In The Name of God The Most Compassionate, The Most Merciful

دانشگاه صنعتی شیراز

Electric Machines I

Table of Contents

Chapter 4
Principle of Direct Current (DC) Machines

4.1. Structure of DC Machines

4.2. Calculation of Induced Voltage (Generating mode)

4.3. Calculation of Developed Torque (Motoring mode)

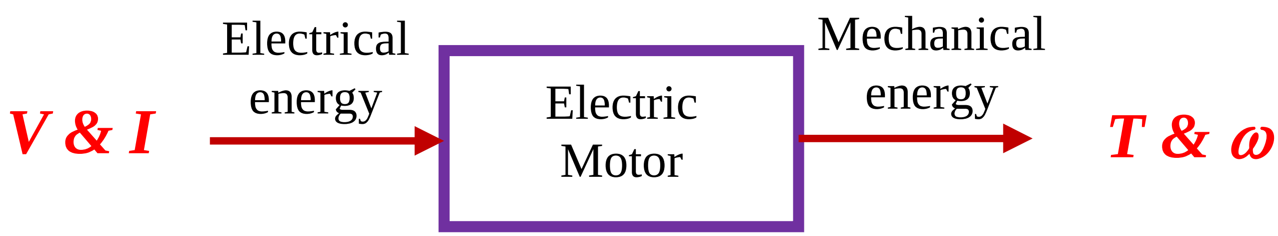
4.4. Armature Winding Topology of DC Machines

4.5. Commutation Process

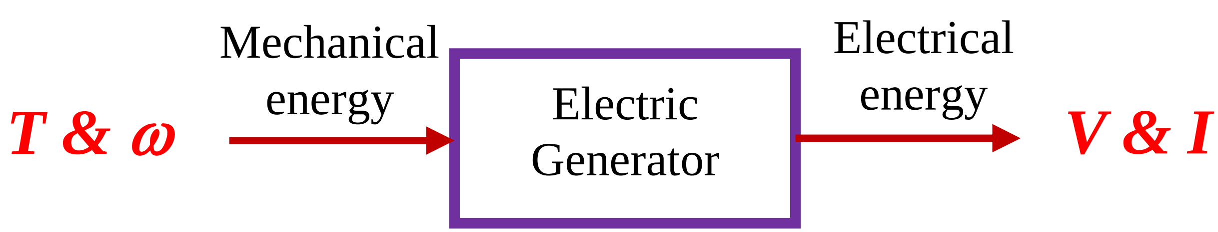
Electric Motors and Generators

An electric machine can be used as an electric motor or a generator.

- **Electric motors** receive electrical energy as input and provide mechanical energy as output.



- **Electric generators** receive mechanical energy as input and provide electrical energy as output.



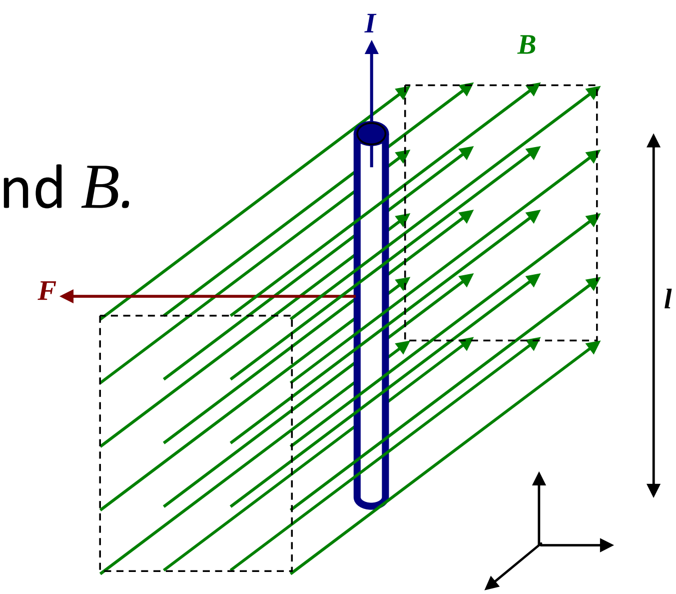
Electric Motor Principle

- Based on the principle theory of electromechanical systems, if a **current-carrying conductor** is located in a **magnetic field**, a **force** is exerted on the conductor.
- The **force**, F, is directly proportional to the **current** I, magnetic **field density** B and the **length** of the conductor l and according to the Lorentz law, the force can be expressed as follow,

$$F = Il \times B \Longrightarrow |F| = IlB\sin(\theta)$$

θ is the angle between the conductor and B.

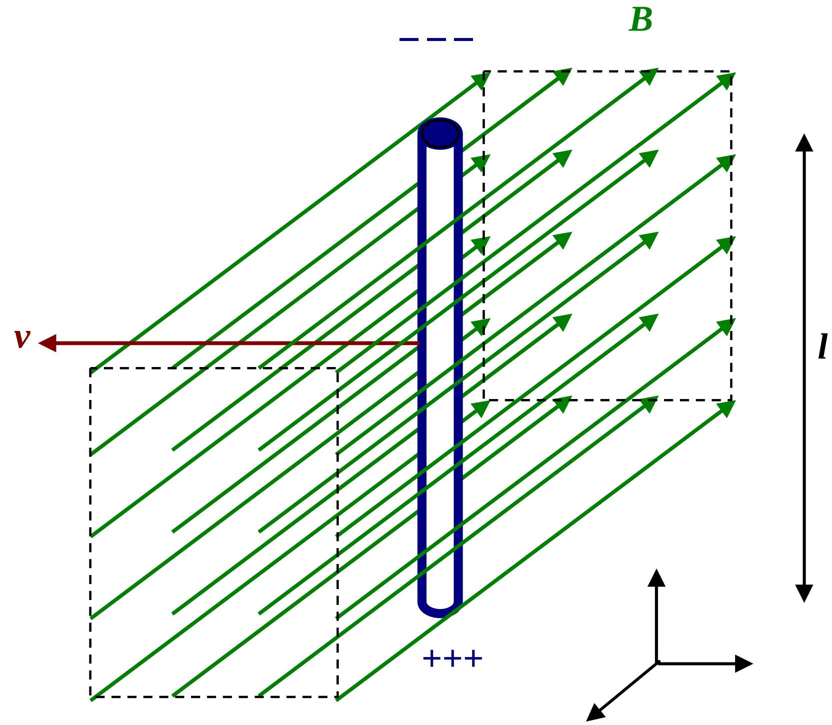
	Right hand	Left hand
Index finger	I	B
Middle finger	B	I
Thumb	F	F



Electric Generator Principle

- Another fundamental theory of electromechanical systems says, if a **conductor moves** with speed *v* inside a **magnetic field** with density *B*, a **voltage** *E* will be induced across the conductor which is expressed as,

$$E = (v \times B) \cdot l$$

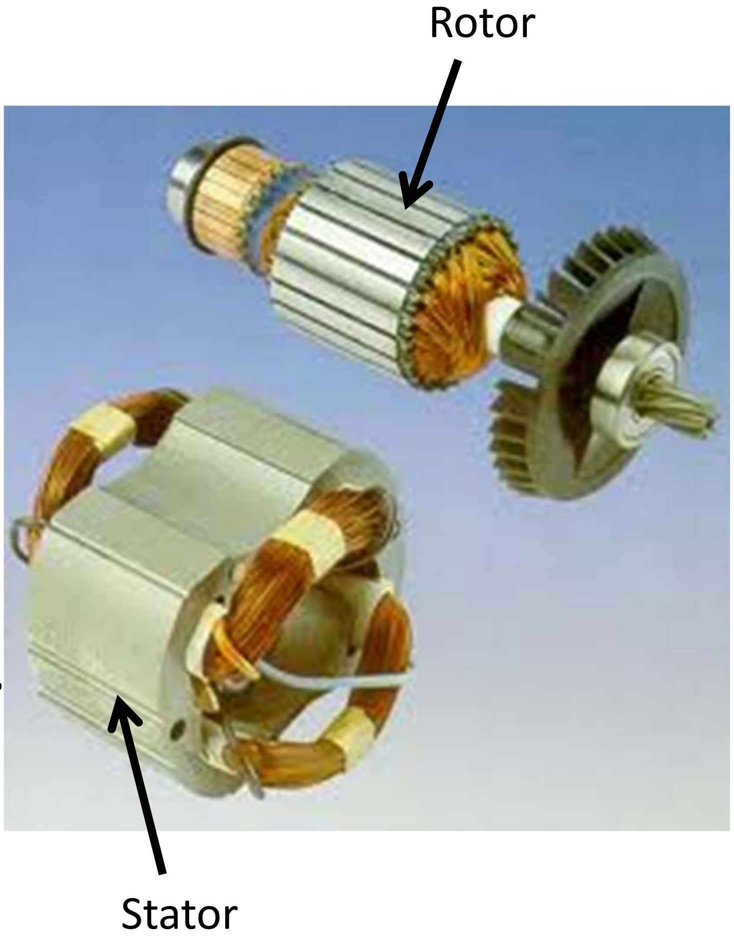


Structure of DC Machines

Rotor vs. stator

Rotor: The rotating part of the machine.

Rotor is usually made of laminated magnetic steel.



Stator: The stationary part of the machine.

Stator is made of cast magnetic steel.

Only the pole shoes are made of laminated magnetic steel.

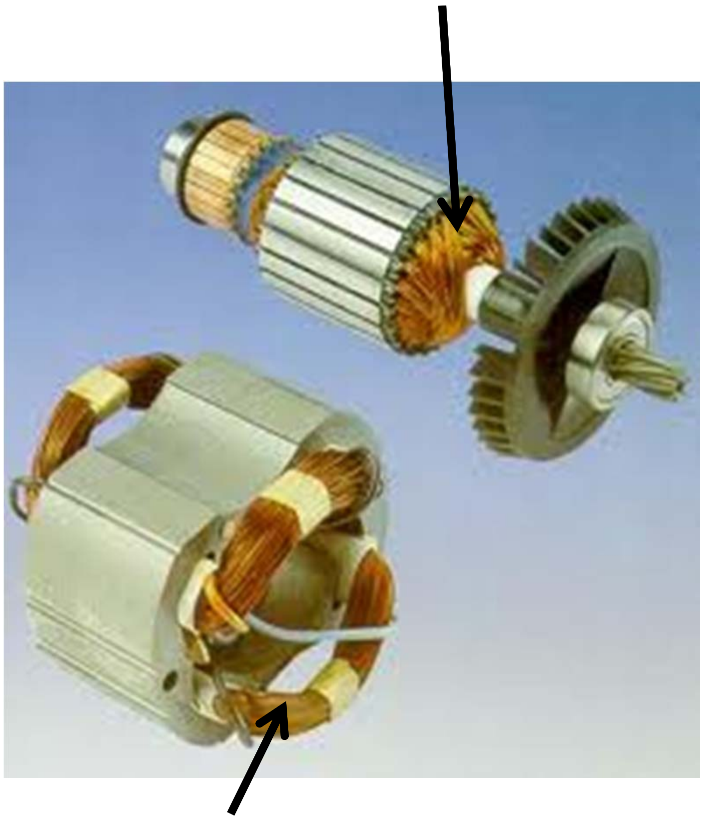
Structure of DC Machines

Armature winding vs. field winding

Armature winding:

- In **generating** mode is the winding when move w.r.t. the magnetic field, a voltage is induced across it.
- In **motoring** mode is the winding that carries current and in the presence of a magnetic field a force is exerted on it.

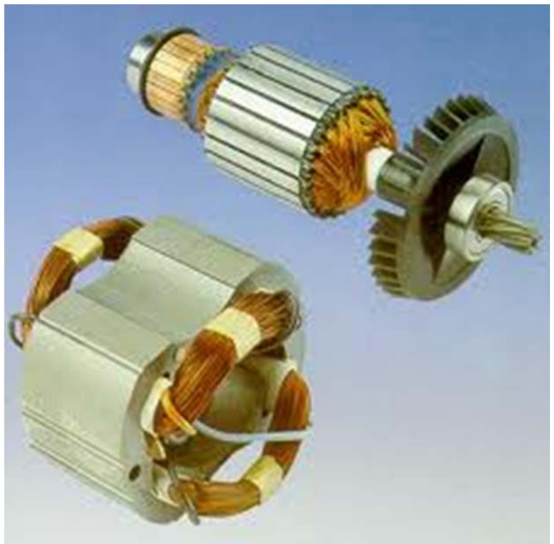
Field Winding: is the winding responsible to produce magnetic field when current passing through it.

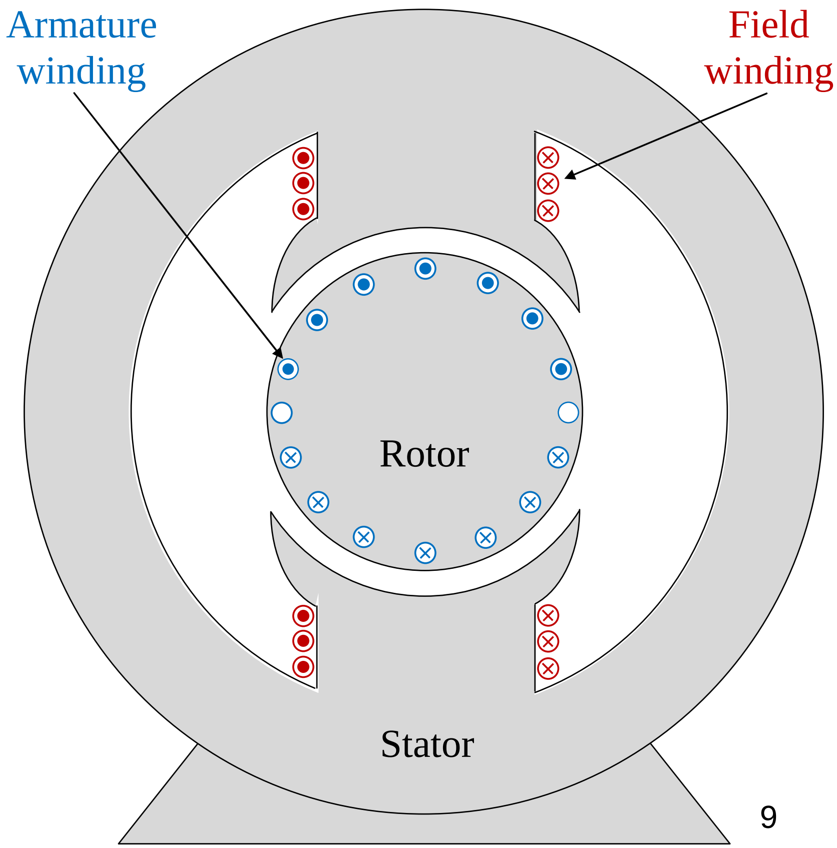


Location of armature and field windings

1. DC Machines

- **Armature** winding is on the **rotor**
- **Field** winding is on the **stator**

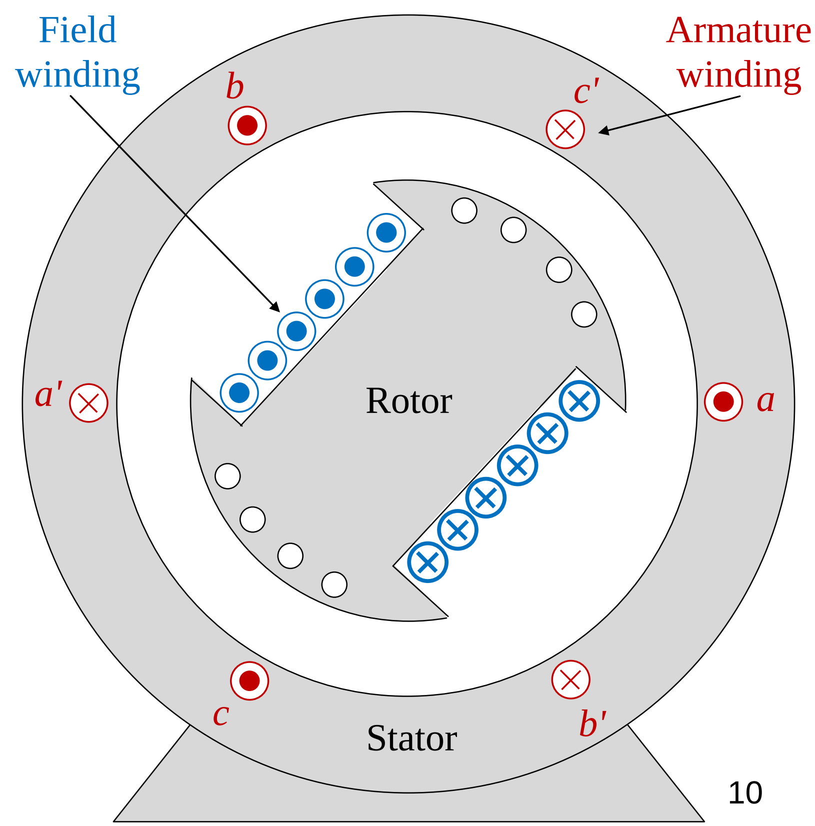




Location of armature and field windings

2. Synchronous Machines

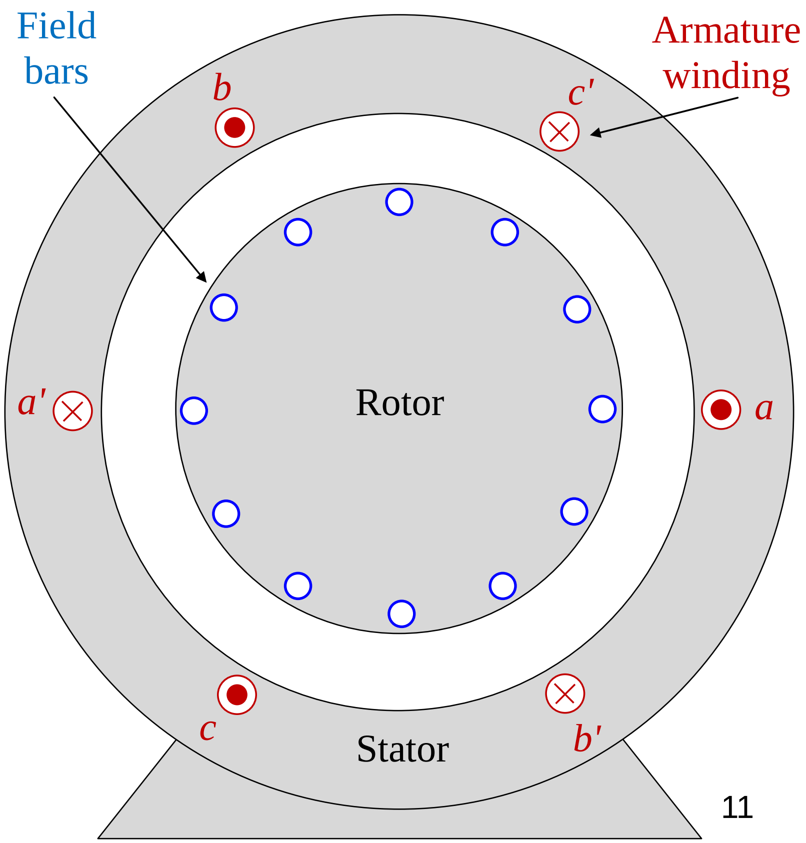
- **Armature** winding is on the **stator**.
- **Field** winding is on the **rotor**.



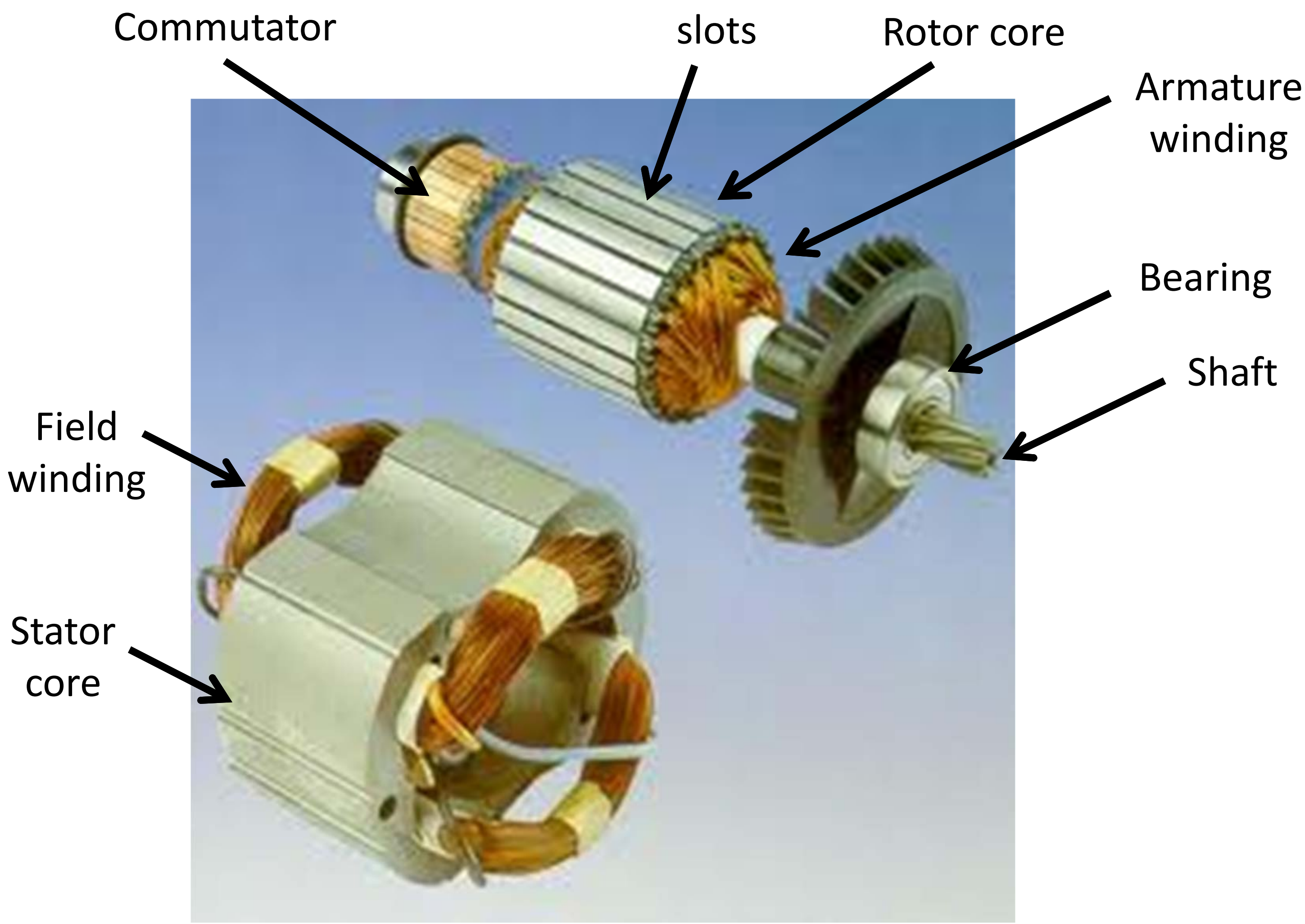
Location of armature and field windings

3. Induction Machines (Squirrel cage rotor)

- **Armature** winding is on the **stator**.
- **Field** bars on the **rotor**.
  The origin of field is from armature winding.



