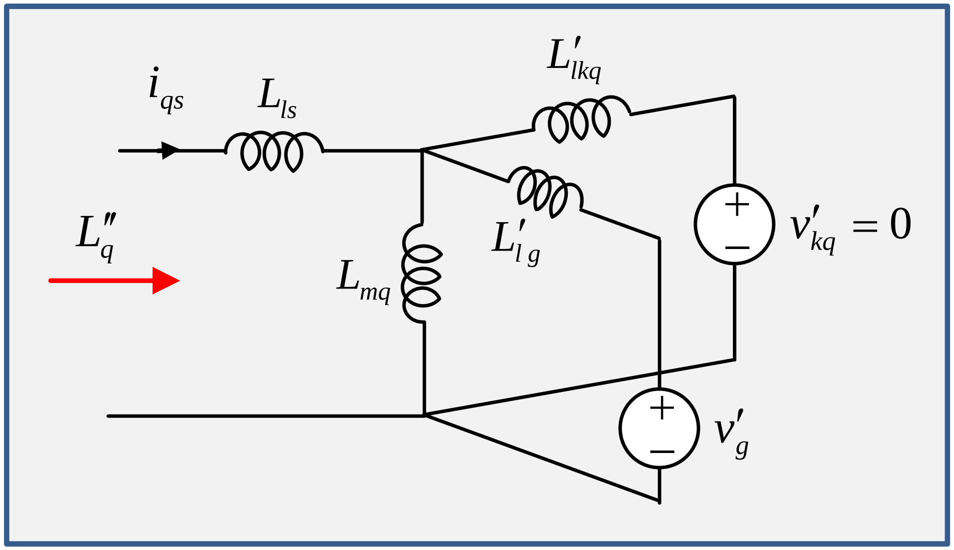
$$\boxed{L'_d = L_{ls} + \frac{L_{md}L'_{lf}}{L_{md} + L'_{lf}}}$$

- Similarly for the q-axis

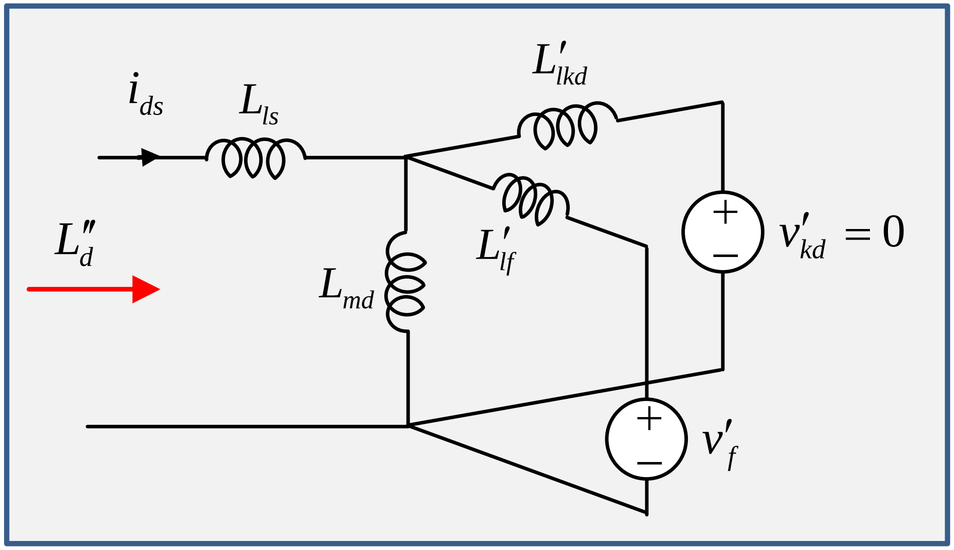
$$\boxed{L'_q = \frac{\Delta\lambda_q}{\Delta i_q} = L_q - \frac{L_{mq}^2}{L'_{gg}}}$$

Machine Parameters

Sub-transient Inductances



$$L''_q = L_{ls} + L_{mq} \,||\, L'_{l\,g} \,||\, L'_{lkq}$$



$$L''_d = L_{ls} + L_{md} \,||\, L'_{lf} \,||\, L'_{lkd}$$

Machine Parameters

Sub-transient Inductances

- During the sub-transient period in d-axis $\Delta\lambda'_f = 0$ and $\Delta\lambda'_{kd} = 0$

$$\boxed{\Delta\lambda'_f = L_{md}\Delta i_d + L'_{ff}\Delta i'_f + L_{md}\Delta i'_{kd} = 0}$$

$$\boxed{\Delta\lambda'_{kd} = L_{md}\Delta i_d + L_{md}\Delta i'_f + L_{kdkd}\Delta i'_{kd} = 0}$$

Writing $\Delta i'_f$ and $\Delta i'_{kd}$ in terms of Δi_d and substituting in

$$\boxed{\Delta\lambda_d = L_d\Delta i_d + L_{md}\Delta i'_f + L_{md}\Delta i'_{kd}}$$

yields

$$\boxed{L''_d = \frac{\Delta\lambda_d}{\Delta i_d} = L_d - \frac{L^2_{md}(L'_{lkd} + L'_{lf})}{L'_{ff}L'_{kdkd} - L^2_{md}}}$$

which is the same as

$$\boxed{L''_d = L_{ls} + \frac{L_{md}L'_{lkd}L'_{lf}}{L'_{lkd}L'_{lf} + L_{md}(L'_{lkd} + L'_{lf})}}$$