Structure of DC Machines

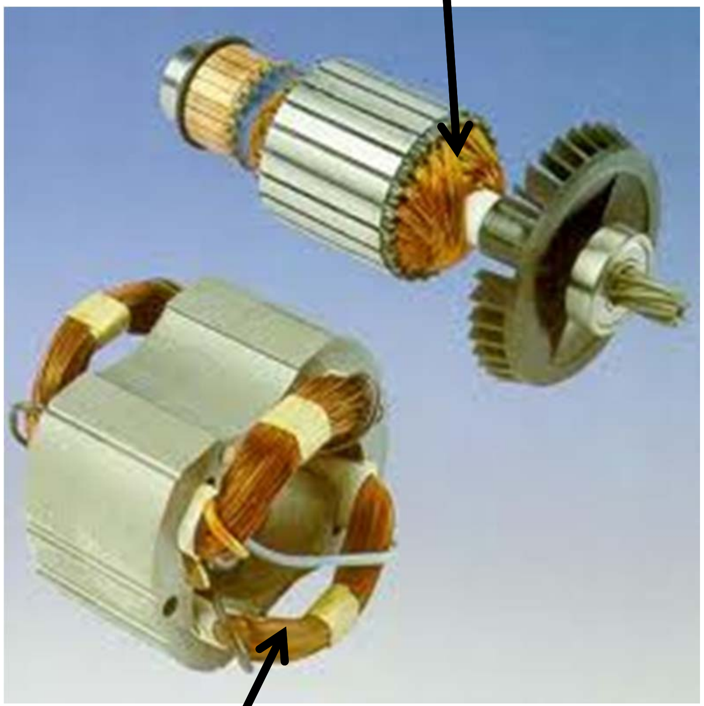


Structure of DC Machines

Distributed vs. concentrated winding

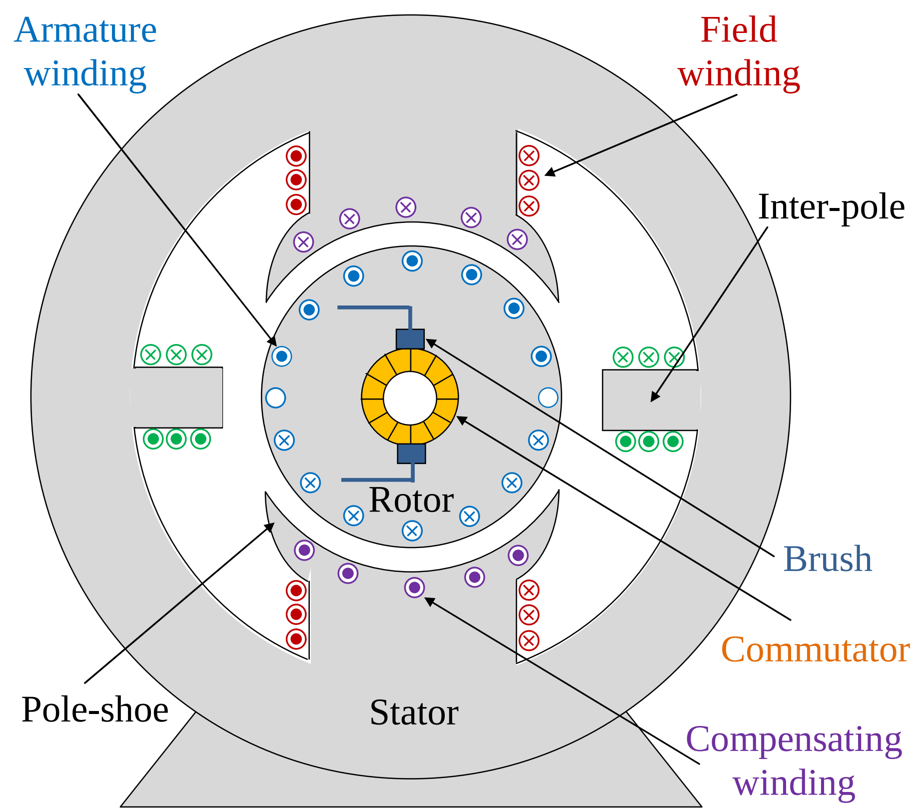
Armature winding is located in the rotor slots and is of distributed type. The induced voltage in armature winding is AC and converted to DC using commutators and brushes.

Field Winding is wound around stator poles and is of concentrated type.



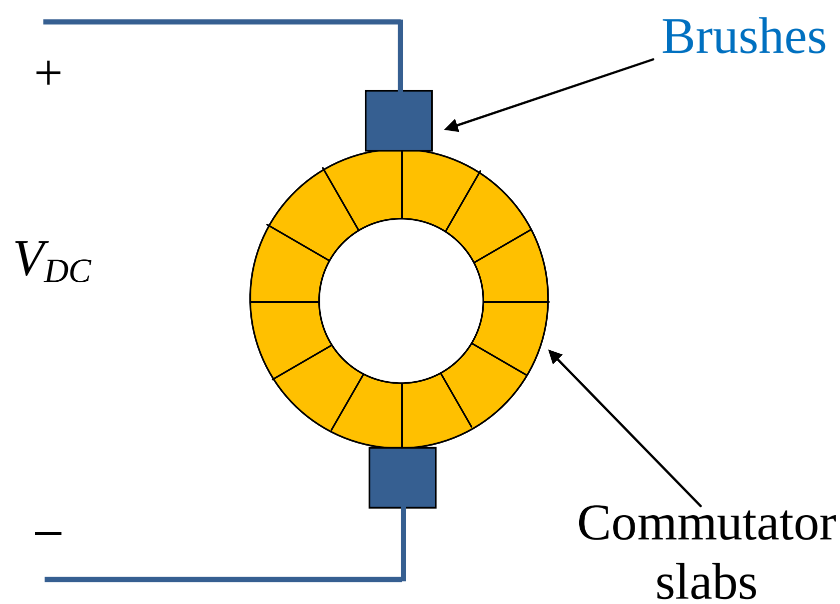
Structure of DC Machines

- The **pole shoes** cause to reduce the magnetic reluctance and to mechanically protect the field winding.
- **Inter-poles** and **compensating winding** improve the performance of DC machines.
- **Commutator slabs** and the **brushes** are together called commutators.



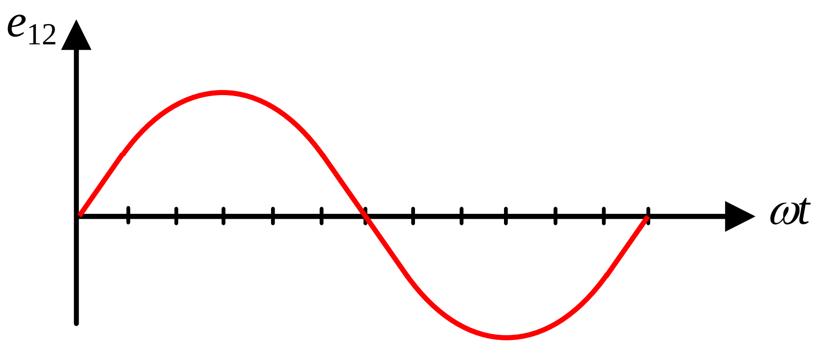
Structure of DC Machines

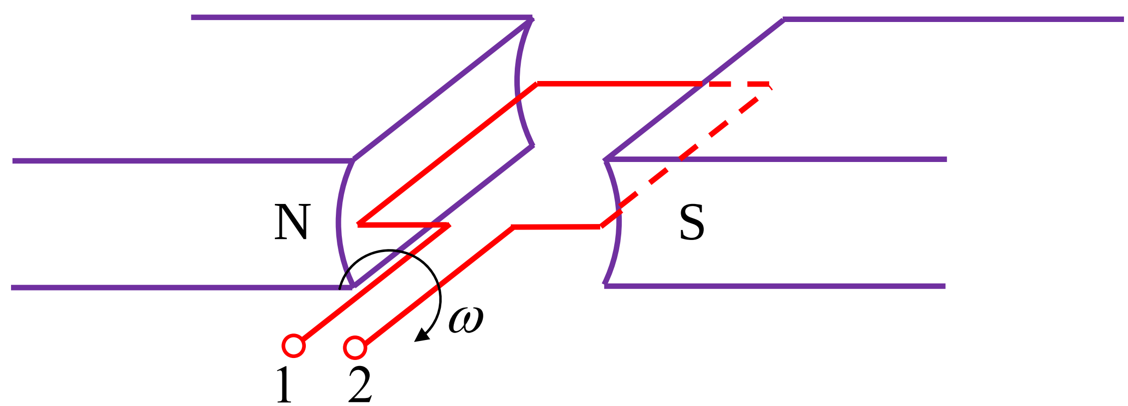
- **Commutators** in generating mode convert the AC voltage to DC and in motoring mode convert the DC voltage to AC.
- **Commutator slabs** are made of **copper** and the slabs are insulated by mica.
- **Brushes** are made of **carbon** or **graphite** or a combination of the two



Structure of DC Machines

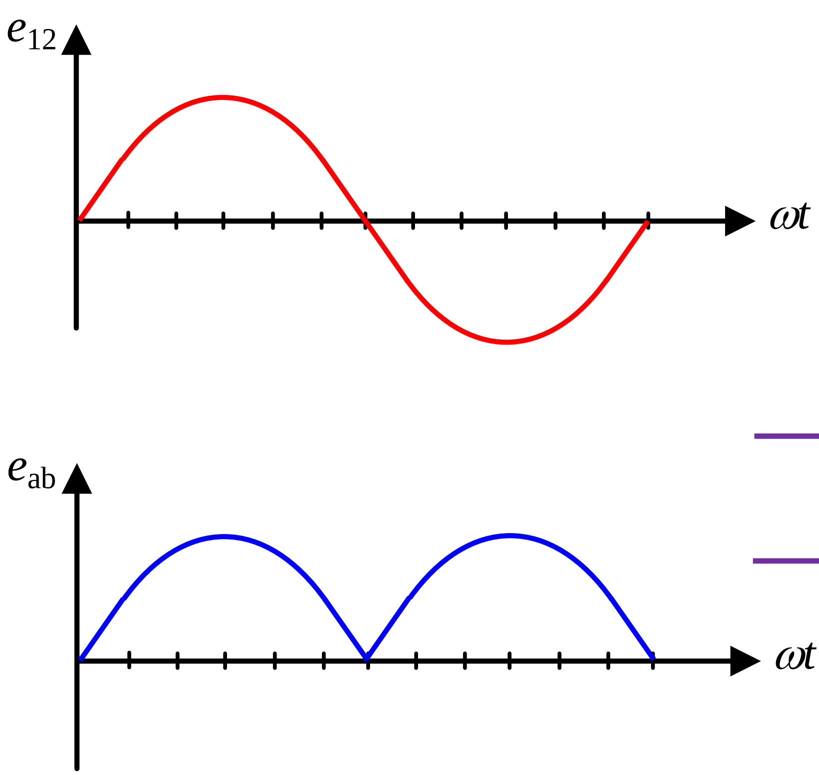
- Consider a single conducting loop rotating with rotational velocity of ω in a constant magnetic field, the induced voltage across the loop is ideally in sinusoidal form

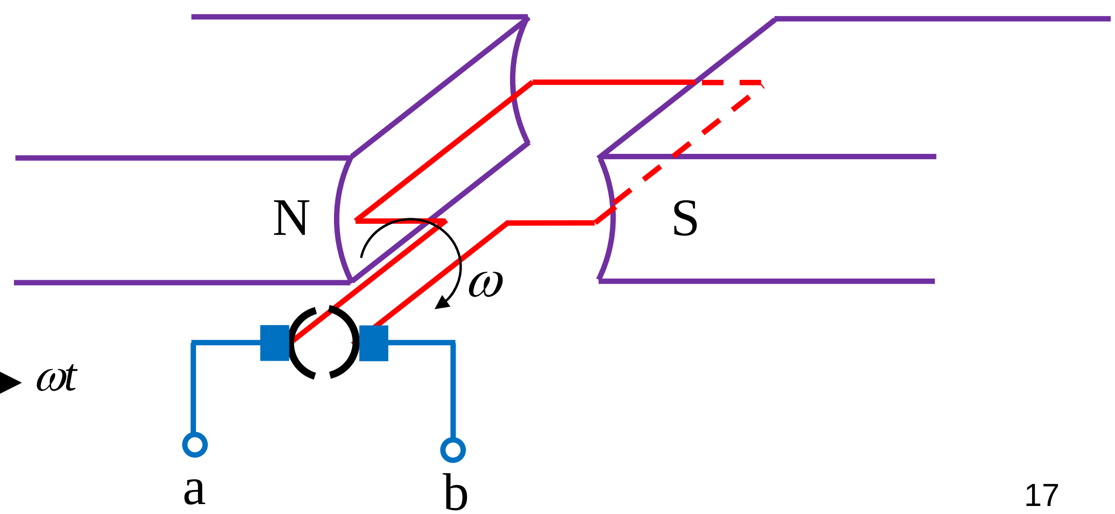




Structure of DC Machines

- Now assume a two-slab commutator is connected to the conducting loop and rotating with the conductor. If two brushes are fixed in place





Rectification of Commutator

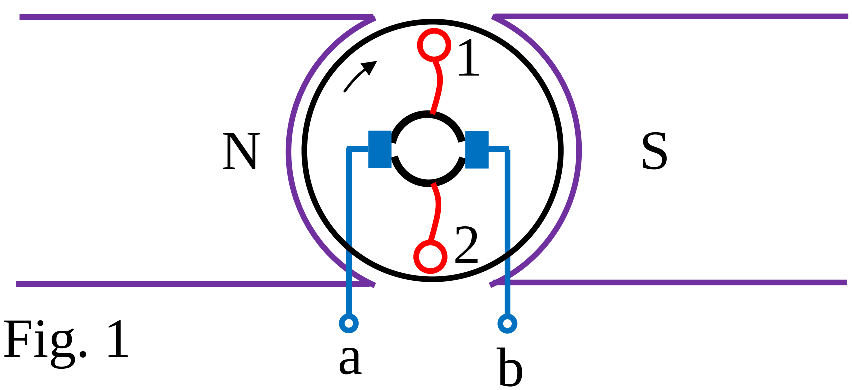


Fig. 1

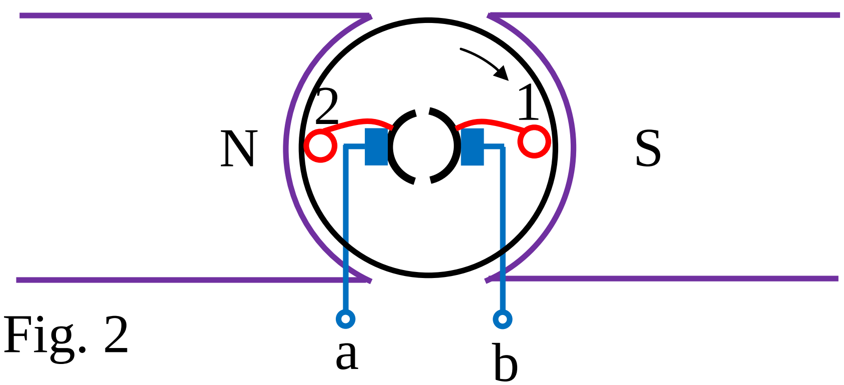


Fig. 2

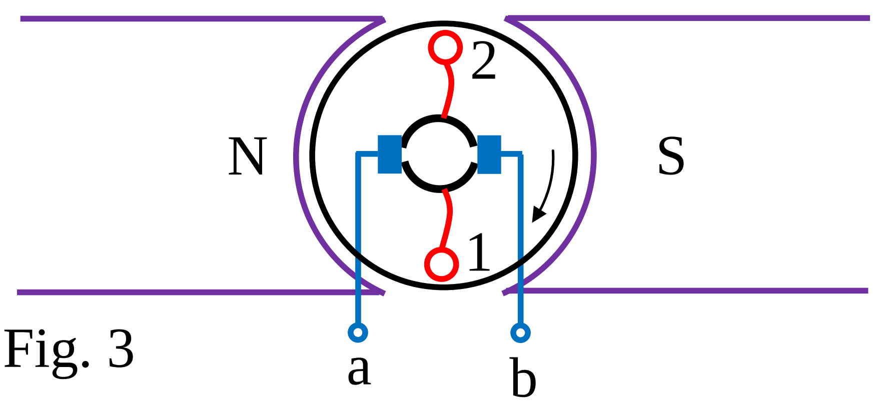


Fig. 3

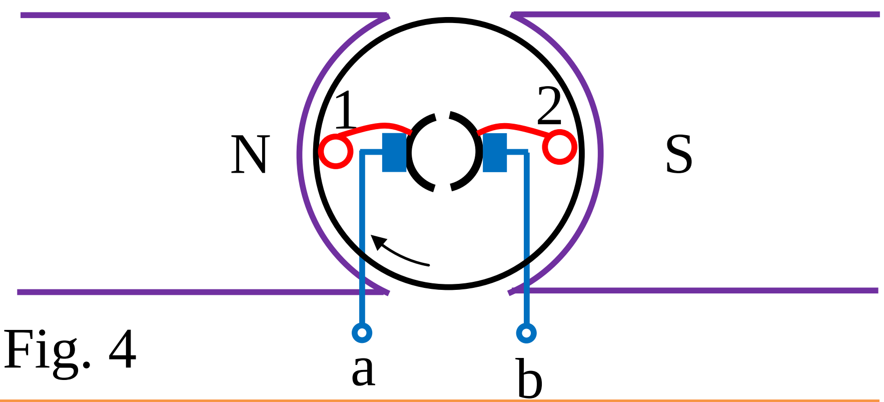
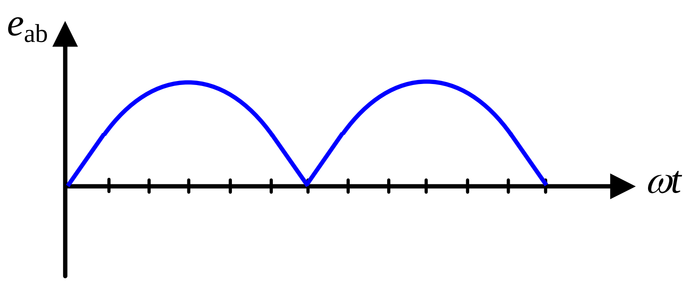
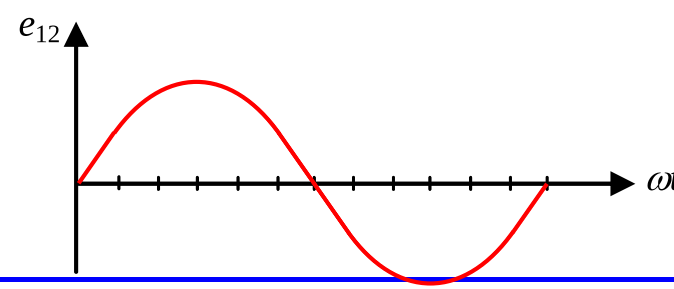


Fig. 4

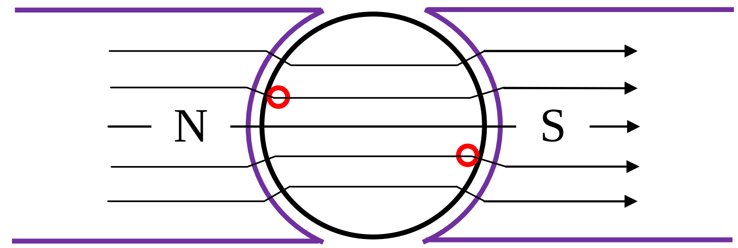


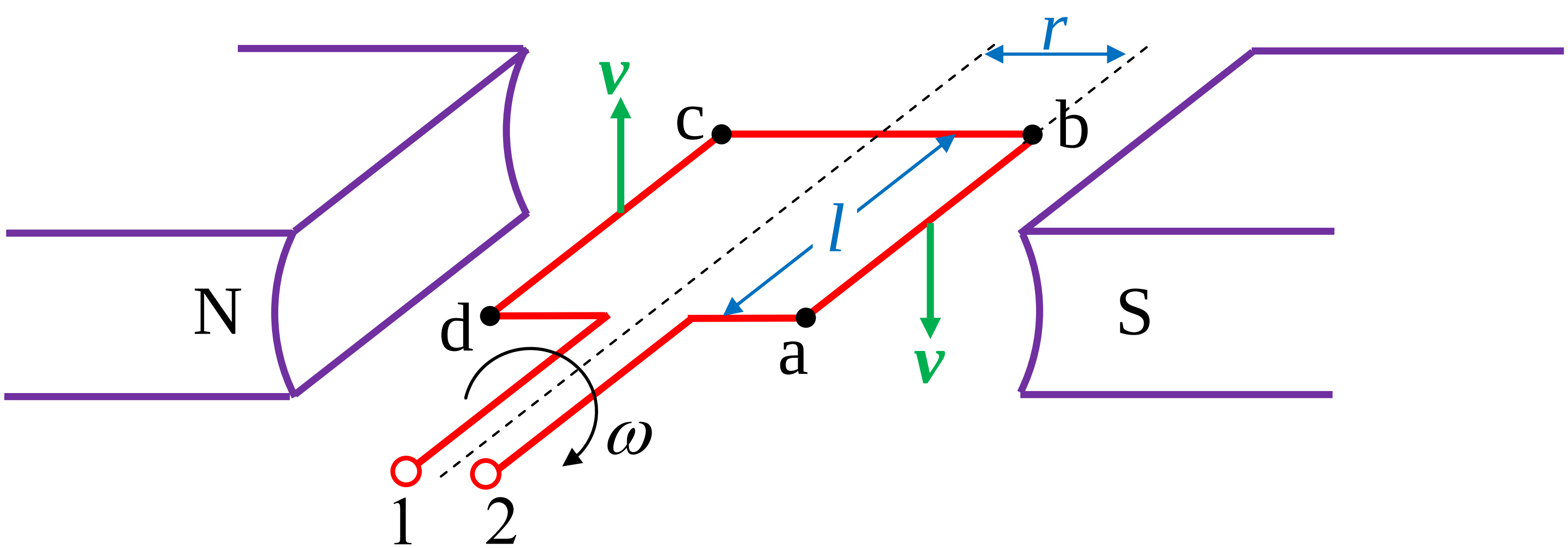


Calculation of Induced Voltage

- If the conducting loop **abcd** is rotating with rotational velocity of ω in a magnetic field B, a voltage is induced across the loop