Machine Parameters

Sub-transient Inductances

- During the sub-transient period in q-axis $\Delta\lambda'_g = 0$ and $\Delta\lambda'_{kq} = 0$

$$\boxed{\Delta\lambda'_g = L_{mq}\Delta i_q + L'_{gg}\Delta i'_g + L_{mq}\Delta i'_{kq} = 0}$$

$$\boxed{\Delta\lambda'_{kq} = L_{mq}\Delta i_q + L_{mq}\Delta i'_g + L_{kqkq}\Delta i'_{kq} = 0}$$

Writing $\Delta i'_g$ and $\Delta i'_{kq}$ in terms of Δi_q and substituting in

$$\boxed{\Delta\lambda_q = L_q\Delta i_q + L_{mq}\Delta i'_g + L_{mq}\Delta i'_{kq}}$$

yields

$$\boxed{L''_q = \frac{\Delta\lambda_q}{\Delta i_q} = L_q - \frac{L_{mq}^2(L'_{lkq} + L'_{l\,g})}{L'_{gg}L'_{kqkq} - L_{mq}^2}}$$

which is the same as

$$\boxed{L''_q = L_{ls} + \frac{L_{mq}L'_{lkq}L'_{l\,g}}{L'_{lkq}L'_{l\,g} + L_{mq}(L'_{lkq} + L'_{l\,g})}}$$

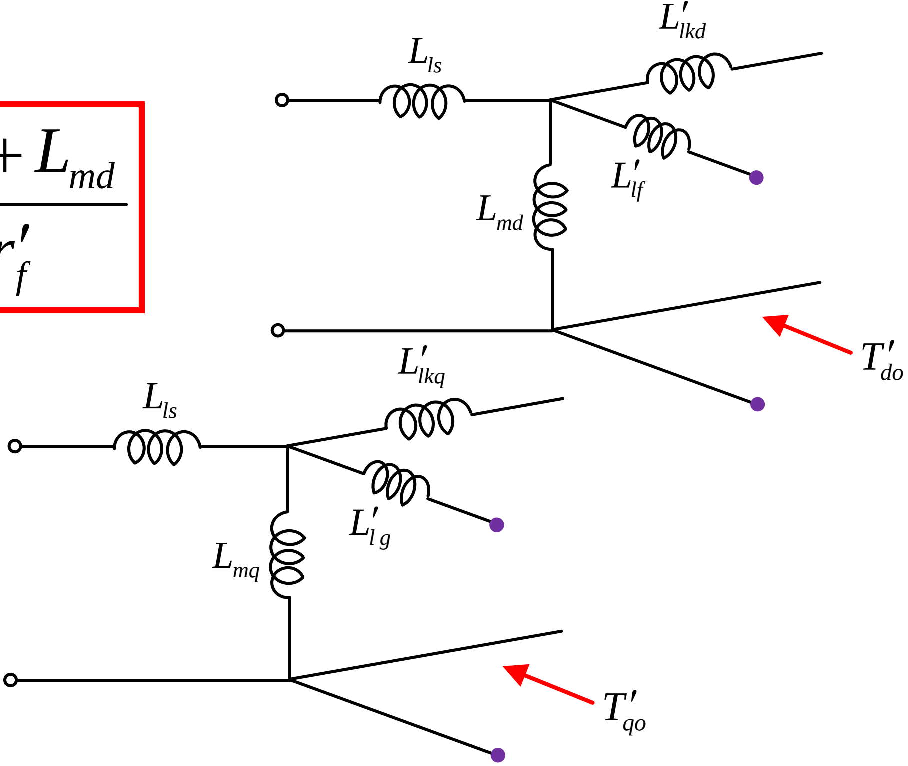
Machine Parameters

Open-circuit transient time constant

- When the stator is open-circuited and the effects of higher winding resistance, damper windings, are neglected, the **open-circuit time constants** of the **field** winding are defined as:

$$\boxed{T'_{do} = \frac{L'_{ff}}{r'_f} = \frac{L'_{lf} + L_{md}}{r'_f}}$$

$$\boxed{T'_{qo} = \frac{L'_{gg}}{r'_g} = \frac{L'_{l\,g} + L_{mq}}{r'_g}}$$

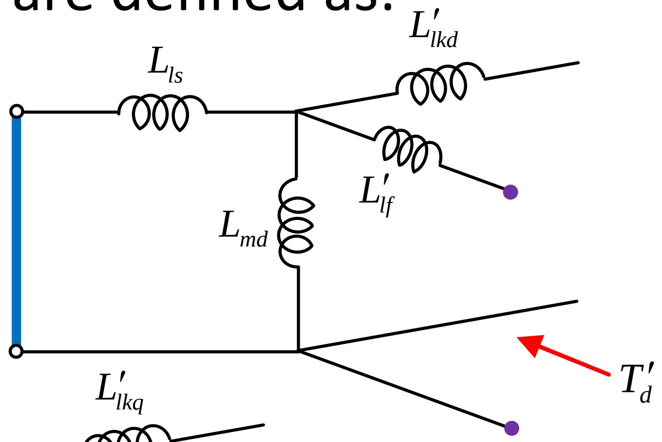


Machine Parameters

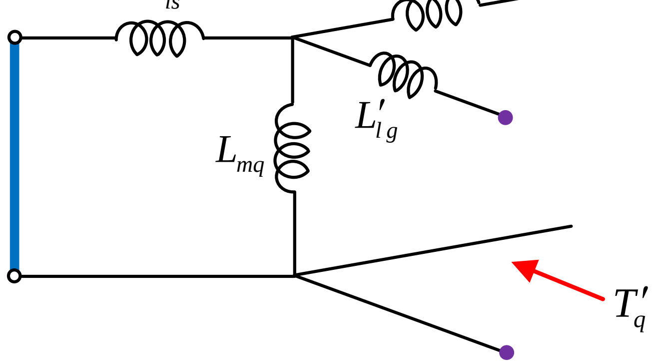
Short-circuit transient time constant

- When the stator is short-circuited and the effects of higher winding resistance, damper windings, are neglected, the **short-circuit time constants** of the **field** winding are defined as:

$$\boxed{T'_d = \frac{L'_{lf} + L_{md} \,||\, L_{ls}}{r'_f}}$$



$$\boxed{T'_q = \frac{L'_{lg} + L_{mq} \,||\, L_{ls}}{r'_g}}$$



Machine Parameters

Transient time constant

- The ratio of the time constants of the field winding with short-circuited stator windings to that with open-circuit stator winding is equal to the ratio of the apparent inductance seen by the stator current with field short-circuited to that with field open-circuited:

$$\boxed{\frac{T'_d}{T'_{do}} = \frac{L'_d}{L_d}} \quad \text{where} \quad \boxed{L_d = L_{ls} + L_{md}} \quad \text{and} \quad \boxed{L'_d = L_{ls} + \frac{L_{md} L'_{lf}}{L_{md} + L'_{lf}}}$$

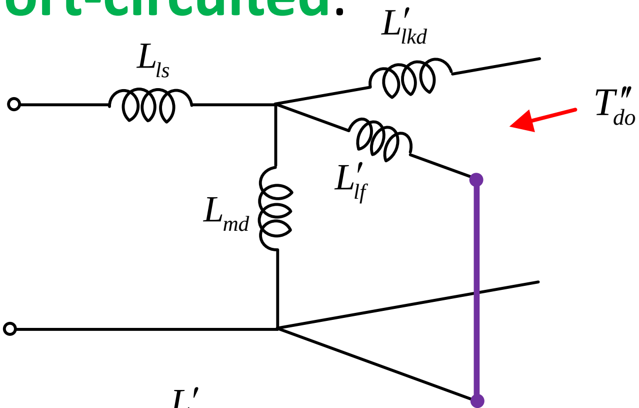
$$\boxed{\frac{T'_q}{T'_{qo}} = \frac{L'_q}{L_q}} \quad \text{where} \quad \boxed{L_q = L_{ls} + L_{mq}} \quad \text{and} \quad \boxed{L'_q = L_{ls} + \frac{L_{mq} L'_{lg}}{L_{mq} + L'_{lg}}}$$

Why?

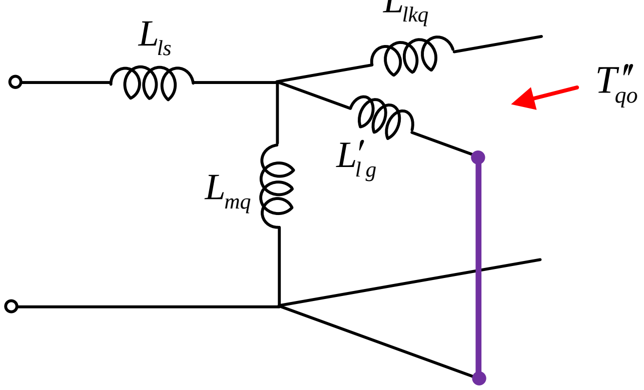
Machine Parameters

Open-circuit sub-transient time constant

- The open-circuit sub-transient time constants are the time constants of the **damper winding current** when the **stator is open-circuited** and the **field winding is short-circuited**.