$$E_{12} = (v \times B) \cdot l$$

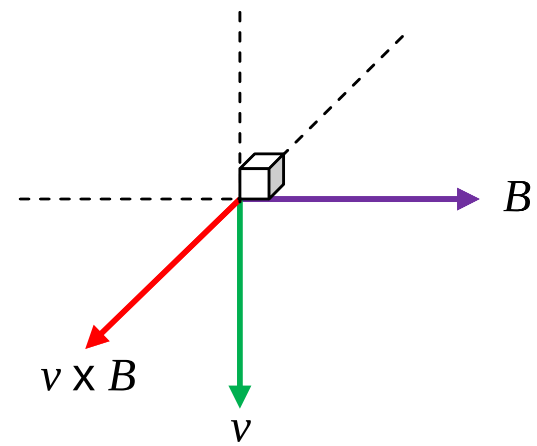


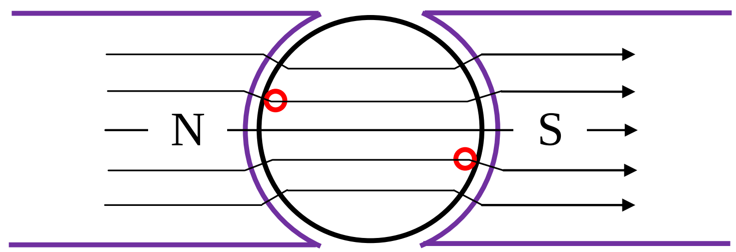


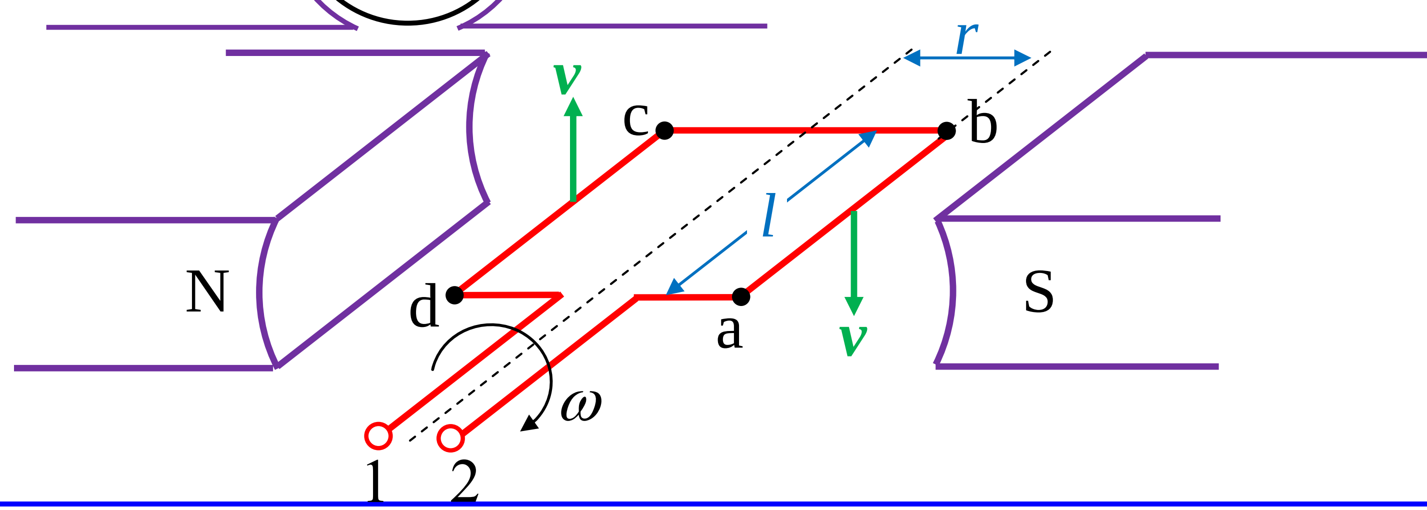
Calculation of Induced Voltage

- **Side ab** : under the pole, B is perpendicular to v

$$E_{ab}=\begin{cases} vBl & \text{under the pole} \\ 0 & \text{outside the pole} \end{cases}$$



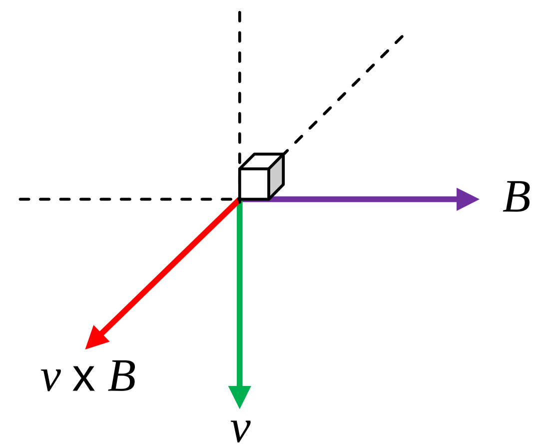


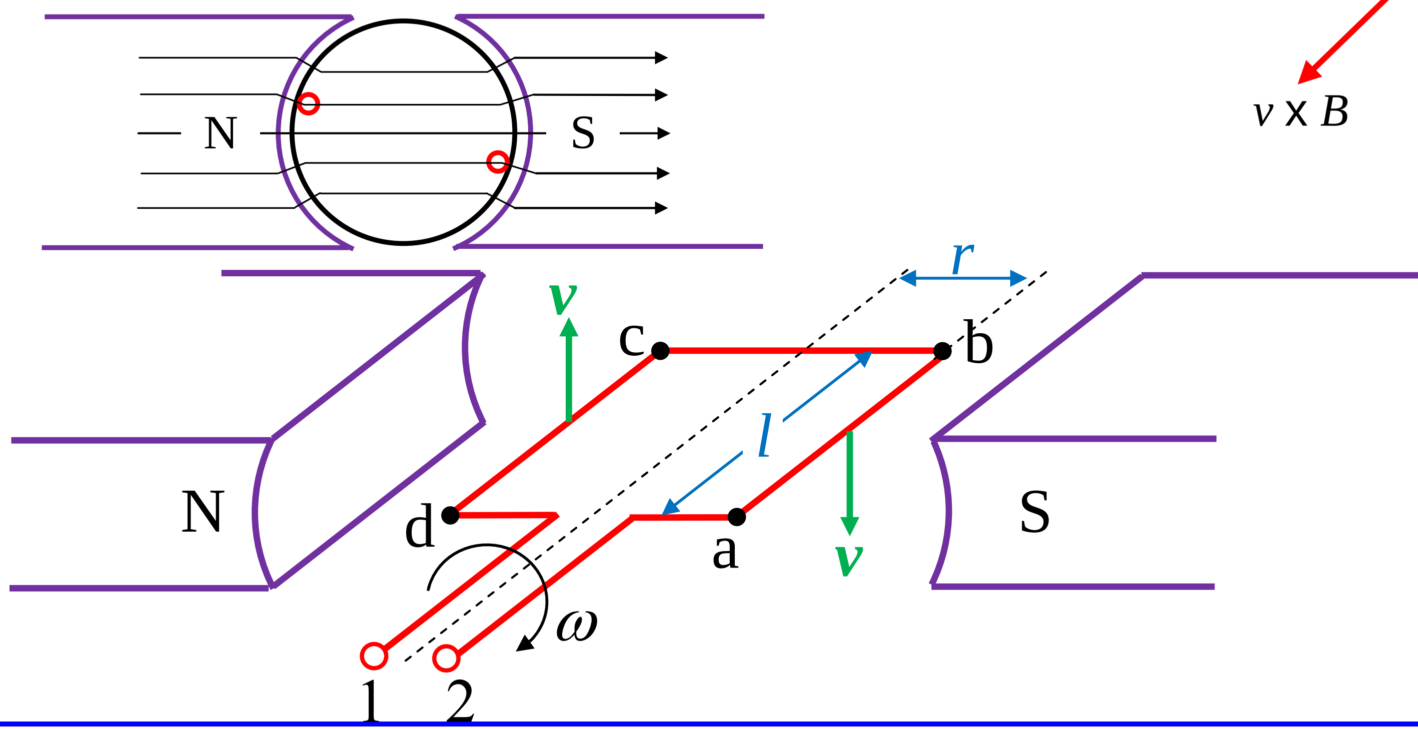


Calculation of Induced Voltage

- **Side bc**: its length is perpendicular to (v x B)

$$E_{bc} = 0$$

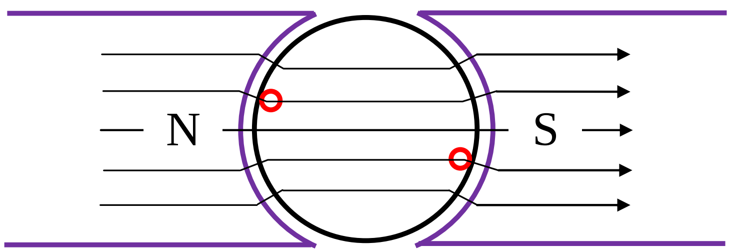


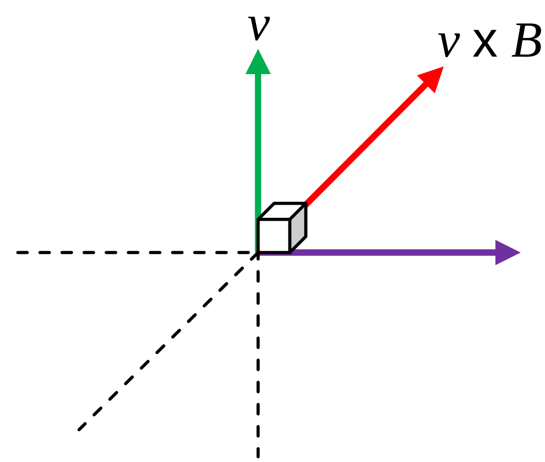


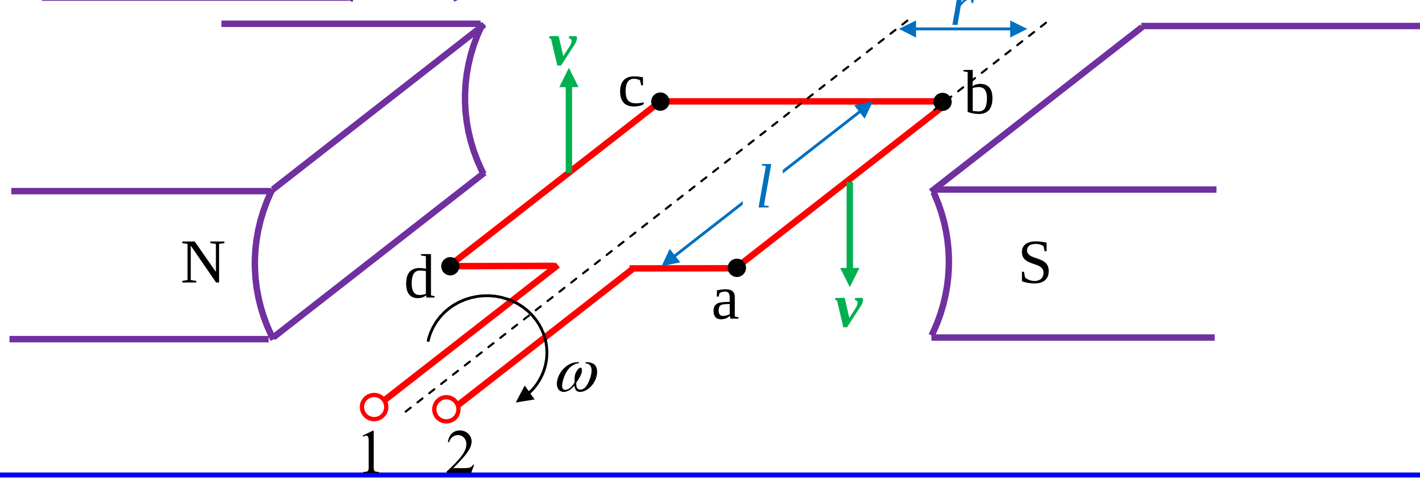
Calculation of Induced Voltage

- **Side cd**: under the pole, B is perpendicular to v

$$E_{cd} = \begin{cases} vBl & \text{under the pole} \\ 0 & \text{outside the pole} \end{cases}$$



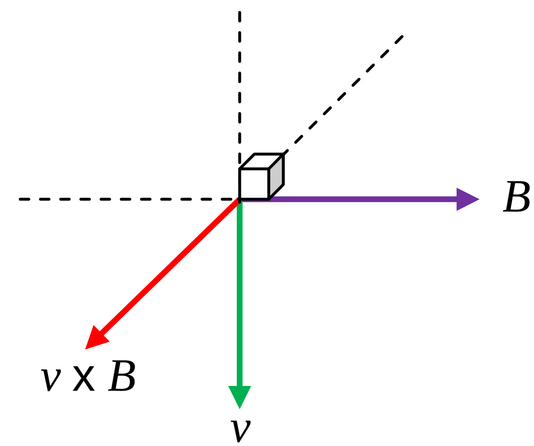


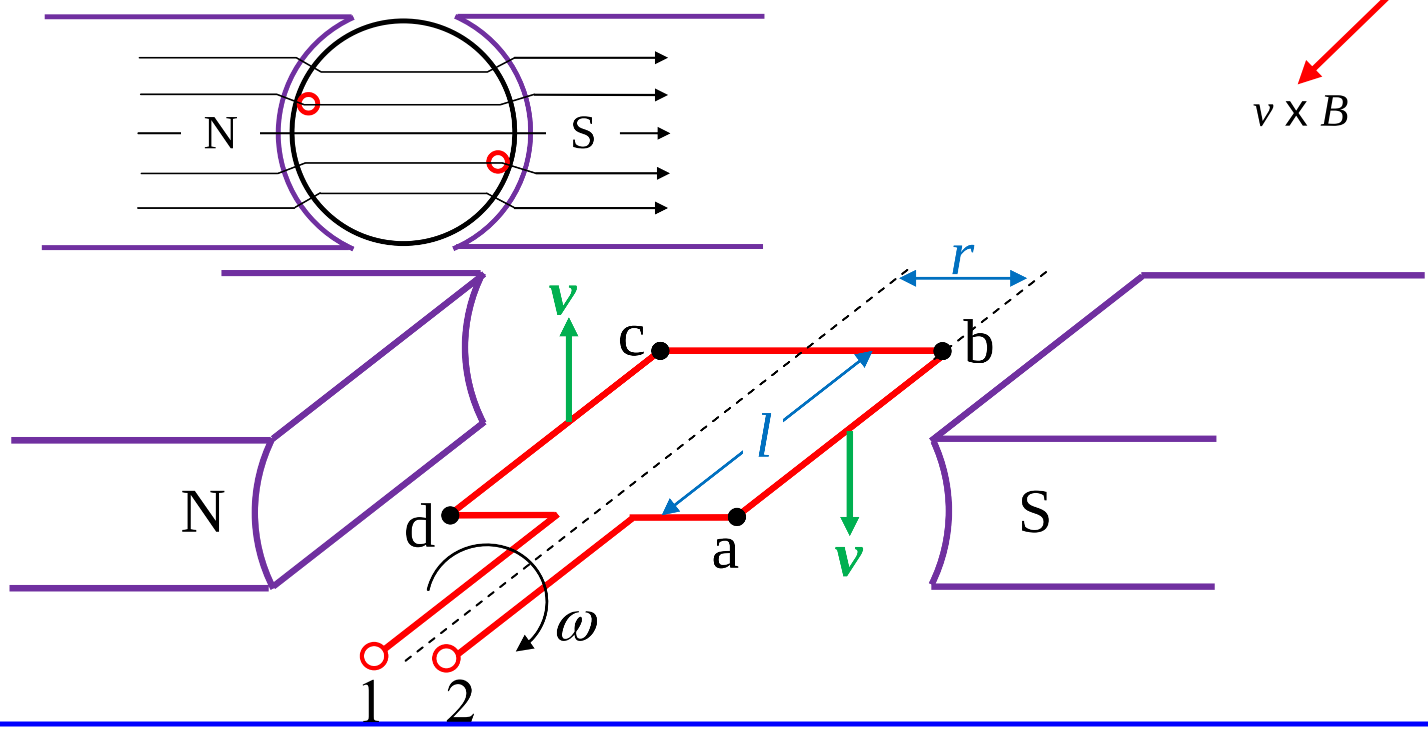


Calculation of Induced Voltage

- **Side da**: its length is perpendicular to (v x B)

$$E_{da} = 0$$





Calculation of Induced Voltage

- KVL:

$$E_{21} = E_{ab} + E_{bc} + E_{cd} + E_{da} = \begin{cases} 2vBl & \text{under the pole} \\ 0 & \text{outside the pole} \end{cases}$$

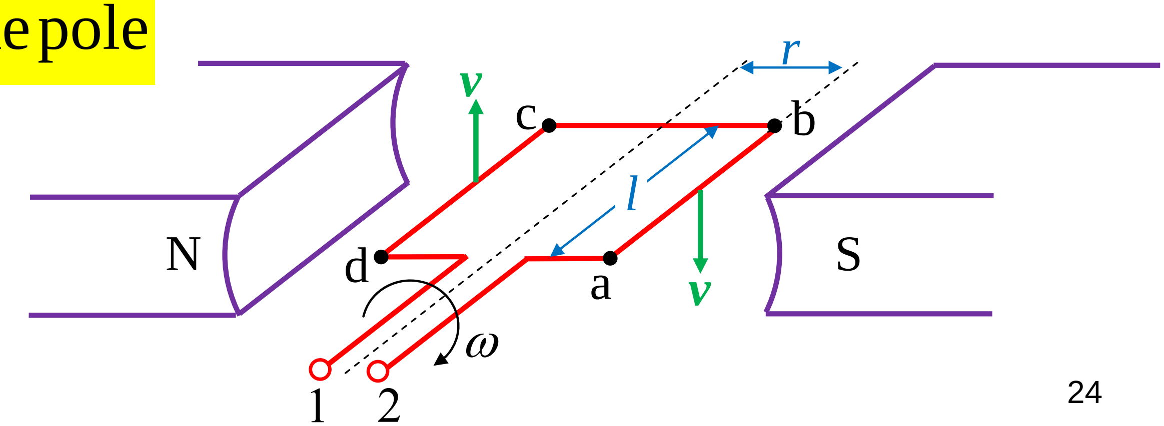
$$v = r\omega$$

$$E_{21} = \begin{cases} 2r\omega Bl & \text{under the pole} \\ 0 & \text{outside the pole} \end{cases}$$

$$A_p = \pi r l$$

$$E_{21} = \begin{cases} \frac{2}{\pi} A_p B\omega & \text{under the pole} \\ 0 & \text{outside the pole} \end{cases}$$

$$\phi = A_p B$$



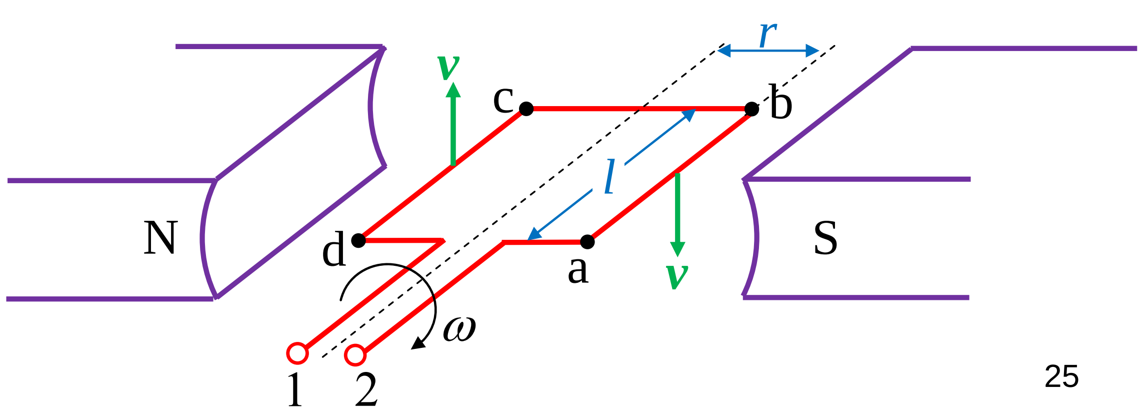
Calculation of Induced Voltage

$$E_{21} = \begin{cases} \frac{2}{\pi} A_p B\omega & \text{under the pole} \\ 0 & \text{outside the pole} \end{cases}$$

$$\phi = A_p B$$

$$E_{21} = \begin{cases} \frac{2}{\pi} \phi\omega & \text{under the pole} \\ 0 & \text{outside the pole} \end{cases}$$

Note that this voltage is for single loop, in the case of a coil with *N* loops the voltage should be multiplied by *N*.



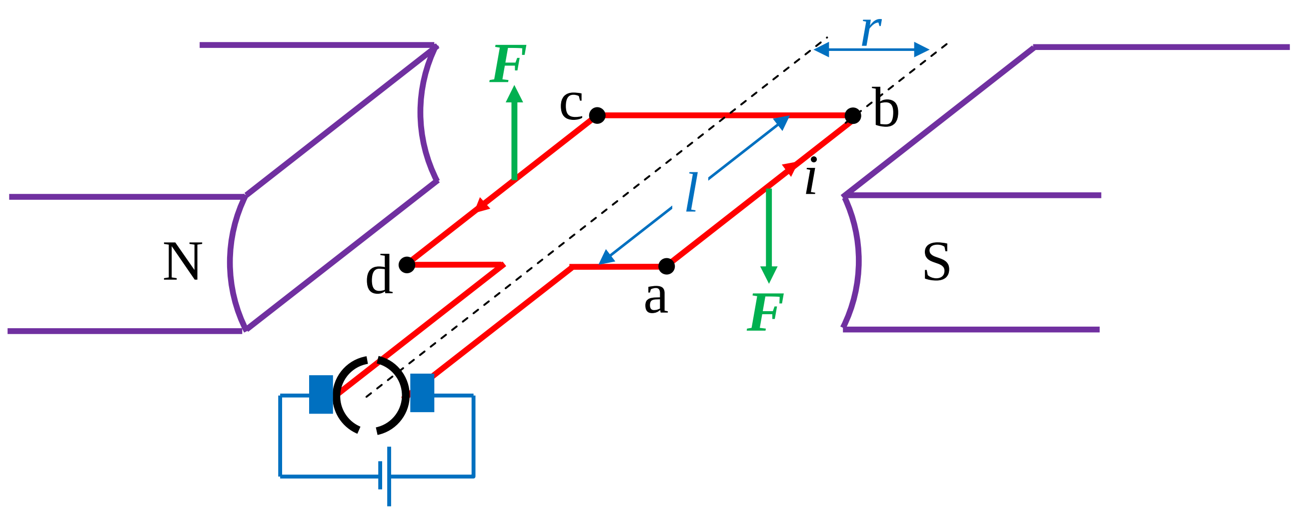
Calculation of Developed Torque

- If the current i is passing through the conducting loop **abcd,** located in a magnetic field B, a force is exerted to the loop

$$F = i(l \times B)$$

- Consequently the torque is calculated as

$$T = rF\sin\theta$$ where θ is the angle between r and F.