$$\boxed{T''_{do} = \frac{L'_{lkd} + L_{md} \,||\, L'_{lf}}{r'_{kd}}}$$



$$\boxed{T''_{qo} = \frac{L'_{lkq} + L_{mq} \,||\, L'_{l\,g}}{r'_{kq}}}$$

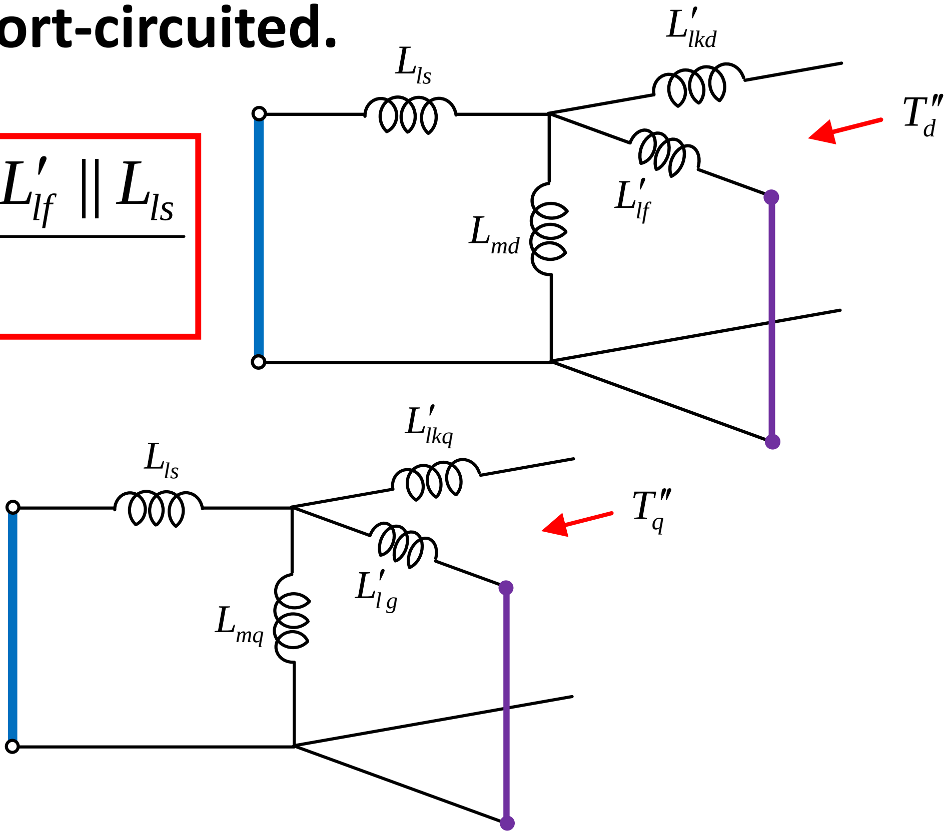
Machine Parameters

Short-circuit sub-transient time constant

- The short-circuit sub-transient time constants are the time constants of the damper winding current when the stator and the field windings **are both short-circuited.**

$$T_d'' = \frac{L_{lkd}' + L_{md} \,||\, L_{lf}' \,||\, L_{ls}}{r_{kd}'}$$

$$T_q'' = \frac{L_{lkq}' + L_{mq} \,||\, L_{lg}' \,||\, L_{ls}}{r_{kq}'}$$



Machine Parameters

Calculations of Machine Parameters

- Assume the following parameters are given by the manufacturers:

$$x_d \;,\; x_q \;,\; x_d' \;,\; x_q' \;,\; x_d'' \;,\; x_q'' \;,\; x_{ls} \;,\; r_s \;,\; T_{do}' \;,\; T_d' \;,\; T_{do}'' \;,\; T_{qo}'' \;,\; T_d'' \;,\; T_q''$$

- Based on the above given parameters, the following parameters are calculated:

$$\boxed{x_{md} \;,\; x_{mq} \;,\; x_{lf}' \;,\; x_{lkd}' \;,\; x_{lkq}' \;,\; r_f' \;,\; r_{kd}' \;,\; r_{kq}' \;.}$$

- The armature leakage reactance is normally given; but if it is not given, the value of the zero-sequence reactance may be used instead:

$$\boxed{x_{ls} = x_0}$$

Machine Parameters

Calculations of Machine Parameters (x_{md} , x_{mq} , x'_{lf})

- The **direct and quadrature magnetizing reactances** are obtained:

$$\boxed{x_{mq} = x_q - x_{ls}} \qquad \boxed{x_{md} = x_d - x_{ls}}$$

- The **field leakage reactance** is obtained using:

$$\boxed{x'_d = x_{ls} + \frac{x_{md}x'_{lf}}{x_{md} + x'_{lf}}} \Rightarrow \boxed{x'_{lf} = \frac{x_{md}(x'_d - x_{ls})}{x_{md} - (x'_d - x_{ls})}}$$

- If $x_{md} >> x'_{lf}$: $\Rightarrow$ $\boxed{x'_{lf} \approx x'_d - x_{ls}}$

Machine Parameters

Calculations of Machine Parameters (x'_{lkd})

- The **leakage reactance of the d-axis damper winding** is calculated using:

$$\boxed{x''_d = x_{ls} + \frac{x_{md}x'_{lf}x'_{lkd}}{x_{md}x'_{lf} + x_{md}x'_{lkd} + x'_{lkd}x'_{lf}}} \Rightarrow$$

$$\boxed{x'_{lkd} = \frac{(x''_d - x_{ls})x_{md}x'_{lf}}{x_{md}x'_{lf} - (x''_d - x_{ls})(x_{md} + x'_{lf})}}$$

- If $x_{md} >> x'_{lf}$ $\Rightarrow$ $\boxed{x'_{lkd} \approx \frac{(x''_d - x_{ls})x'_{lf}}{x'_{lf} - (x''_d - x_{ls})}}$

Machine Parameters

Calculations of Machine Parameters (x'_{lkq})

- Having no g-winding in the q-axis ($x_{lg} \to \infty$), the **leakage reactance of the q-axis damper winding** is calculated using:

$$\boxed{x''_q = x_{ls} + \frac{x_{mq} x'_{lkq}}{x_{mq} + x'_{lkq}}} \Rightarrow \boxed{x'_{lkq} = \frac{x_{mq}(x''_q - x_{ls})}{x_{mq} - (x''_q - x_{ls})}}$$

- If $x_{mq} >> x'_{lkq}$ $\Rightarrow$ $\boxed{x'_{lkq} \approx x''_q - x_{ls}}$

- The time constants are used to calculate the resistances of the rotor windings.

Machine Parameters

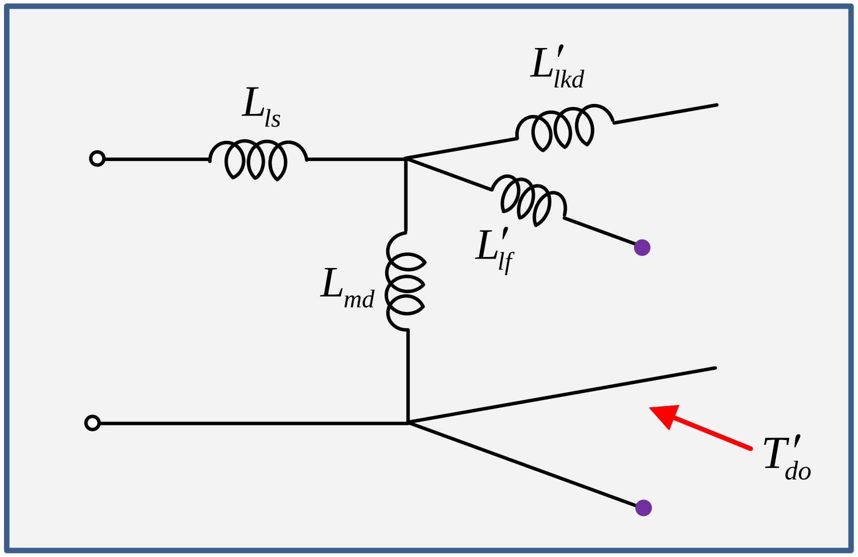
Calculations of Machine Parameters (r'_f)

- The **field resistance** is calculated from the **d-axis transient open-circuit time constant**:

$$T'_{do} = \frac{1}{\omega_b \, r'_f}(x'_{lf} + x_{md})$$

$$\Rightarrow$$

$$\boxed{r'_f = \frac{1}{\omega_b T'_{do}}(x'_{lf} + x_{md})}$$



Machine Parameters

Calculations of Machine Parameters (r'_{kd})

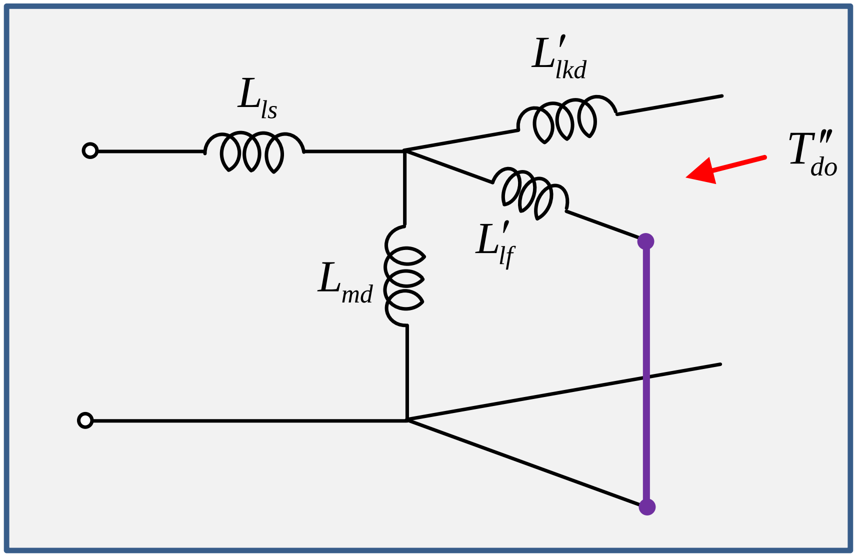
- The **d-axis damper resistance** is calculated from the **d-axis sub-transient open-circuit time constant**:

$$T''_{do} = \frac{1}{\omega_b\, r'_{kd}}(x'_{lkd} + \frac{x_{md}\, x'_{lf}}{x_{md} + x'_{lf}})$$

or

$$T''_{do} = \frac{1}{\omega_b\, r'_{kd}}(x'_{lkd} + x'_d - x_{ls})$$

$$r'_{kd} = \frac{1}{\omega_b\, T''_{do}}(x'_{lkd} + x'_d - x_{ls})$$

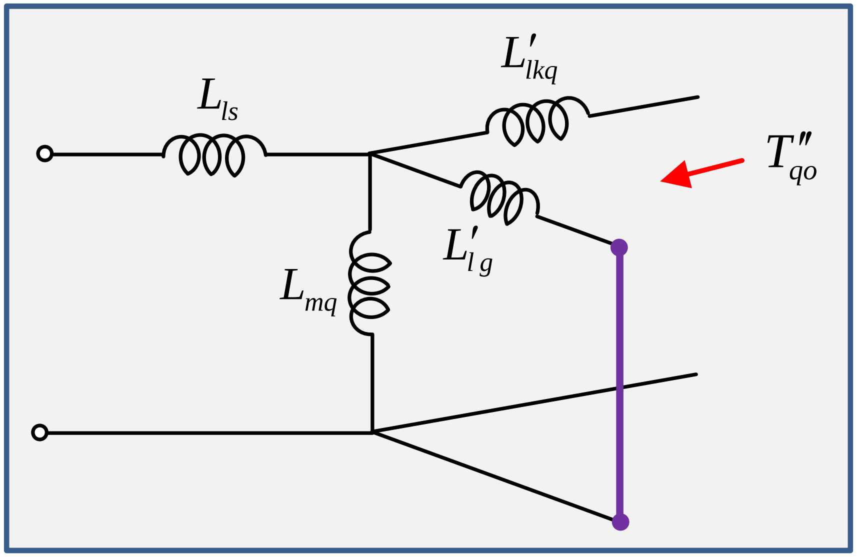


Machine Parameters

Calculations of Machine Parameters (r'_{kq})

- The **q-axis damper resistance** is calculated from the **q-axis sub-transient open-circuit time constant**:

$$\boxed{T''_{qo} = \frac{1}{\omega_b \, r'_{kq}} (x'_{lkq} + x_{mq})} \Rightarrow \boxed{r'_{kq} = \frac{1}{\omega_b \, T''_{qo}} (x'_{lkq} + x_{mq})}$$