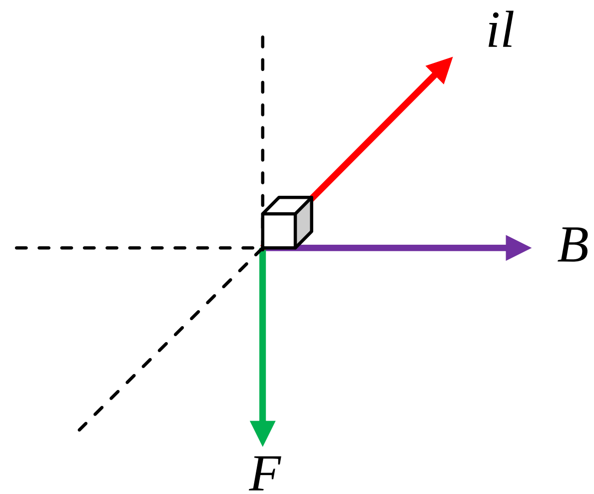


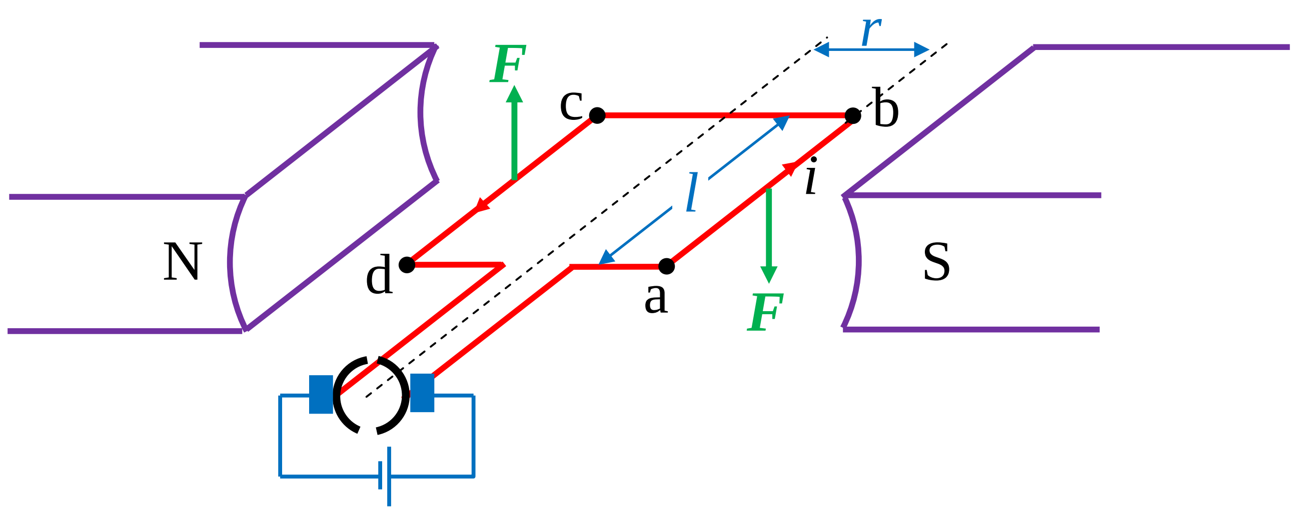
Calculation of Developed Torque

- **Side ab**: il is perpendicular to B

$$F_{ab} = ilB$$

$$T_{ab} = rilB$$



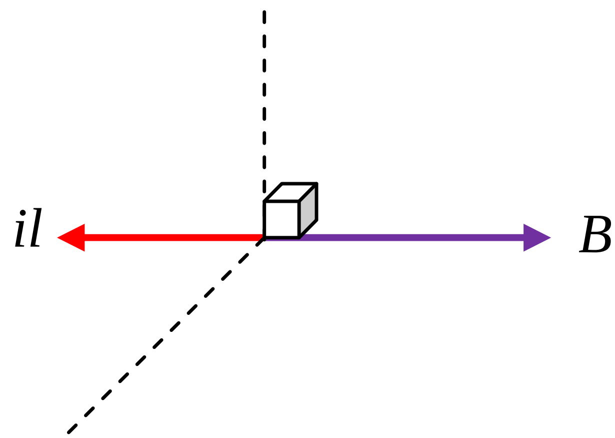


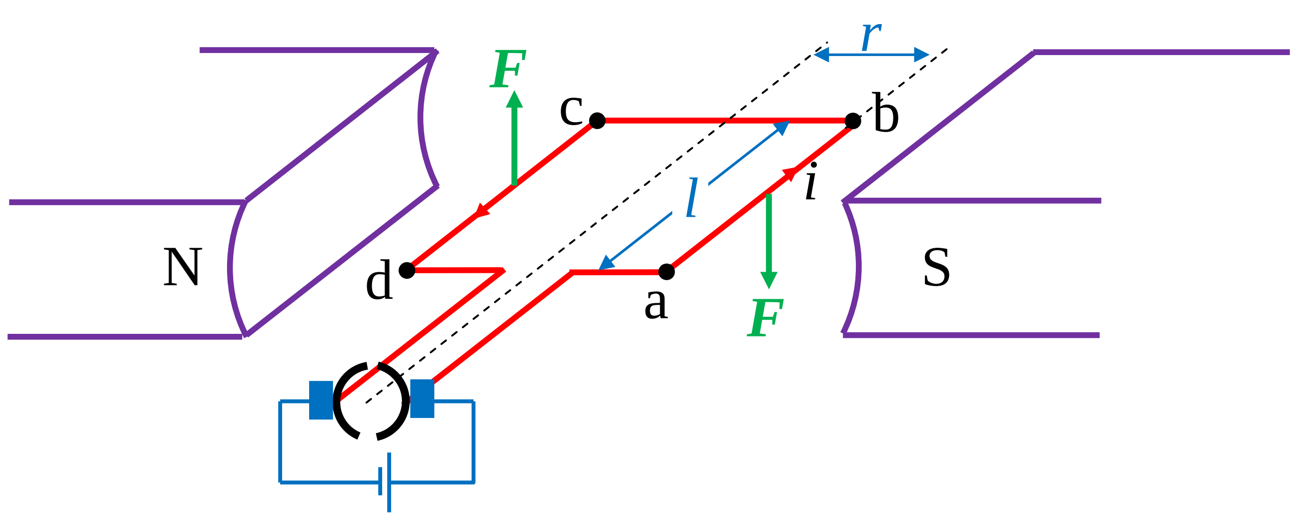
Calculation of Developed Torque

- **Side bc**: il is parallel to B

$$F_{bc} = 0$$

$$T_{bc} = 0$$



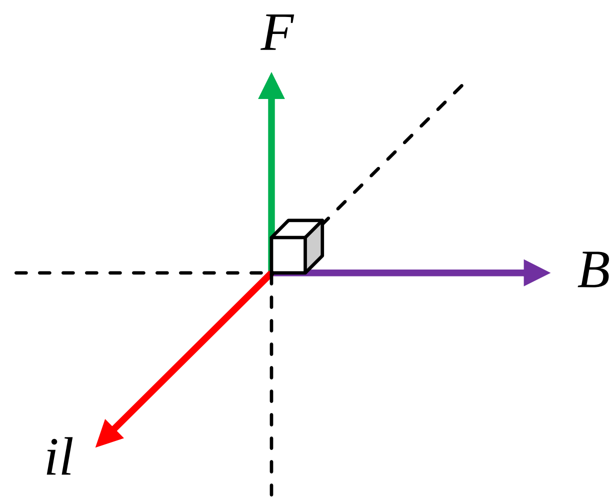


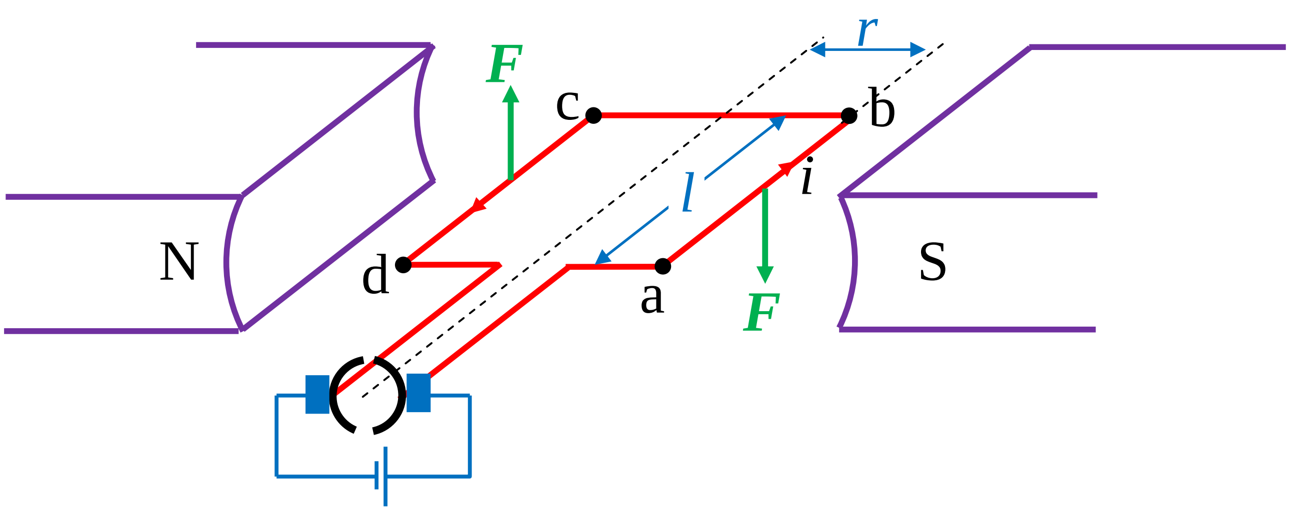
Calculation of Developed Torque

- **Side cd**: il is perpendicular to B

$$F_{cd} = ilB$$

$$T_{cd} = rilB$$



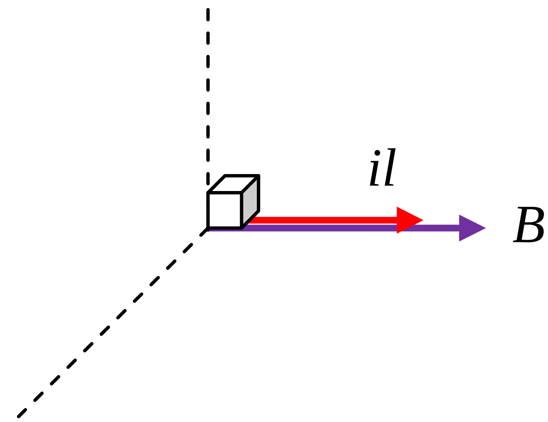


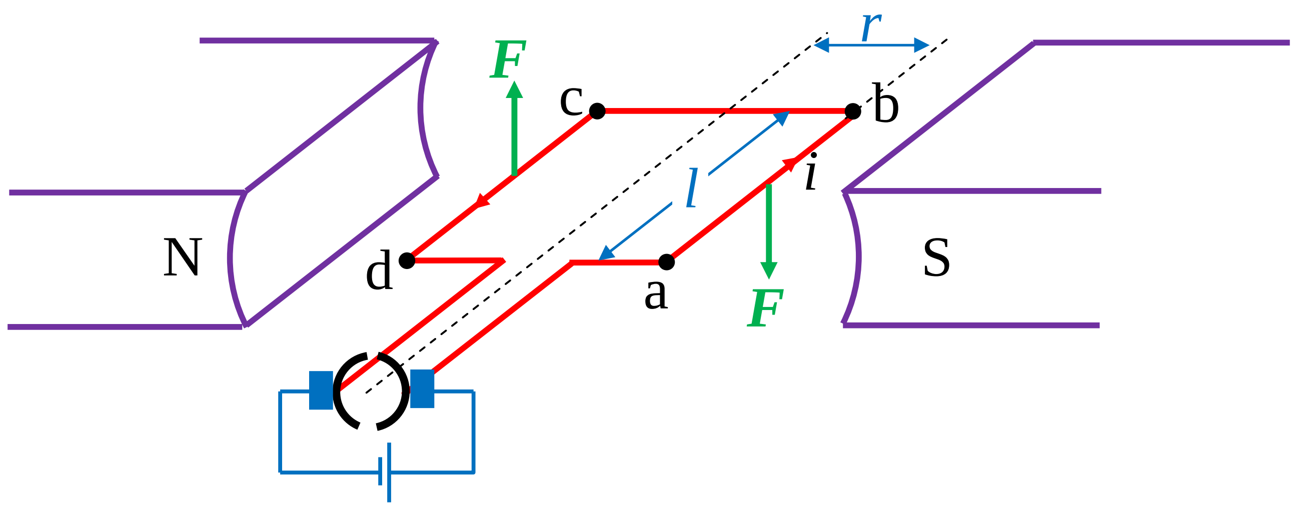
Calculation of Developed Torque

- **Side da**: il is parallel to B

$$F_{da} = 0$$

$$T_{da} = 0$$



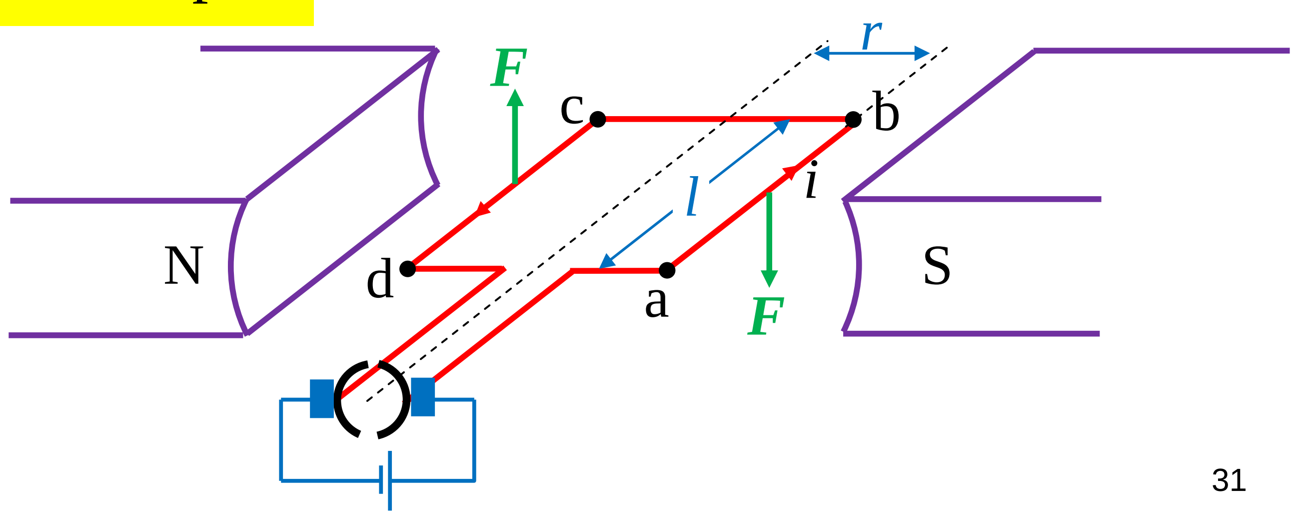


Calculation of Developed Torque

$$T_{total} = T_{ab} + T_{bc} + T_{cd} + T_{da} = \begin{cases} 2rilB & \text{under the pole} \\ 0 & \text{outside the pole} \end{cases}$$

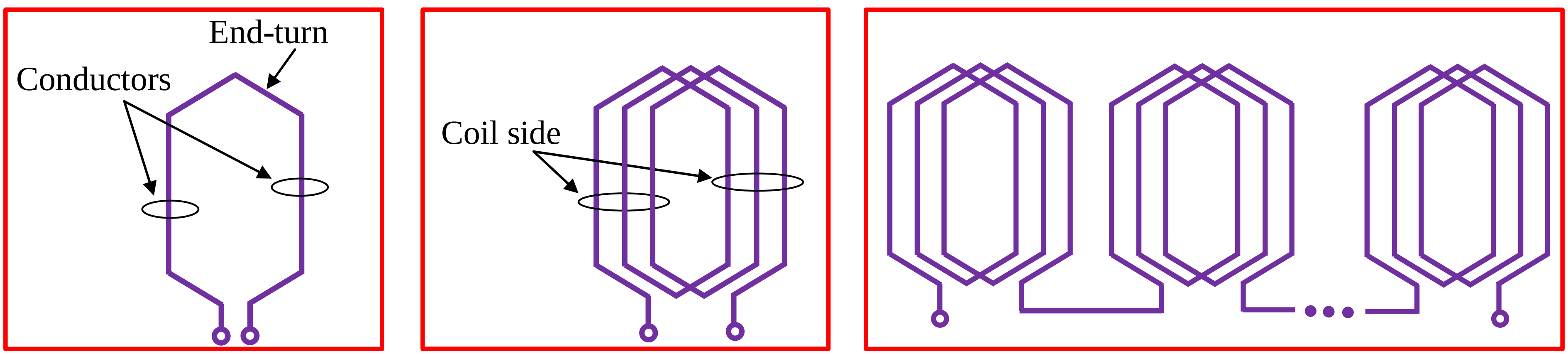
$$A_p = \pi r l \qquad \phi = A_p B$$

$$T_{total} = \begin{cases} \frac{2}{\pi}\phi i & \text{under the pole} \\ 0 & \text{outside the pole} \end{cases}$$



Armature Winding Topology

- **Turn:** consists of two conductors connected by an end-turn.
- **Coil**: consists of a number of turns connected in series.
- **Winding**: consists of several coils connected in series (or parallel)



Turn **coil** **Winding**

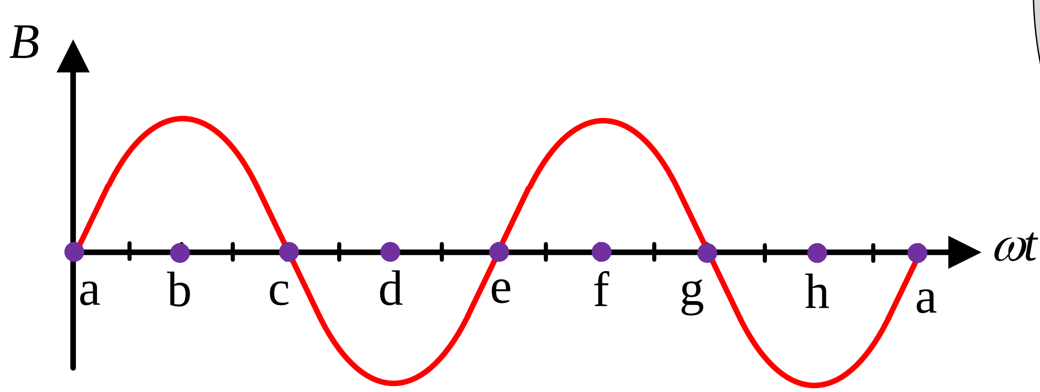
Electrical vs. Mechanical Angles

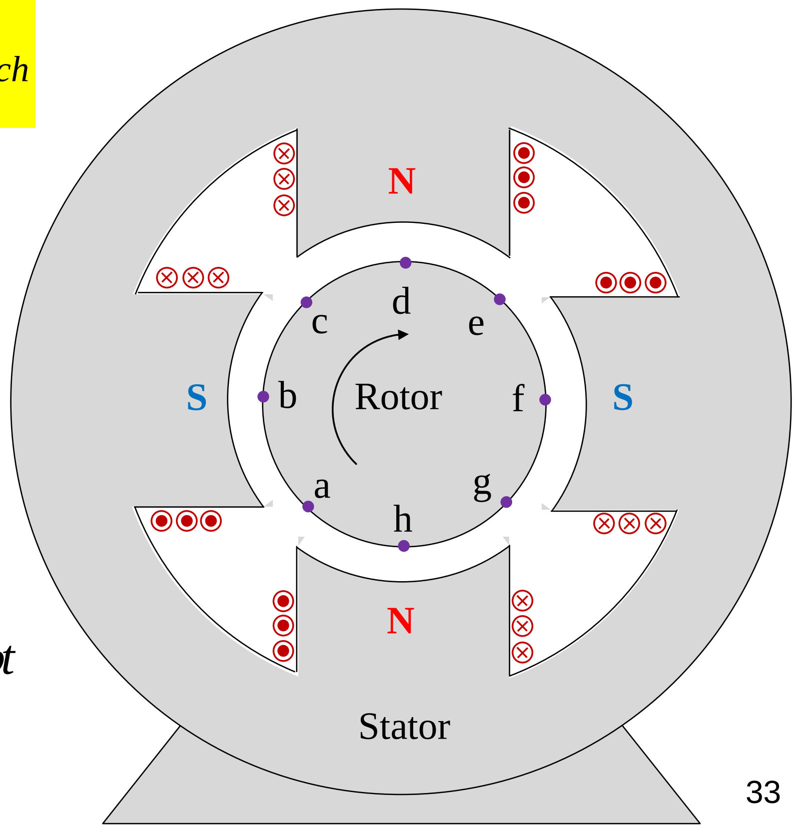
- In the following 4-pole machine, one revolution of rotor is equivalent of two cycles of flux density seen by a coil.

$$\theta_{elec} = \frac{P}{2}\theta_{mech}$$

$$\omega_{elec} = \frac{P}{2}\omega_{mech}$$

where P is the number of poles.



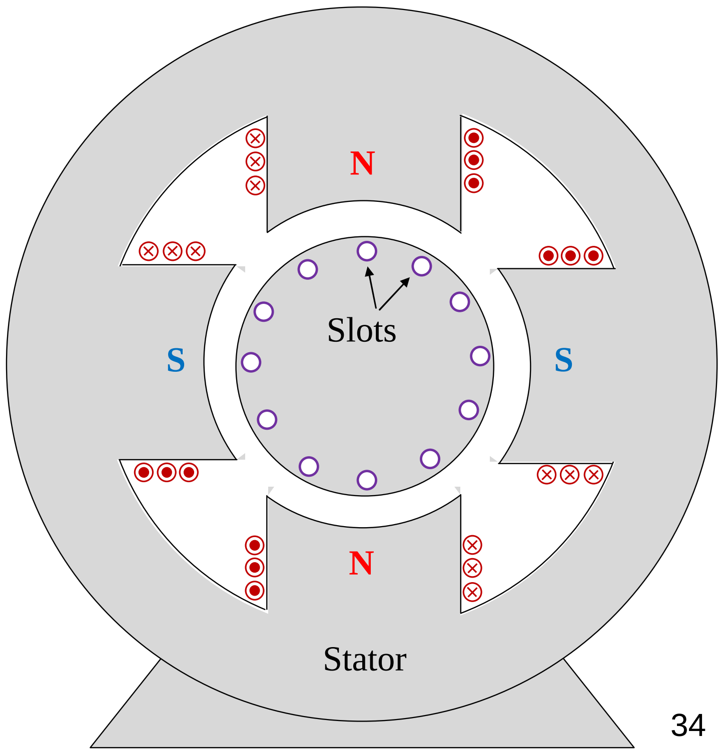


Definitions

- **Pole pitch**: is the angle between two adjacent (stator) poles in electrical angle. Pole pitch is always 180 electrical degrees regardless of the number of poles.
- **Pole pitch in number of slots**: is the number of rotor slots dedicated to each (stator) pole.

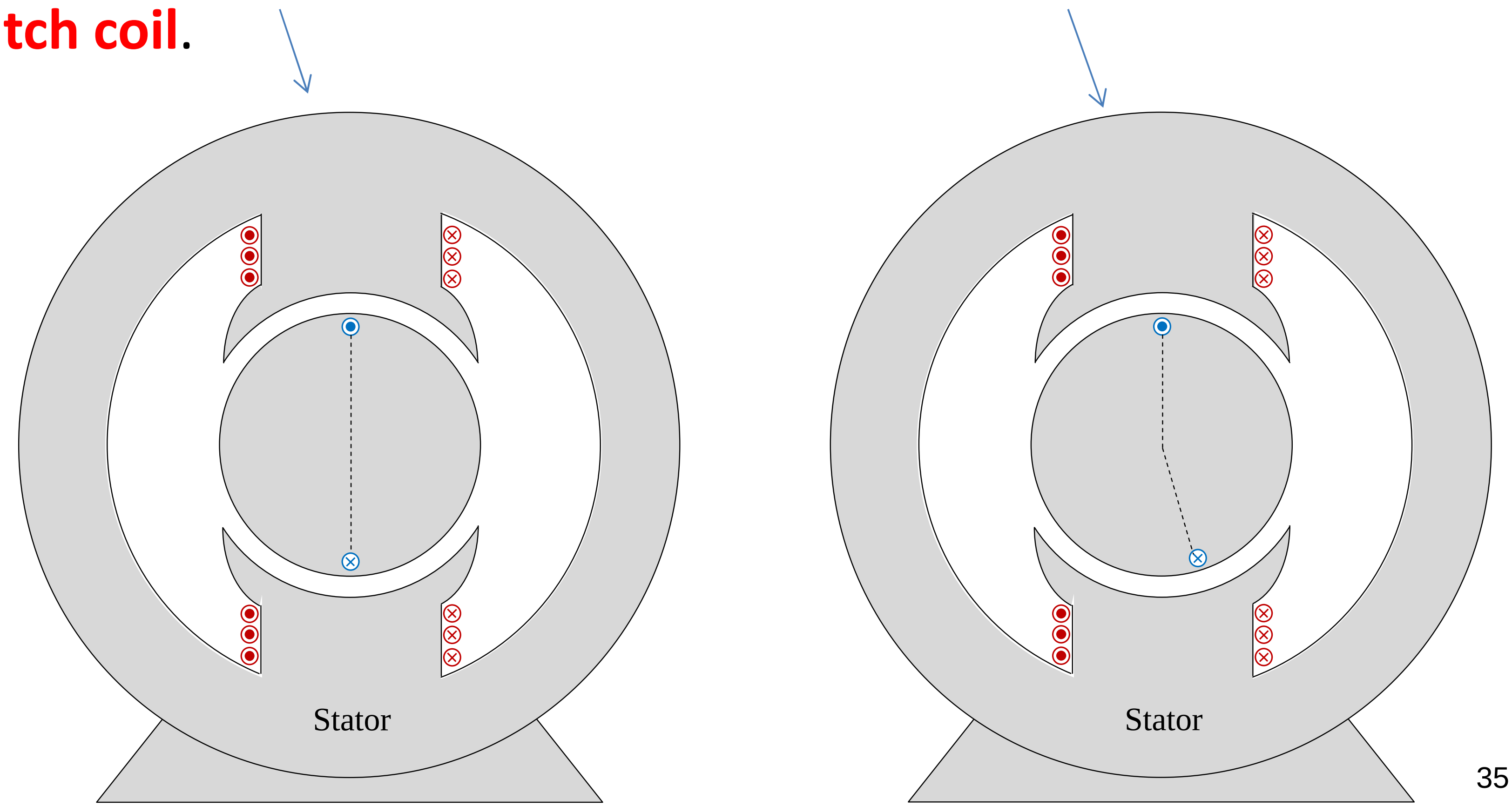
$$\text{Pole pitch in number of slots} = \frac{n_s}{P}$$

where n_s is the number of slots and P is the number of poles



Definitions

- **Coil pitch**: is the angle between two sides of one armature coil in electrical angle. If the coil pitch is 180 electrical degrees, the coil is a **full-pitch coil**; otherwise it is called **short**- or **chorded-pitch coil**.



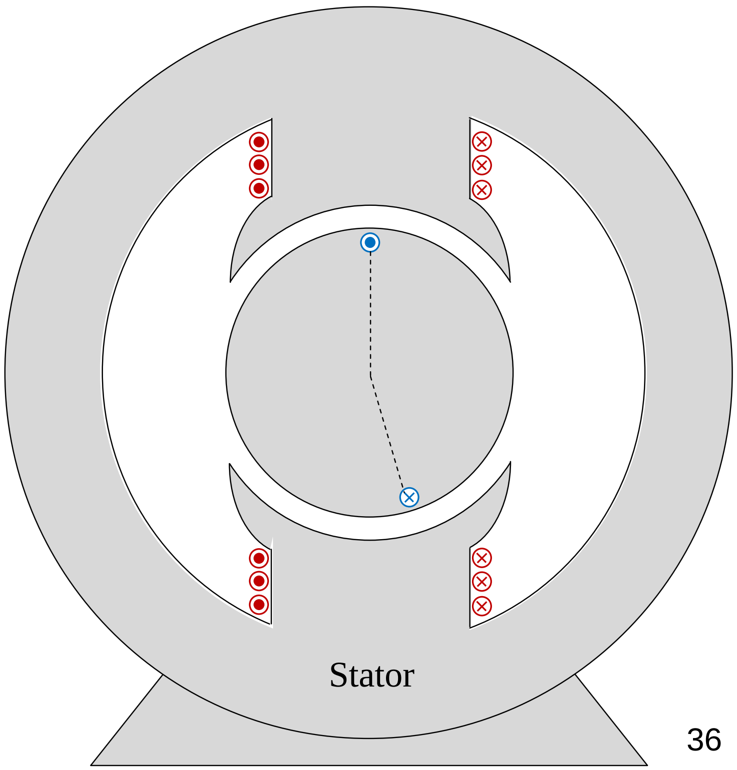
Definitions

- **Why is short-pitch coil used?**

- Although short-pitch coil decreases the induced voltage (by about 3%), it **decreases** the **disturbing harmonics** significantly (by about 70 to 80%).

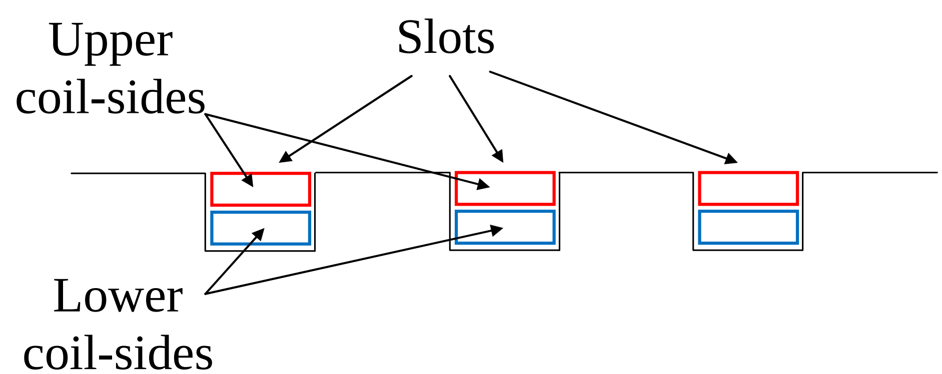
- Disturbing harmonics in electric machines are 5 and 7;

- In transformers, harmonic 3 is the most disturbing one.

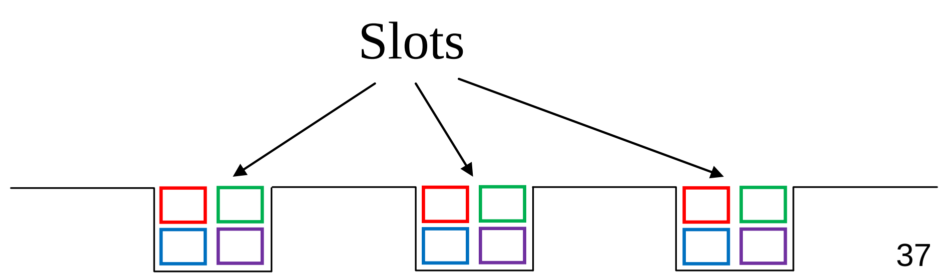


Multi-layer Armature Winding

- It is normal to have more than one coil-side in each rotor slot:
- **2-layer** (Double-layer) armature winding:



- **4-layer** armature winding:



Different Types of Armature Winding in DC Machines

- Two widely used winding topologies in DC machines are:
  1. **Simplex Lap winding**
  2. **Simplex wave winding**

- The difference between the two topologies is due to different connection of the coils to the commutator slabs.

- Assume each slot can accommodate two sides of two different coils. In this case the winding is a **double-layer** winding.

- In 2-layer winding, the **upper coil-sides** are shown by solid lines and numbered by odd numbers and the **lower coil-sides** are shown by dashed lines and numbered by even numbers.

Simplex Lap Winding

- In this case the **two ends of each coil** are connected to the **two adjacent commutator slabs**. Also the end-side of each coil is connected to the start-side of the next coil.

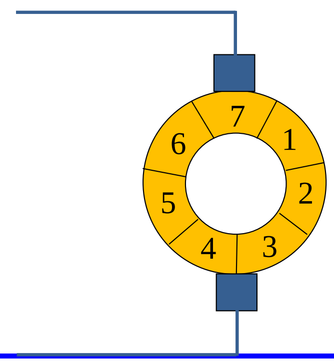
- **Example**

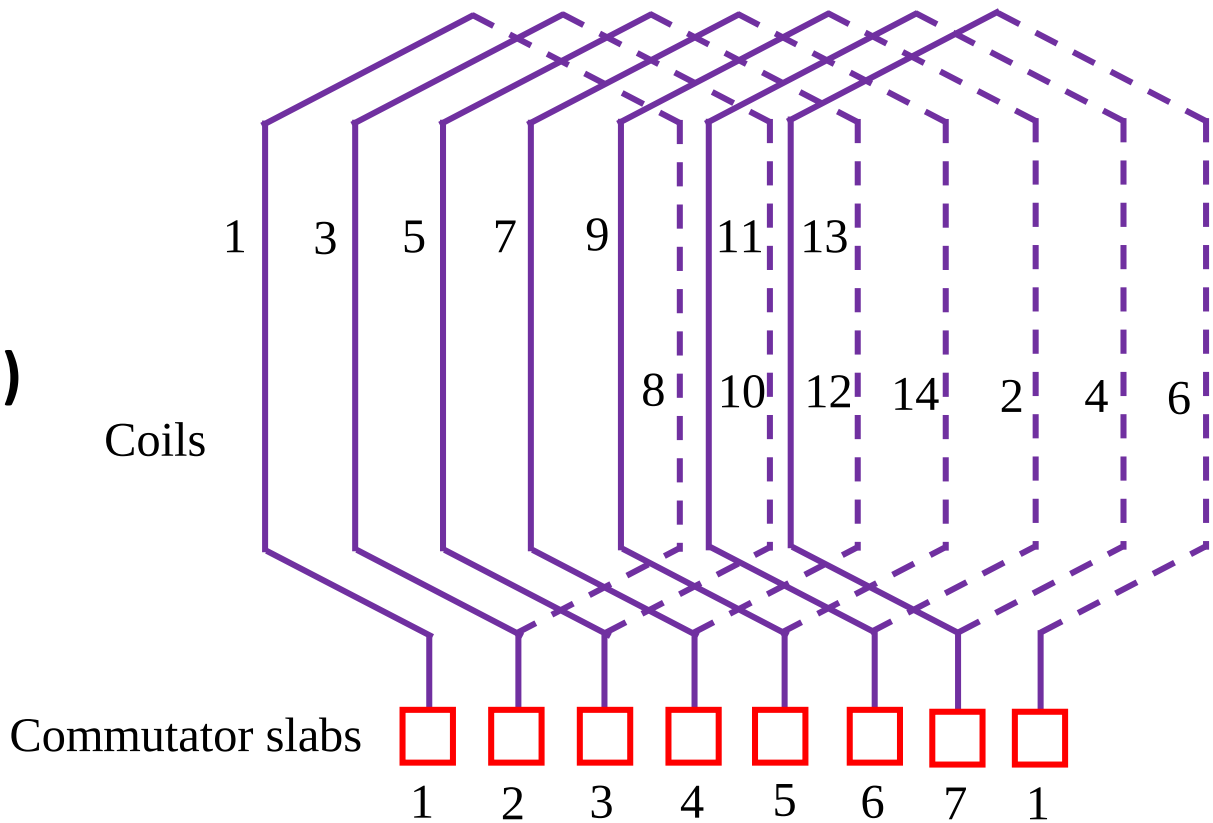
  7 Slots

  2-layer slot

  7 Coils (14 coil sides)

  7 commutator slabs





Simplex Wave Winding

- In this case the **two ends of each coil** are connected to the **two commutator slabs** which are far by about **360** electrical degrees.

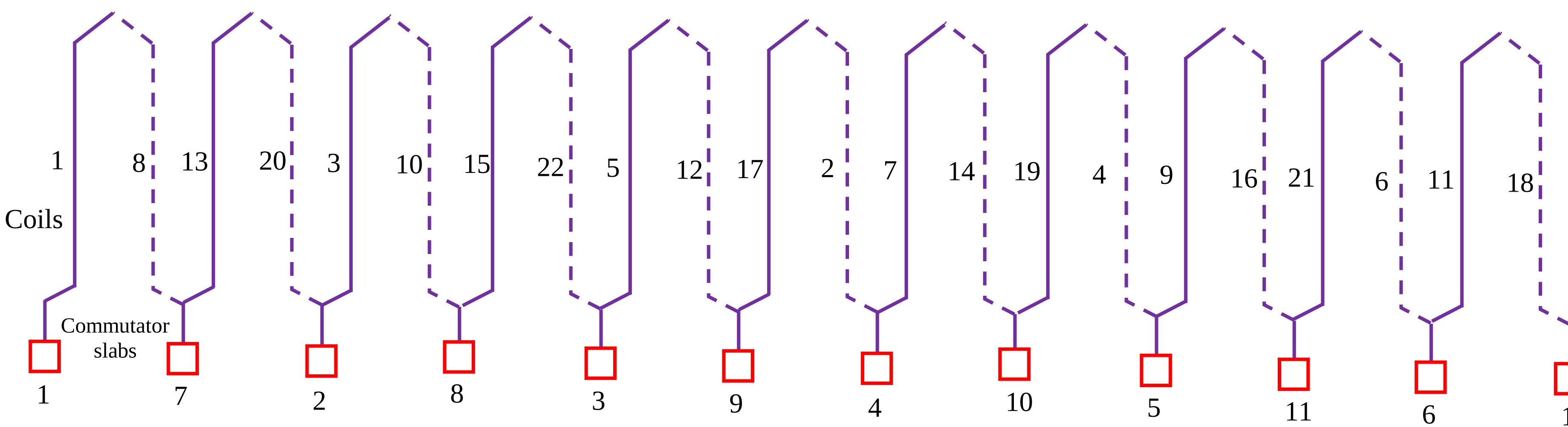
- **Example**

11 Slots;

2-layer slot;

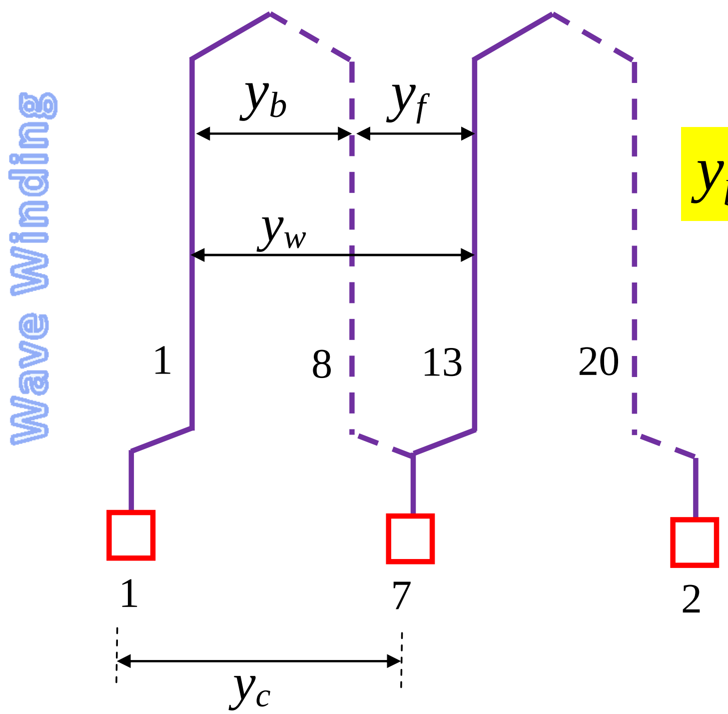
11 Coils (22 coil sides);

11 commutator slabs

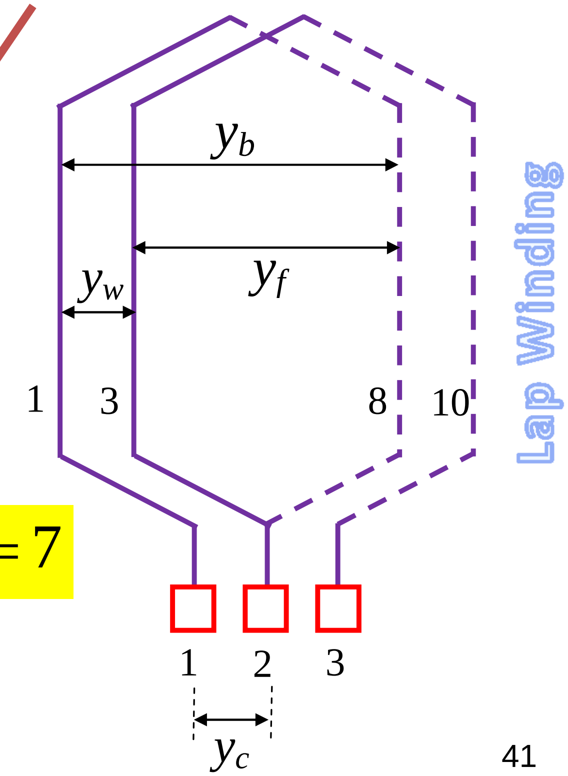


Specific Winding Parameters of DC Machines

- **Back-pitch**: (y_b) the distance between upper and lower coil-sides in terms of coil-side number. Back-pitch is always an odd number because upper side number is odd and lower side number is even.



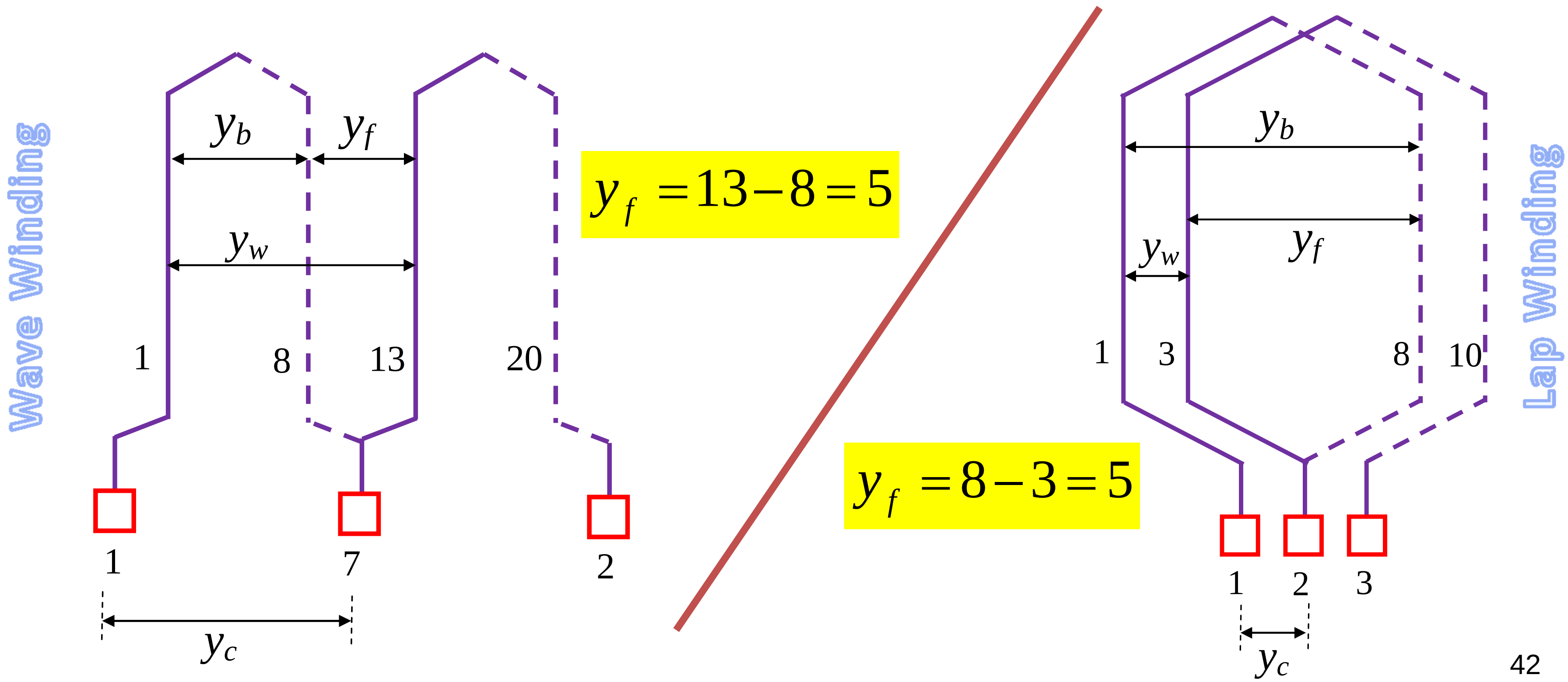
$$y_b = 8 - 1 = 7$$



$$y_b = 8 - 1 = 7$$

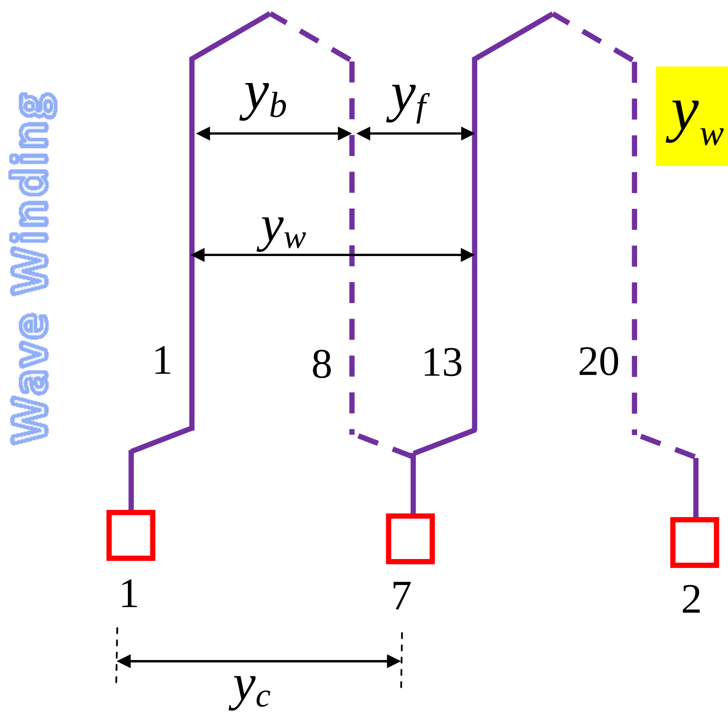
Specific Winding Parameters of DC Machines

- **Forward-pitch**: (y_f) the distance between two coil-sides of two coils connected to a commutator slab in terms of coil-side number.