Machine Parameters

Calculations of Machine Parameters

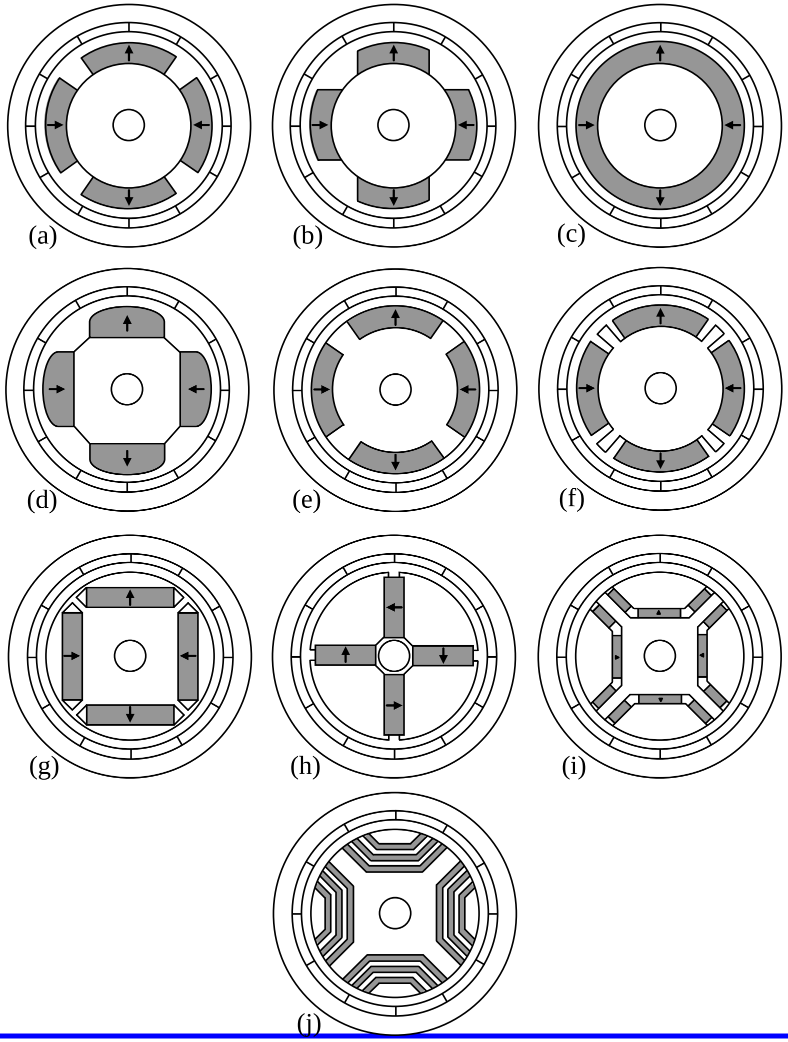
- Alternatively, the short-circuit time constants can be used to calculate the rotor resistances:

$$\boxed{r'_f = \frac{1}{\omega_b T'_d}(x'_{lf} + \frac{x_{md}\, x_{ls}}{x_{md} + x_{ls}})}$$

$$\boxed{r'_{kd} = \frac{1}{\omega_b T''_d}(x'_{lkd} + \frac{x_{md}\, x_{ls}\, x'_{lf}}{x_{md}\, x_{ls} + x_{md}\, x'_{lf} + x_{ls}\, x'_{lf}})}$$

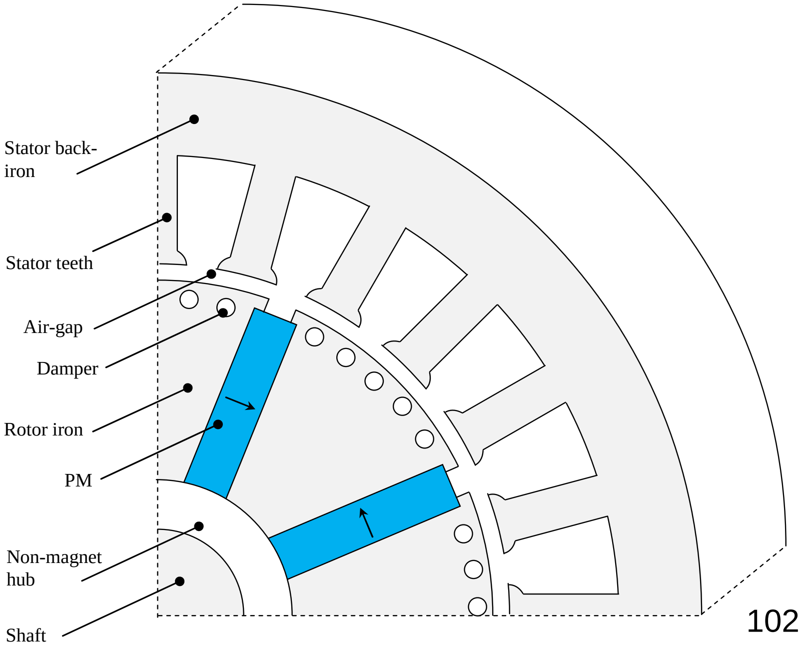
$$\boxed{r'_{kq} = \frac{1}{\omega_b T''_q}(x'_{lkq} + \frac{x_{mq}\, x_{ls}}{x_{mq} + x_{ls}})}$$

Permanent Magnet Synchronous Machines



PM Synchronous Machines

- The DC excitation of the field winding in a synchronous machine can be provided by **permanent magnets**.
- So the **rotor copper losses** are **eliminated**.
- PMSMs can offer **simpler construction**, **lower weight and size**, reduced losses and thus higher efficiency.
- The disadvantages of PMSMs are the **higher prices** if rare-earth magnets are used.