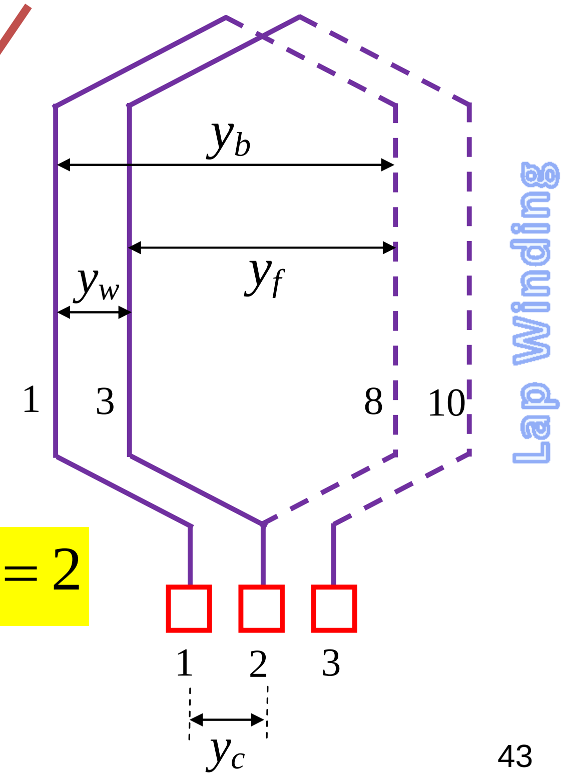
Specific Winding Parameters of DC Machines

- **Winding-pitch**: (y_w) the distance between two upper (or lower) coil-sides of two adjacent coils in terms of coil-side number. Winding-pitch is an even number. It is always 2 in lap winding.



$$y_w = y_b + y_f = 12$$

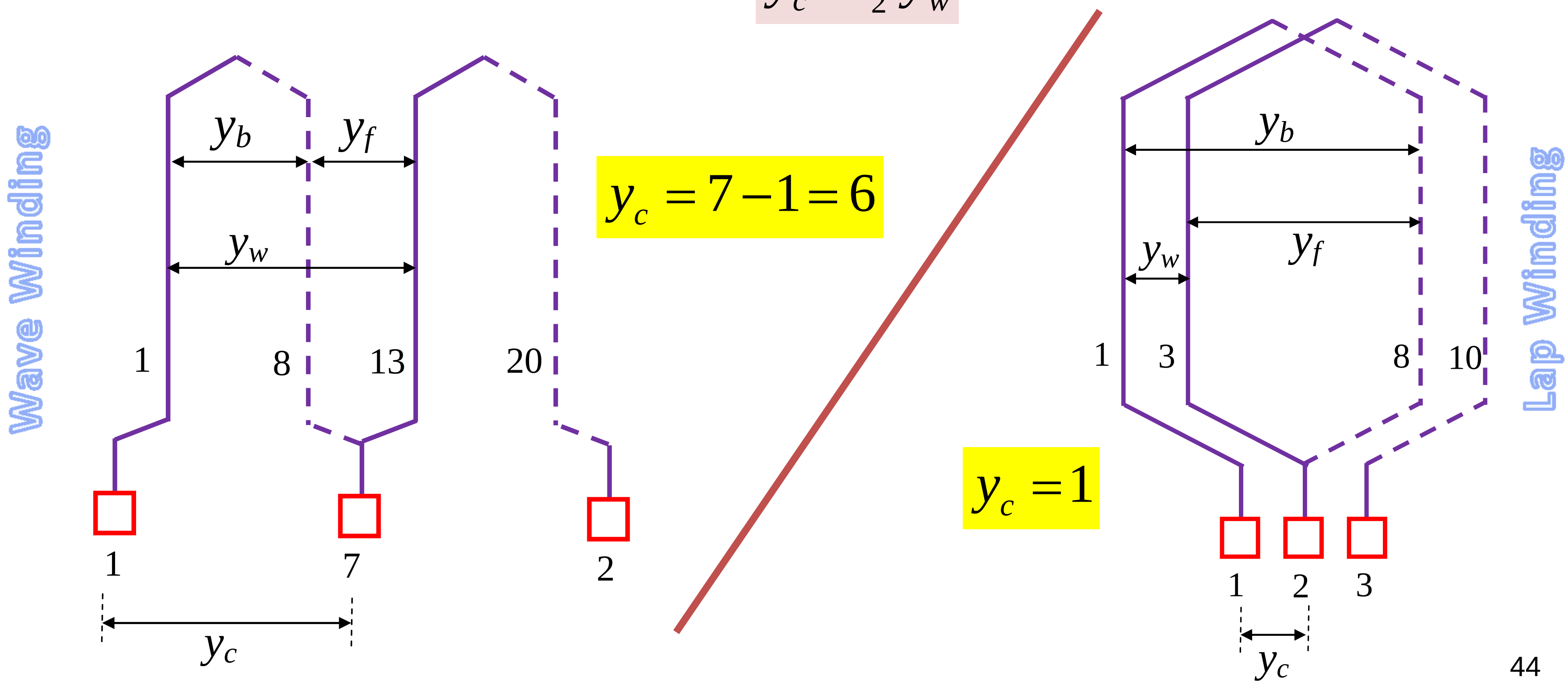
$$y_w = y_b - y_f = 2$$



Specific Winding Parameters of DC Machines

- **Commutator-pitch**: (y_c) the distance between two commutator slabs connected to a coil. It is always 1 for lap winding.

$$y_c = \tfrac{1}{2} y_w$$



Some Points

- The **number of commutator slabs** is the same as the **number of coils**, except for one-coil machines.
- In **lap winding** the **winding-pitch** is always **2**.
- In **lap winding** the **commutator-pitch** is always **1**.
- **Back-pitch** is an **odd** number.
- **Winding pitch** is an **even** number.
- In **2-layer** slot machines, the **number of coils** is the same as the **number of slots**.
- In general $n_s = 2n_c / n_l$ where n_s is the number of slots, n_c is the number of coils and n_l is the number of layers.

Armature Winding Procedure
Lap Winding Method

Assume a DC Machine with ***P* poles** and **n_s slots** with **2-layer**:

1. Therefore the **number of coils**, n_c, is the same as n_s.

2. Also the **number of commutators** is the same as n_c.

3. The following expression calculates the **back-pitch**

$$y_b = \frac{n_c}{P/2} \pm k$$

Since back-pitch should be an integer odd number, k is used to serve the purpose.

Armature Winding Procedure

Lap Winding Method

Assume a DC Machine with P **poles** and n_s **slots** with **2-layer**:

4. **Winding-pitch** is always 2 in lap winding: $y_w = 2$

5. **Forward-pitch** is obtained as: $y_f = y_b - y_w$

6. **Commutator-pitch** is always 1 in lap winding: $y_c = 1$

Lap Winding

Example: Consider a DC Machine with **6 poles** and **40 slots** with **2-layer**. Obtain the lap winding parameters.

1. Number of coils: $n_c = n_s = 40$
2. Number of commutator slabs is 40.
3. Back-pitch $$y_b = \frac{n_c}{P/2} \pm k = \frac{40}{3} - 0.3 = 13$$
4. Winding-pitch $y_w = 2$
5. Forward-pitch $y_f = y_b - y_w = 13 - 2 = 11$
6. Commutator-pitch $y_c = 1$

Lap Winding

Example: Consider a DC Machine with **4 poles** and **12 slots** with **2-layer**. Perform the lap winding.

1. Number of coils: $n_c = \frac{1}{2} n_l n_s = 12$
2. Number of commutator slabs is 12.
3. Back-pitch $$y_b = \frac{n_c}{P/2} \pm k = \frac{12}{2} + 1 = 7$$
4. Winding-pitch $y_w = 2$
5. Forward-pitch $y_f = y_b - y_w = 7 - 2 = 5$
6. Commutator-pitch $y_c = 1$

Lap Winding

Solution: $n_c = 12$ $y_b = 7$ $y_w = 2$ $y_f = 5$ $y_c = 1$

Commutator table: In this table, we start from the first commutator slab number (1) and add y_c to reach to the same slab:

$$1 \xrightarrow{+y_c} 2 \xrightarrow{+y_c} 3 \xrightarrow{+y_c} 4 \xrightarrow{+y_c} 5 \xrightarrow{+y_c} 6 \xrightarrow{+y_c} 7 \xrightarrow{+y_c} 8$$

$$\xrightarrow{+y_c} 9 \xrightarrow{+y_c} 10 \xrightarrow{+y_c} 11 \xrightarrow{+y_c} 12 \xrightarrow{+y_c} 13 - 12 = 1$$

This table shows the sequence of the coil connections to the commutator slabs.

In the lap winding, since commutator-pitch is 1, the commutator slab table starts from 1 and with incremental of 1 reaches to the last slab.

Lap Winding

Solution: $n_c = 12$ $y_b = 7$ $y_w = 2$ $y_f = 5$ $y_c = 1$

Winding table: In this table, we start from the first coil-side number (1) and add y_b (if the result is greater than $2n_c$, the result is subtracted by $2n_c$). In the next stage, y_f is subtracted from the result. This procedure continues to return to coil-side 1.

$$(1 \xrightarrow{+y_b=7} 8) \xrightarrow{-y_f=-5} (3 \xrightarrow{+y_b} 10) \xrightarrow{-y_f} (5 \xrightarrow{+y_b} 12) \xrightarrow{-y_f} (7 \xrightarrow{+y_b} 14)$$
$$\xrightarrow{-y_f} (9 \xrightarrow{+y_b} 16) \xrightarrow{-y_f} (11 \xrightarrow{+y_b} 18) \xrightarrow{-y_f} (13 \xrightarrow{+y_b} 20) \xrightarrow{-y_f}$$
$$(15 \xrightarrow{+y_b} 22) \xrightarrow{-y_f} (17 \xrightarrow{+y_b} 24) \xrightarrow{-y_f} (19 \xrightarrow{+y_b} 26 - 24 = 2) \xrightarrow{-y_f}$$
$$(21 \xrightarrow{+y_b} 28 - 24 = 4) \xrightarrow{-y_f} (23 \xrightarrow{+y_b} 30 - 24 = 6) \xrightarrow{-y_f} (1$$

The pair of values in round brackets are corresponding to one coil.

Lap Winding

Solution: $n_c = 12$ $y_b = 7$ $y_w = 2$ $y_f = 5$ $y_c = 1$

$$1 \xrightarrow{+y_c} 2 \xrightarrow{+y_c} 3 \xrightarrow{+y_c} 4 \xrightarrow{+y_c} 5 \xrightarrow{+y_c} 6 \xrightarrow{+y_c} 7 \xrightarrow{+y_c} 8$$

$$\xrightarrow{+y_c} 9 \xrightarrow{+y_c} 10 \xrightarrow{+y_c} 11 \xrightarrow{+y_c} 12$$

$$(1 \xrightarrow{+y_b=7} 8) \xrightarrow{-y_f=-5} (3 \xrightarrow{+y_b} 10) \xrightarrow{-y_f} (5 \xrightarrow{+y_b} 12) \xrightarrow{-y_f} (7 \xrightarrow{+y_b} 14)$$

$$\xrightarrow{-y_f} (9 \xrightarrow{+y_b} 16) \xrightarrow{-y_f} (11 \xrightarrow{+y_b} 18) \xrightarrow{-y_f} (13 \xrightarrow{+y_b} 20) \xrightarrow{-y_f}$$

$$(15 \xrightarrow{+y_b} 22) \xrightarrow{-y_f} (17 \xrightarrow{+y_b} 24) \xrightarrow{-y_f} (19 \xrightarrow{+y_b} 26-24=2) \xrightarrow{-y_f}$$

$$(21 \xrightarrow{+y_b} 28-24=4) \xrightarrow{-y_f} (23 \xrightarrow{+y_b} 30-24=6) \xrightarrow{-y_f} (1$$

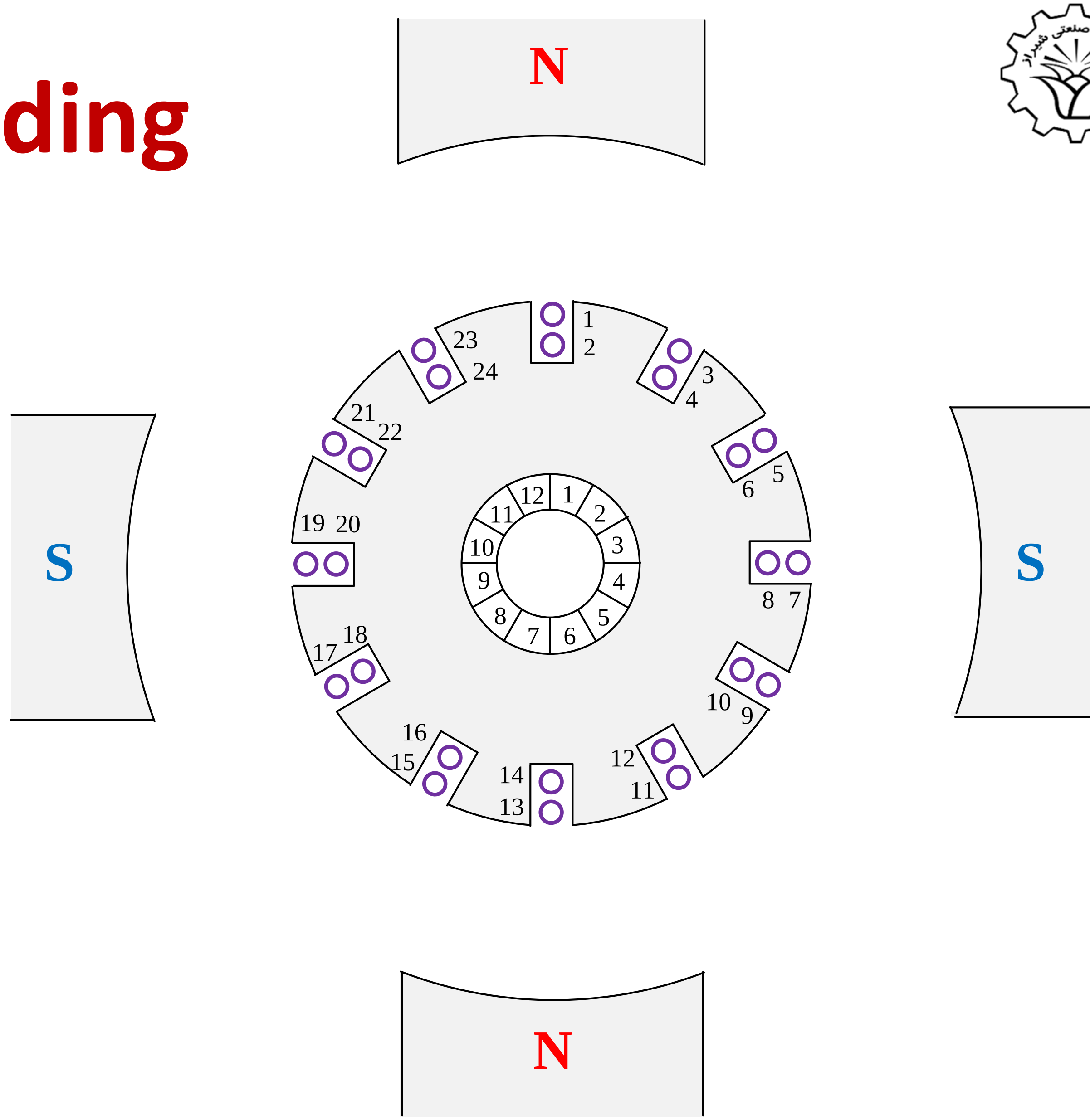
Lap Winding

Machine structure

$n_c = 12$

$p = 4$

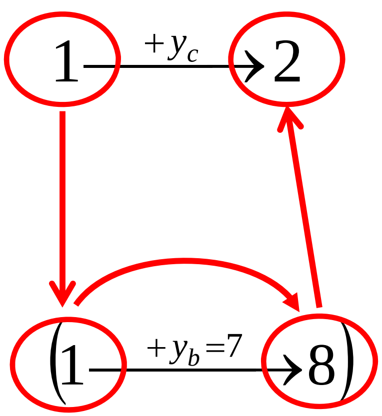
$n_l = 2$



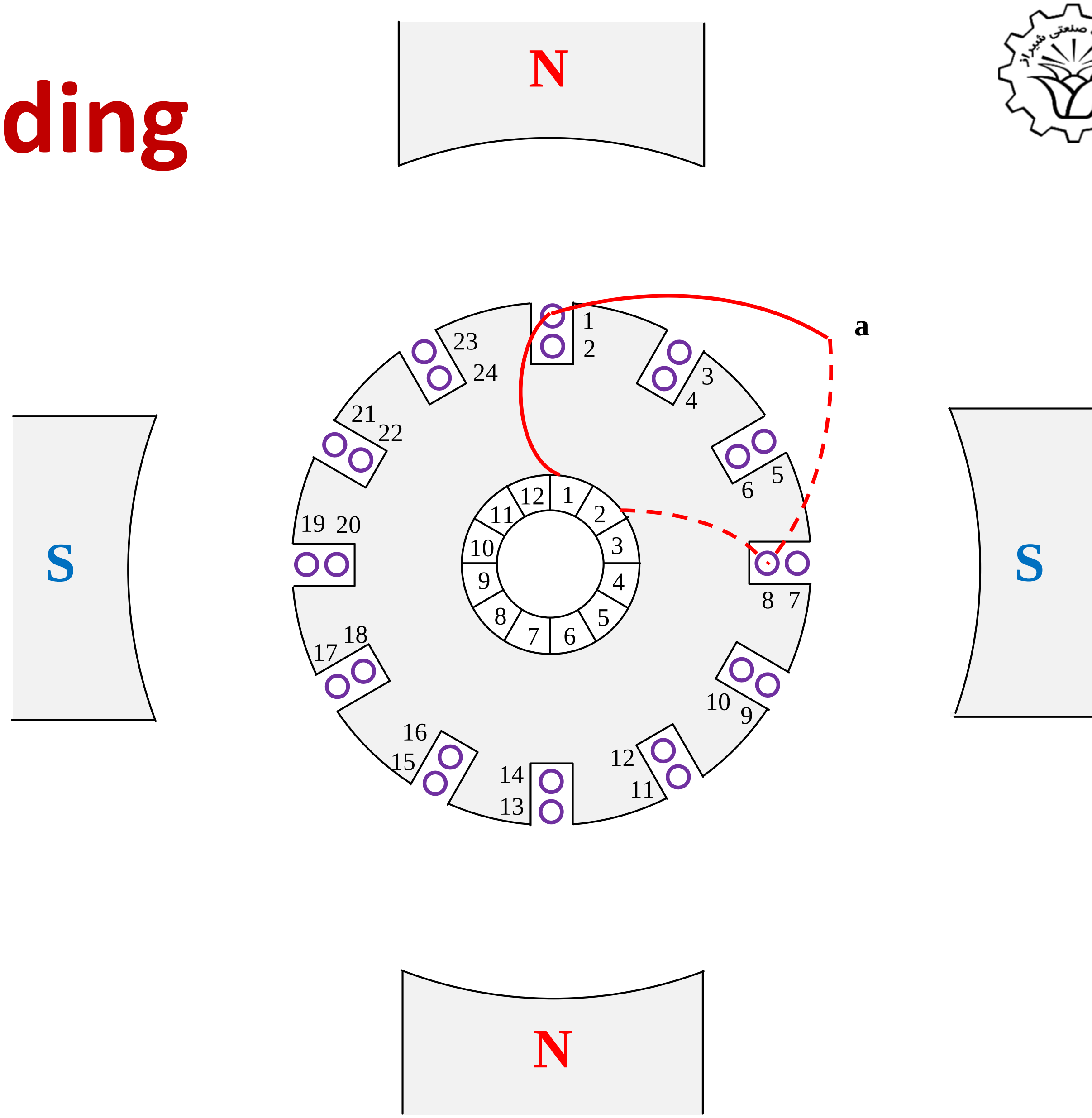
Lap Winding

First coil

Commutator table



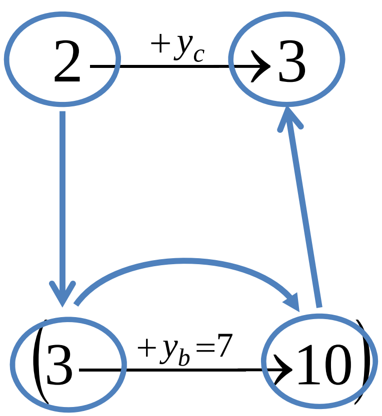
Winding table



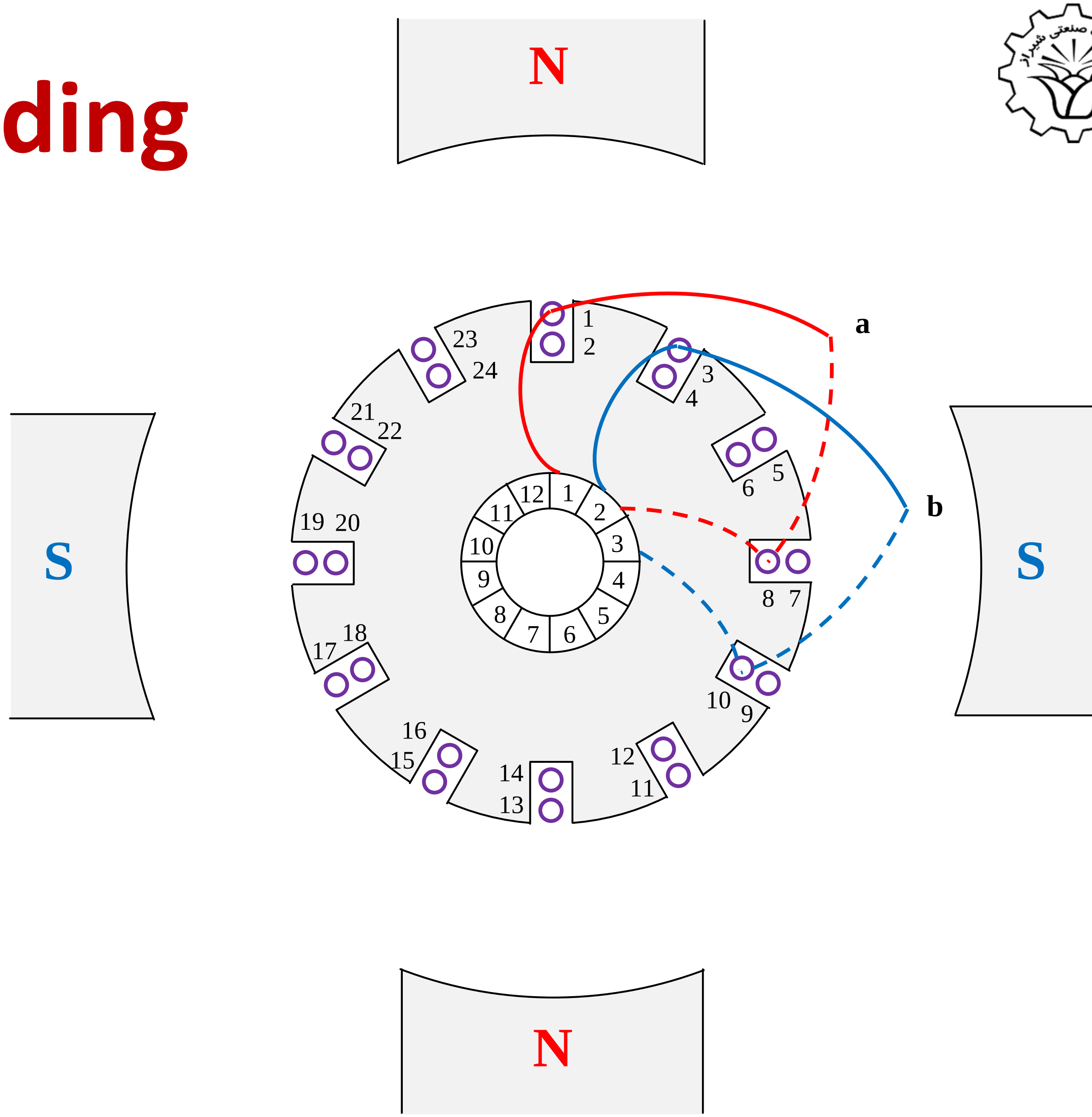
Lap Winding

Second coil

Commutator table

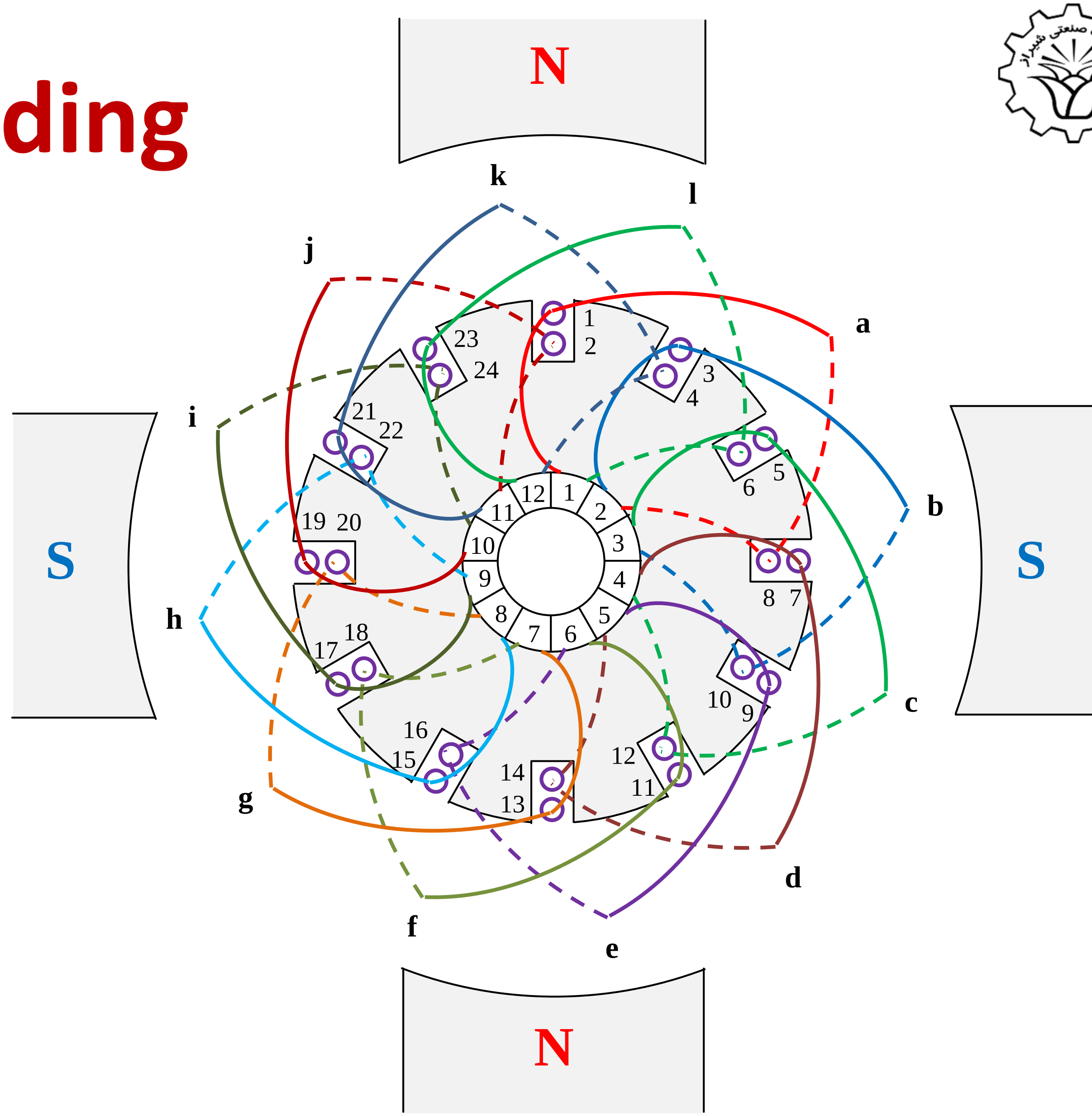


Winding table



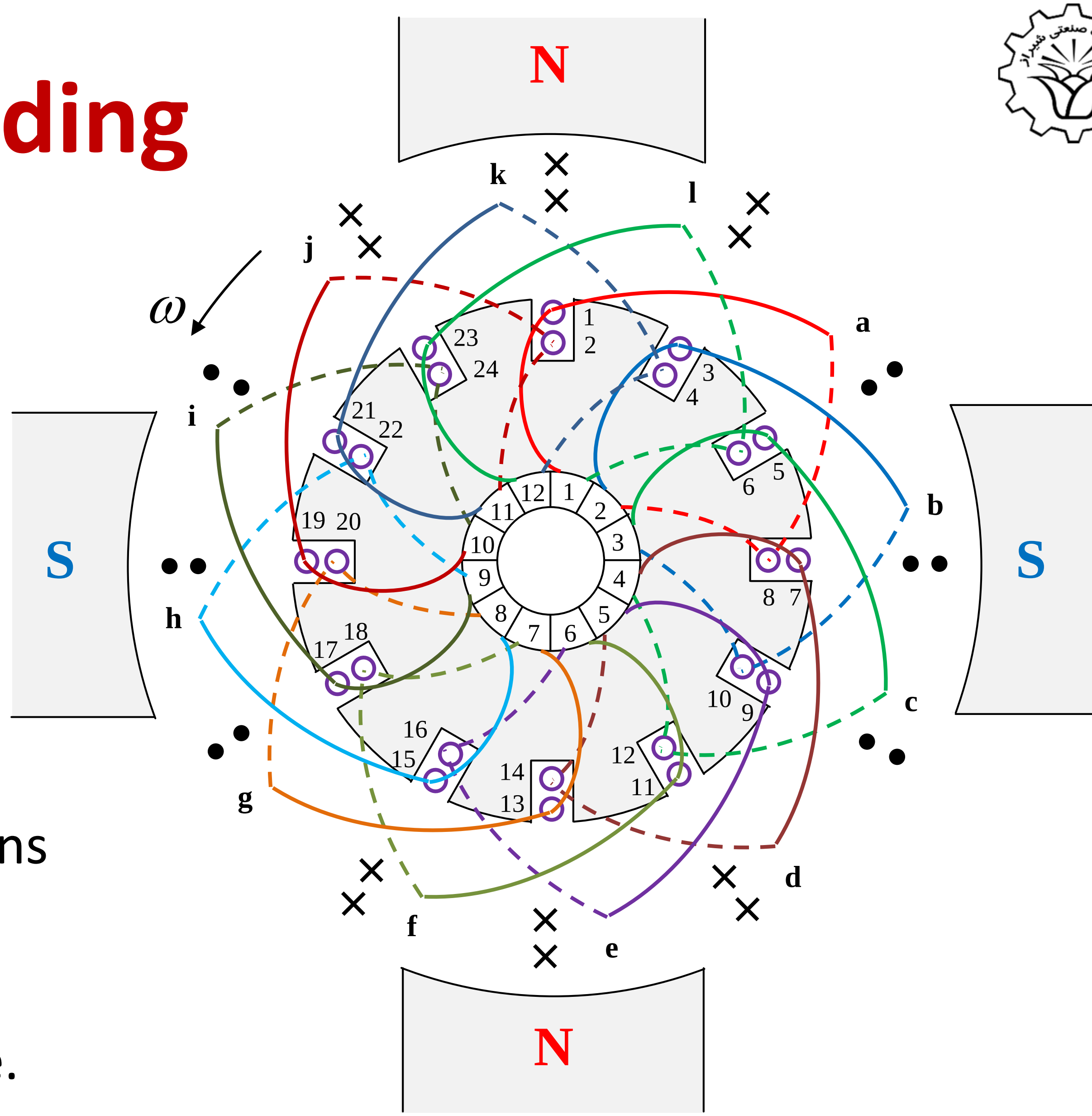
Lap Winding

All coils



Lap Winding

Brushes location:



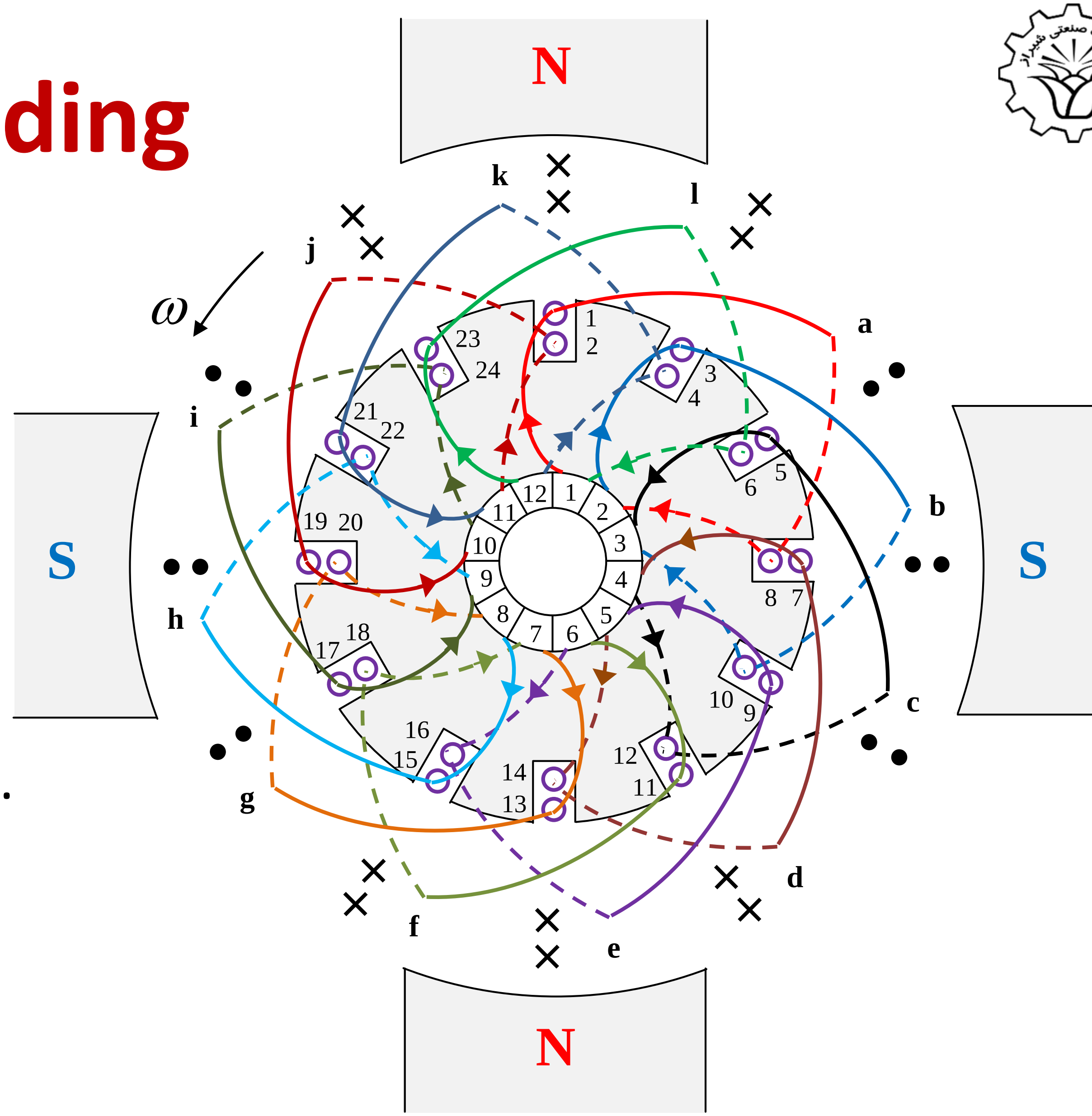
Find the direction of current in each coil based on their positions under N or S pole:

Under N into the page.

Under S out of the page.

Lap Winding

Brushes location:



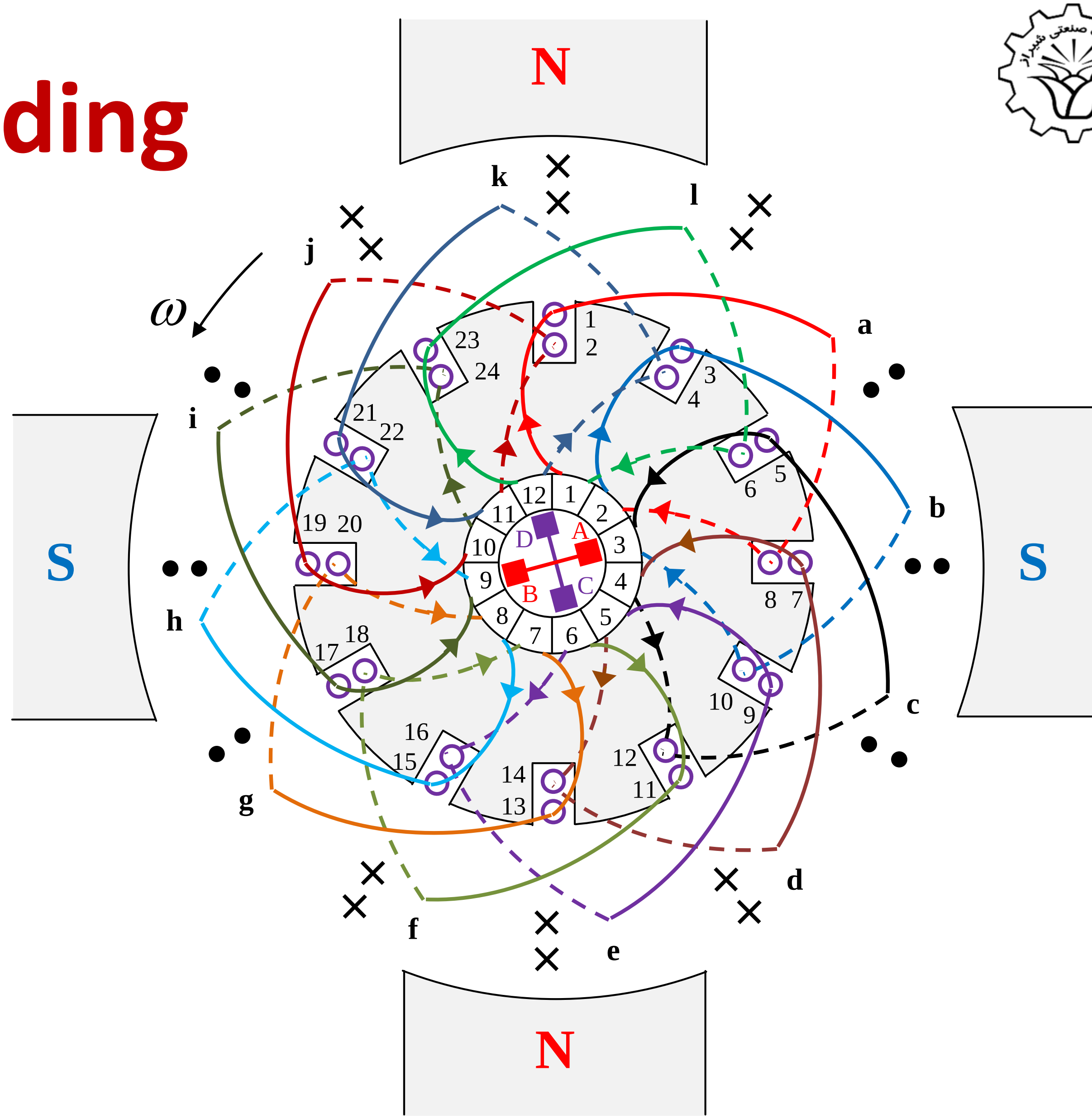
Draw the current directions on the coils.

Find those slabs in which both currents enter or from which both currents leave.

Lap Winding

Brushes location:

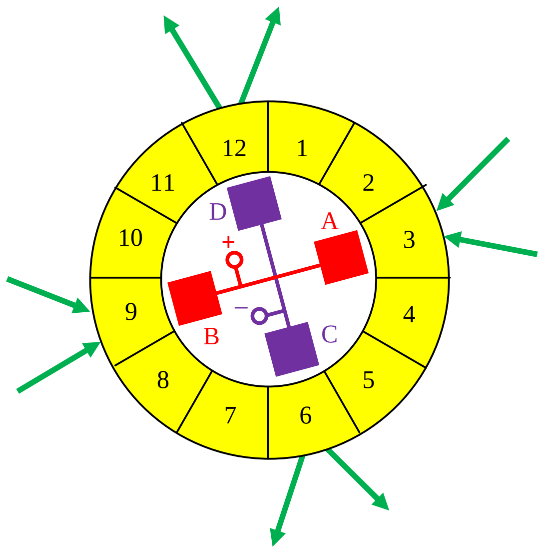
The slabs in which both currents enter or from which both currents leave are good candidates for brushes locations.

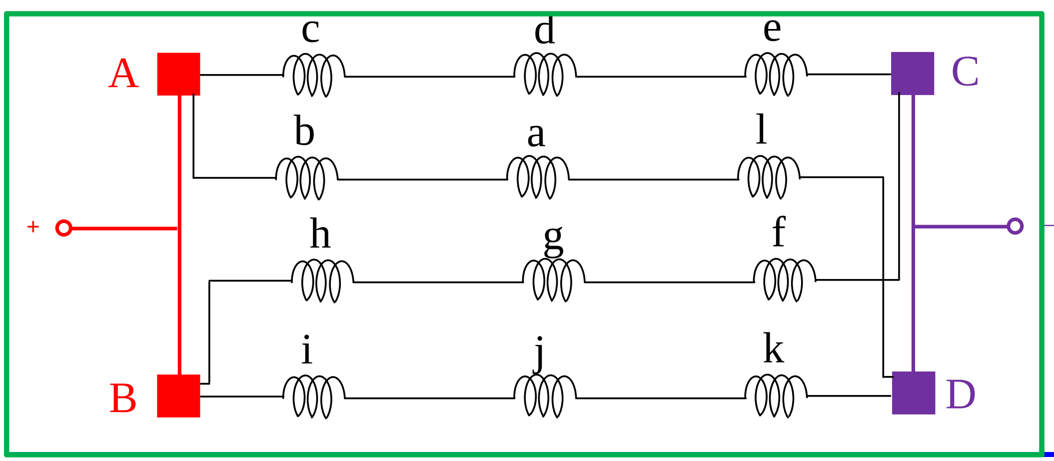


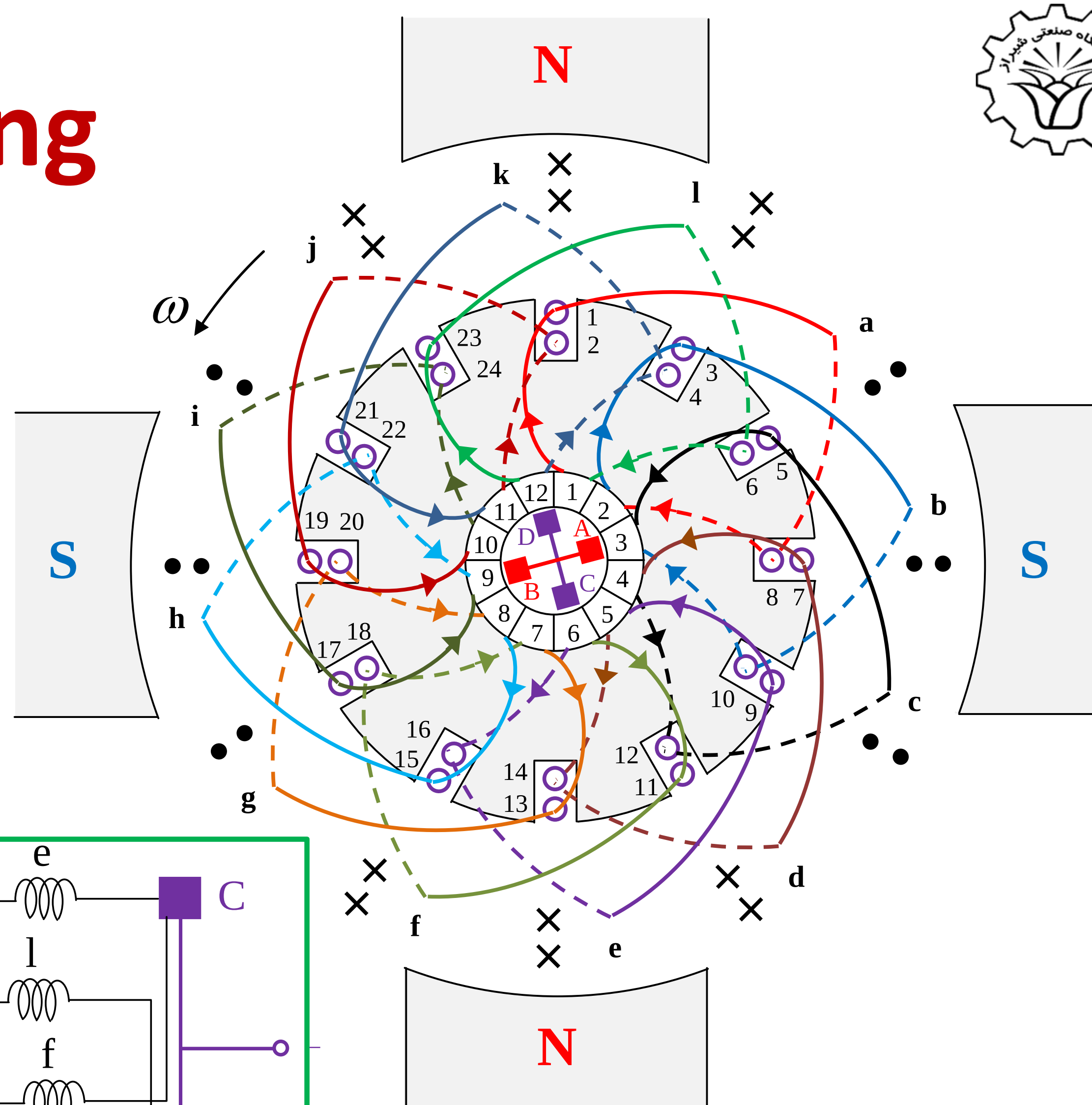
Note that slabs are rotating with rotor but brushes are stationary.