PM Synchronous Machines

*qd*0 equations referred to the stator side

Stator

$$v_q = r_s\, i_q + d\lambda_q / dt + \omega_r \lambda_d$$
$$v_d = r_s\, i_d + d\lambda_d / dt - \omega_r \lambda_q$$
$$v_0 = r_s\, i_0 + d\lambda_0 / dt$$

$$\lambda_q = L_q i_q + L_{mq} i'_{kq}$$
$$\lambda_d = L_d i_d + L_{md} i'_m + L_{md} i'_{kd}$$
$$\lambda_0 = L_{ls} i_0$$

Rotor

$$0 = r'_{kd}\, i'_{kd} + d\lambda'_{kd} / dt$$
$$0 = r'_{kq}\, i'_{kq} + d\lambda'_{kq} / dt$$

$$\lambda'_{kq} = L_{mq}\, i_q + L'_{kqkq}\, i'_{kq}$$
$$\lambda'_{kd} = L_{md}\, i_d + L_{md}\, i'_m + L'_{kdkd}\, i'_{kd}$$

where i'_m is the equivalent magnetizing current of the PMs.

PM Synchronous Machines

q-axis equivalent circuit

Stator

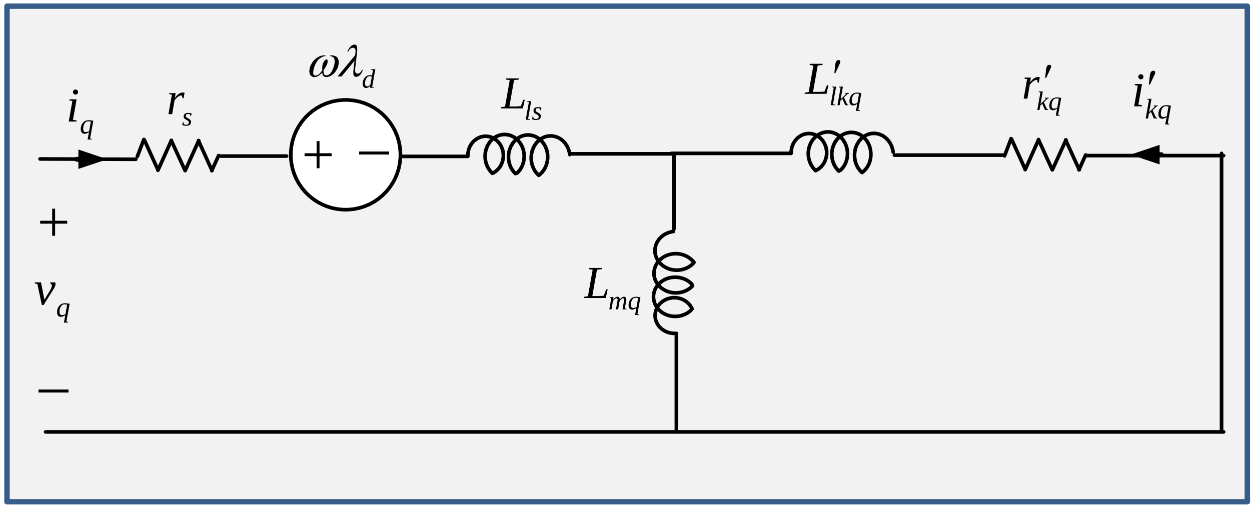
$$\boxed{v_q = r_s\, i_q + d\lambda_q / dt + \omega\, \lambda_d}$$

$$\boxed{\lambda_q = L_{ls}\, i_q + L_{mq}(i_q + i'_{kq})}$$

Rotor

$$\boxed{0 = r'_{kq}\, i'_{kq} + d\lambda'_{kq} / dt}$$

$$\boxed{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}(i_q + i'_{kq})}$$



PM Synchronous Machines

d-axis equivalent circuit

Stator

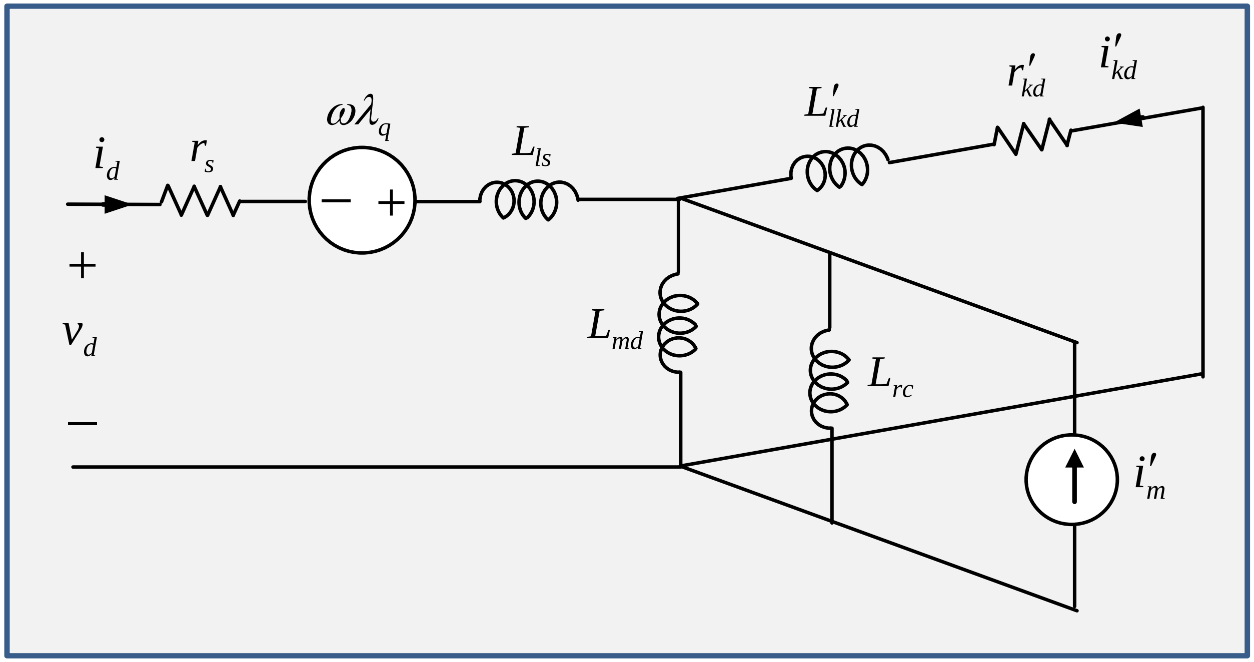
$$\boxed{v_d = r_s\, i_d + d\lambda_d / dt - \omega \lambda_q}$$

$$\boxed{\lambda_d = L_{ls}\, i_d + L_{md}(i_d + i'_m + i'_{kd})}$$

Rotor

$$\boxed{0 = r'_{kd}\, i'_{kd} + d\lambda'_{kd} / dt}$$

$$\boxed{\lambda'_{kd} = L'_{lkd}\, i'_{kd} + L_{md}(i_d + i'_m + i'_{kd})}$$



The PM reluctance can be represented by a virtual inductance denoted by L_{rc} , that is associated with its recoil slope

$$\boxed{L_{md} + L_{rc} \rightarrow L_{md}}$$

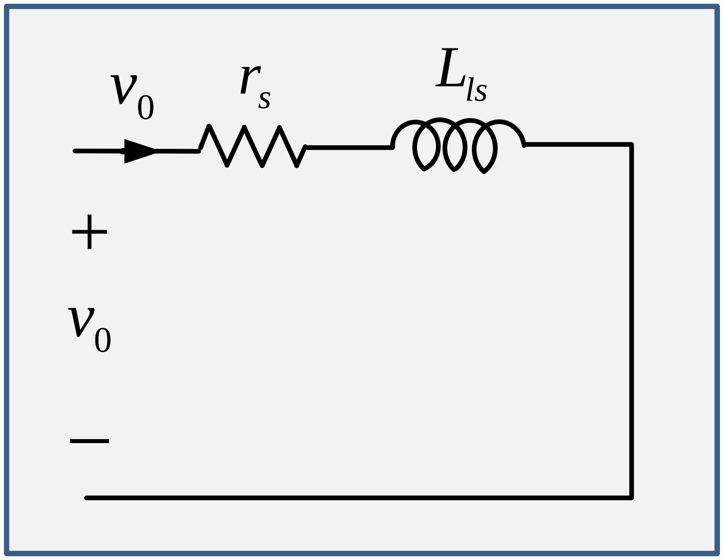
PM Synchronous Machines

Zero-component equivalent circuit

Stator

$$\boxed{v_0 = r_s i_0 + d\lambda_0/dt}$$

$$\boxed{\lambda_0 = L_{ls} i_0}$$



PM Synchronous Machines

Torque Equation

- Similar to wound rotor synchronous machine, the electromagnetic torque of PMSMs is obtained by

$$\boxed{T_{em} = \frac{3}{2}\frac{P}{2}\left(\lambda_d i_q - \lambda_q i_d\right)}$$

where P is the number of poles.

- It can be rewritten as:

$$\boxed{T_{em} = \underbrace{\frac{3}{2}\frac{P}{2}\left(L_d - L_q\right)i_d i_q}_{\text{Reluctance torque}} + \underbrace{\frac{3}{2}\frac{P}{2}\left(L_{md}\, i'_{kd}\, i_q - L_{mq} i'_{kq}\, i_d\right)}_{\text{Induction torque}} + \underbrace{\frac{3}{2}\frac{P}{2} L_{md}\, i'_m\, i_q}_{\text{Excitation torque}}}$$

where $\boxed{L_d = L_{ls} + L_{md}}$ and $\boxed{L_q = L_{ls} + L_{mq}}$

PM Synchronous Machines

Mechanical Dynamic Equation

- Based on Newton's 2nd law for rotational movement, $\sum T = J\dfrac{d\omega_{rm}}{dt}$ the **mechanical dynamic equation** is obtained:

Motoring Mode

$$\boxed{T_{em} - T_{mech} - T_{damp} = J\frac{d\omega_{rm}}{dt}}$$

Generating Mode

$$\boxed{T_{em} + T_{mech} - T_{damp} = J\frac{d\omega_{rm}}{dt}}$$

where T_{mech} is the externally-applied mechanical torque, T_{damp} is the damping torque, J is the moment of inertia and ω_{rm} is the mechanical rotational velocity.