Lap Winding

Coil connections







Simplex Lap Winding Points

- In simplex lap winding:

  The number of **parallel paths** = The number of **poles**

  = The number of **brushes**

- Lap winding is used in machines with **high number of poles** and **high current** with relatively **low voltage**.

- Since in machines with high number of poles, the number of parallel paths is also high, therefore the current level is high.

Armature Winding Procedure
Wave Winding Method

Assume a DC Machine with ***P*** **poles** and n_s **slots** with **2-layer**:

1. Therefore the **number of coils**, n_c, is the same as n_s.

2. Also the **number of commutators** is the same as n_c.

3. The following expression calculates the **commutator-pitch**

$$y_c = \frac{n_c \pm k}{P/2}$$

Since commutator-pitch should be an integer number, k is used to serve the purpose.

Armature Winding Procedure
Wave Winding Method

Assume a DC Machine with P **poles** and n_s **slots** with **2-layer**:

4. Winding-pitch:

$$y_w = 2y_c$$

5. Back-pitch and **forward-pitch** are obtained as: $y_f + y_b = y_w$

Note that both back-pitch and forward-pitch should be odd numbers.

Since there are several option, those with smallest difference between back-pitch and forward-pitch are the best.

Wave Winding

Example: Consider a DC Machine with **4 poles** and **11 slots** with **2-layer**. Obtain the wave winding parameters and winding topology.

1. Number of coils: $n_c = n_s = 11$
2. Number of commutator slabs is 11.
3. Commutator-pitch $y_c = \frac{n_c \pm k}{P/2} = \frac{11+1}{2} = 6$
4. Winding-pitch $y_w = 2y_c = 12$
5. Back-pitch and Forward-pitch

$$y_f + y_b = y_w = 12 \quad \begin{cases} y_b = 7 \\ y_f = 5 \end{cases}$$

Wave Winding

Solution: $n_c = 11$ $y_b = 7$ $y_w = 12$ $y_f = 5$ $y_c = 6$

Commutator table: In this table, we start from the first commutator slab number (1) and add y_c to reach until return to the first slab:

$$1 \xrightarrow{+y_c=6} 7 \xrightarrow{+y_c} 13-11=2 \xrightarrow{+y_c} 8 \xrightarrow{+y_c} 14-11=3$$

$$\xrightarrow{+y_c} 9 \xrightarrow{+y_c} 15-11=4 \xrightarrow{+y_c} 10 \xrightarrow{+y_c} 16-11=5$$

$$\xrightarrow{+y_c} 11 \xrightarrow{+y_c} 17-11=6 \xrightarrow{+y_c} 12-11=1$$

This table shows the sequence of the coil connection to the commutator slabs.

Wave Winding

Solution: $n_c = 11$ $y_b = 7$ $y_w = 12$ $y_f = 5$ $y_c = 6$

Winding table: In this table, we start from the first coil-side number (1) and add y_b (if the result is greater than $2n_c$, the result is subtracted by $2n_c$). In the next stage, y_f is added to the result. This procedure continues to return to coil-side 1.

$$\left(1 \xrightarrow{+y_b=7} 8\right) \xrightarrow{+y_f=5} \left(13 \xrightarrow{+y_b} 20\right) \xrightarrow{+y_f} \left(25-22=3 \xrightarrow{+y_b} 10\right) \xrightarrow{+y_f}$$

$$\left(15 \xrightarrow{+y_b} 22\right) \xrightarrow{+y_f} \left(27-22=5 \xrightarrow{+y_b} 12\right) \xrightarrow{+y_f} \left(17 \xrightarrow{+y_b} 24-22=2\right)$$

$$\xrightarrow{+y_f} \left(7 \xrightarrow{+y_b} 14\right) \xrightarrow{+y_f} \left(19 \xrightarrow{+y_b} 26-22=4\right) \xrightarrow{+y_f} \left(9 \xrightarrow{+y_b} 16\right)$$

$$\xrightarrow{+y_f} \left(21 \xrightarrow{+y_b} 28-22=6\right) \xrightarrow{+y_f} \left(11 \xrightarrow{+y_b} 18\right) \xrightarrow{+y_f} \left(23-22=1\right.$$

The pair of values in round brackets are corresponding to one coil.

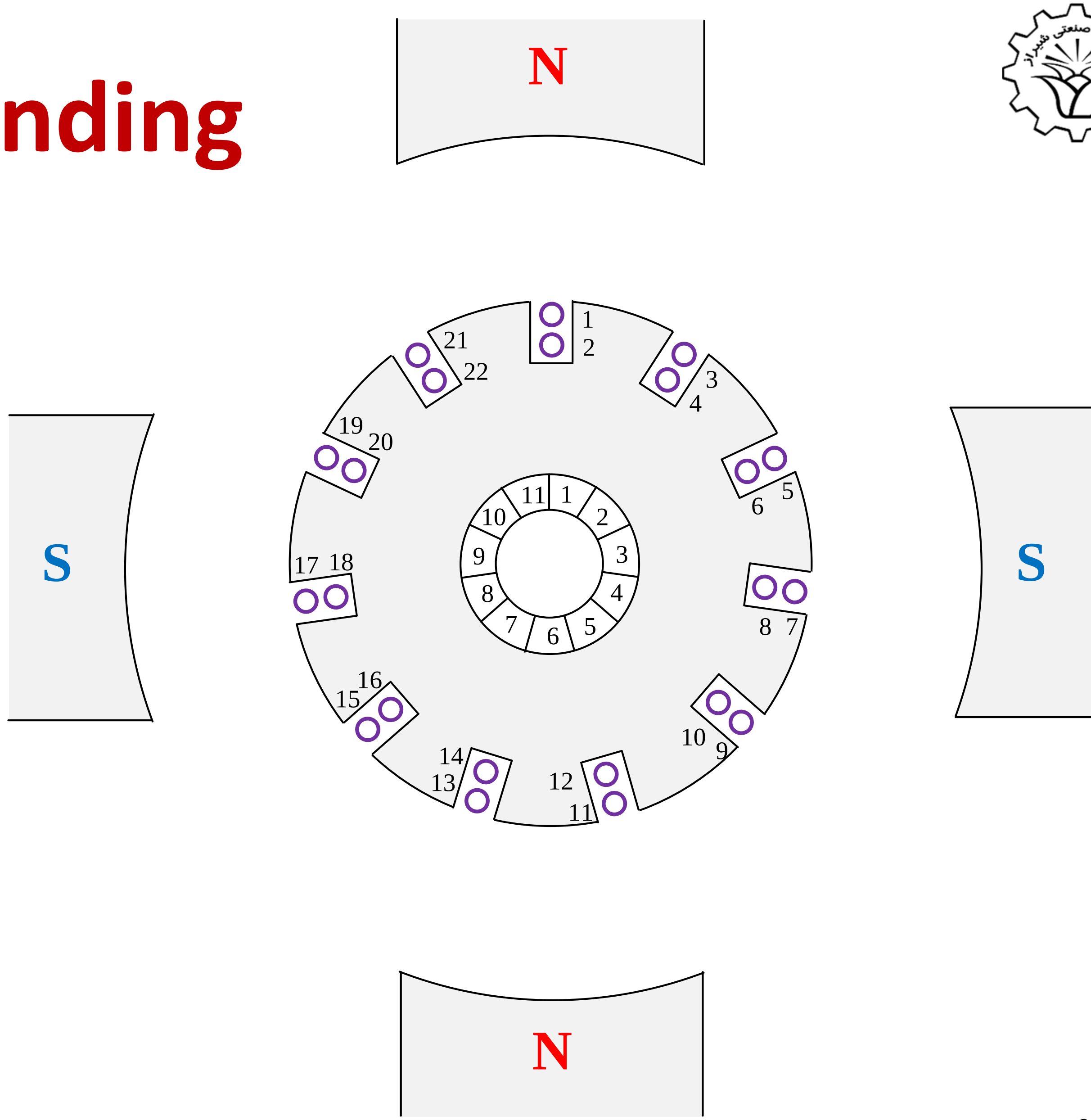
Wave Winding

Machine structure

$n_c = 11$

$p = 4$

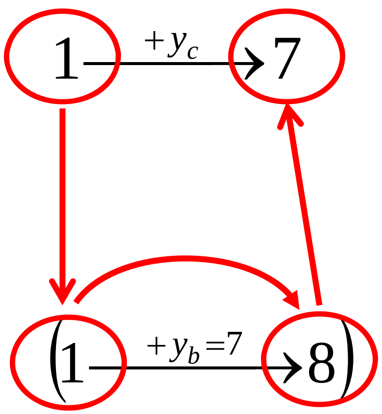
$n_l = 2$



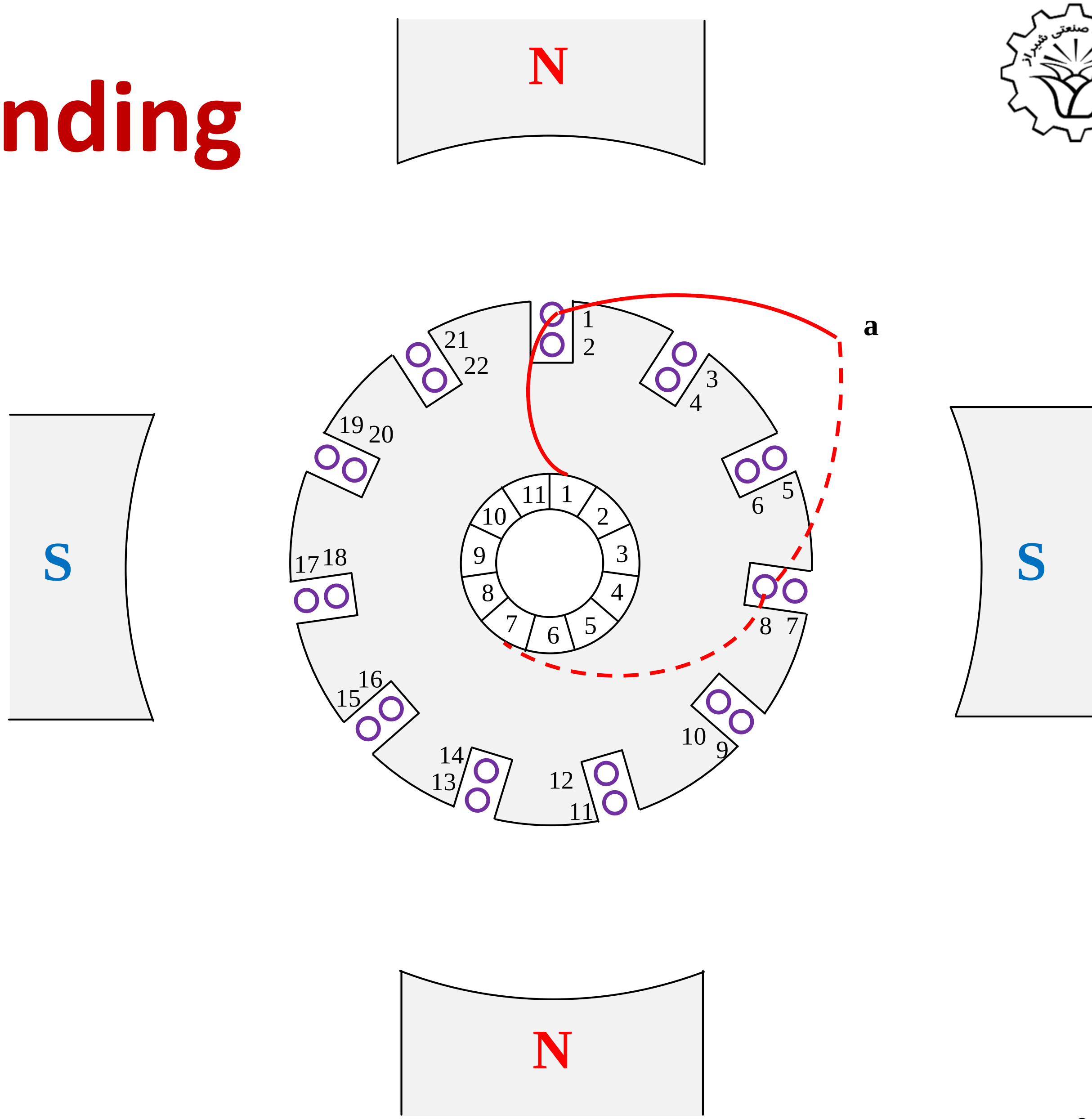
Wave Winding

First coil

Commutator table



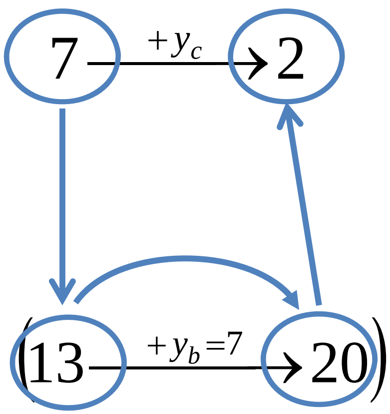
Winding table



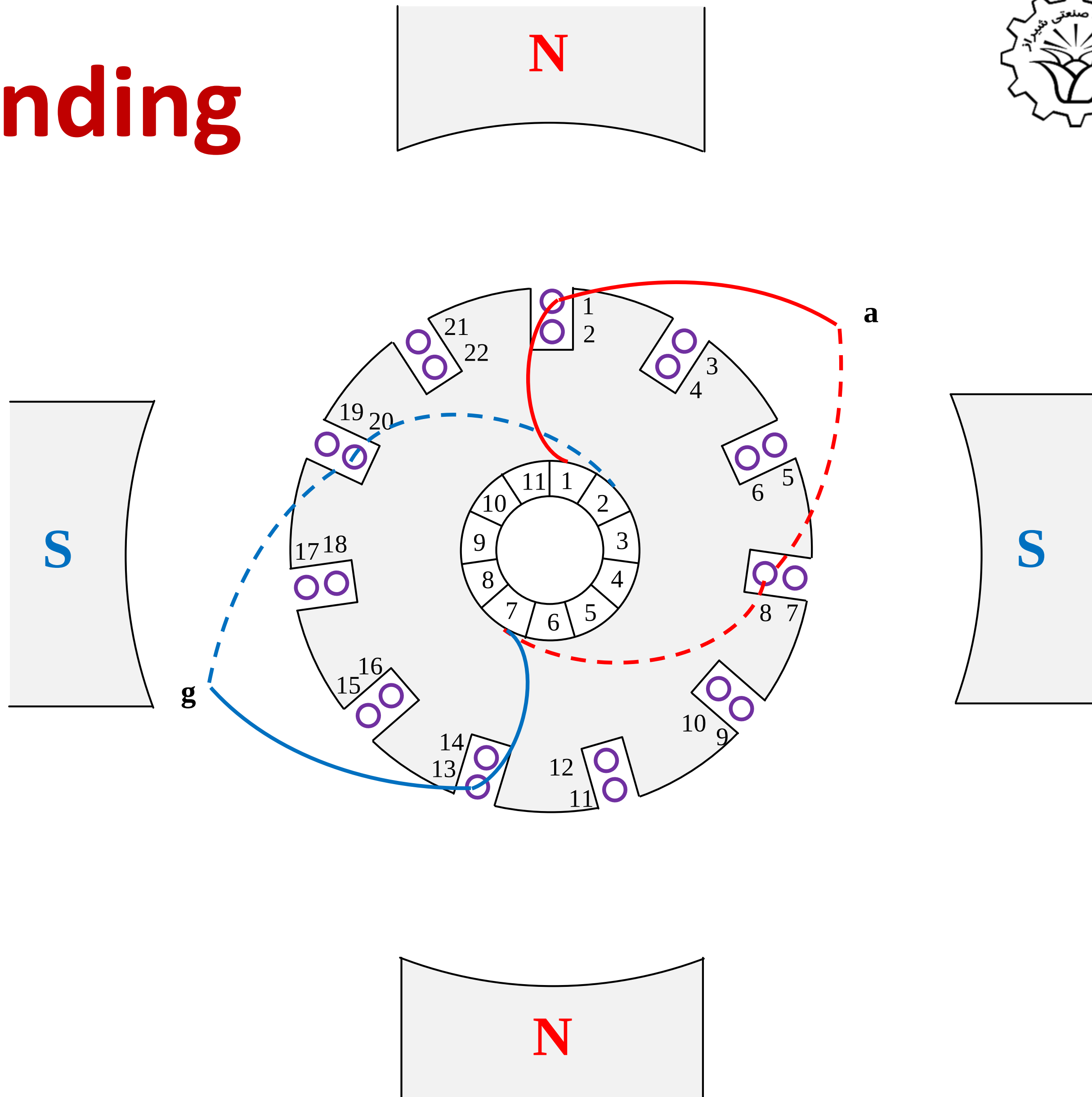
Wave Winding

Second coil

Commutator table

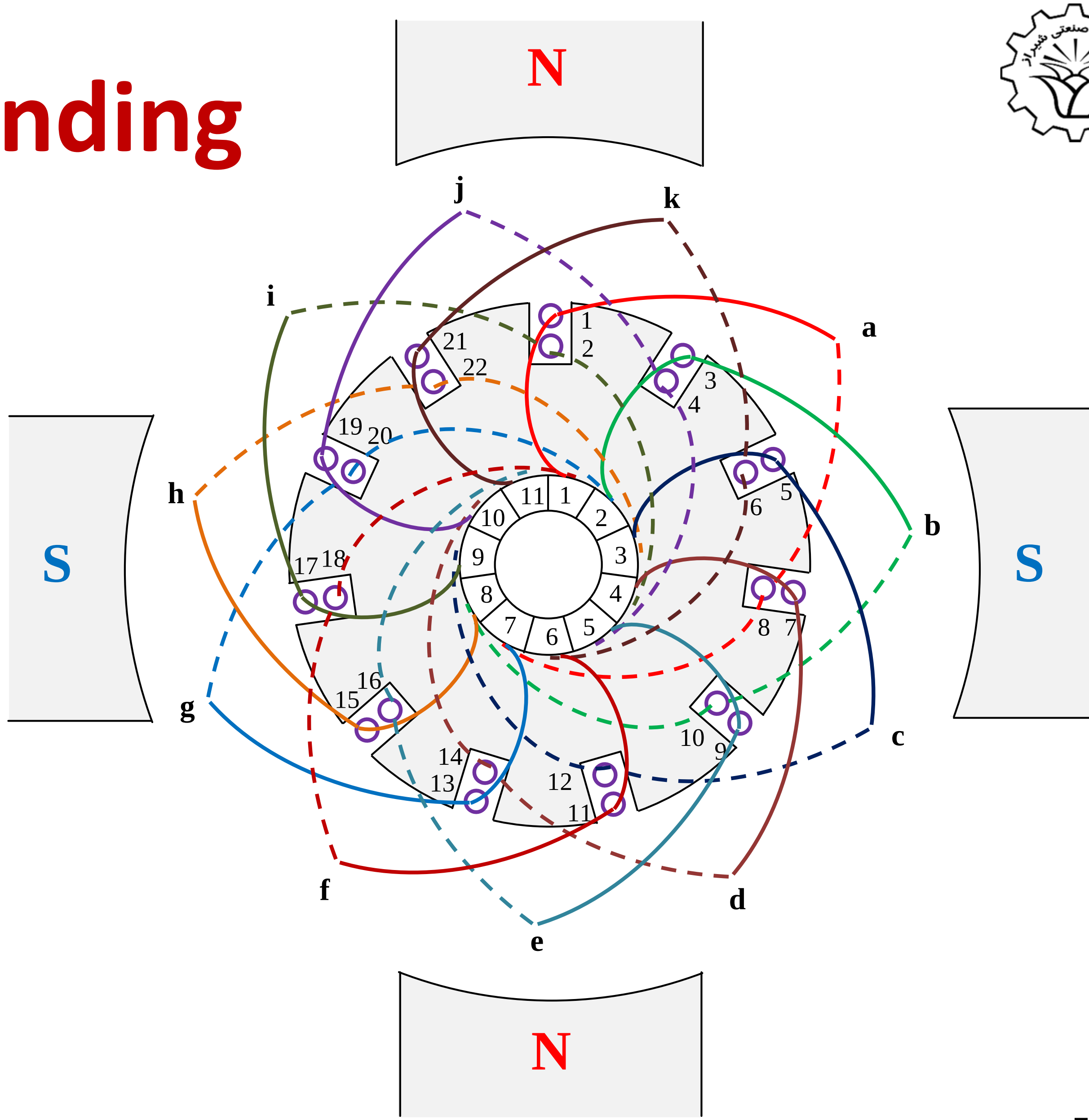


Winding table



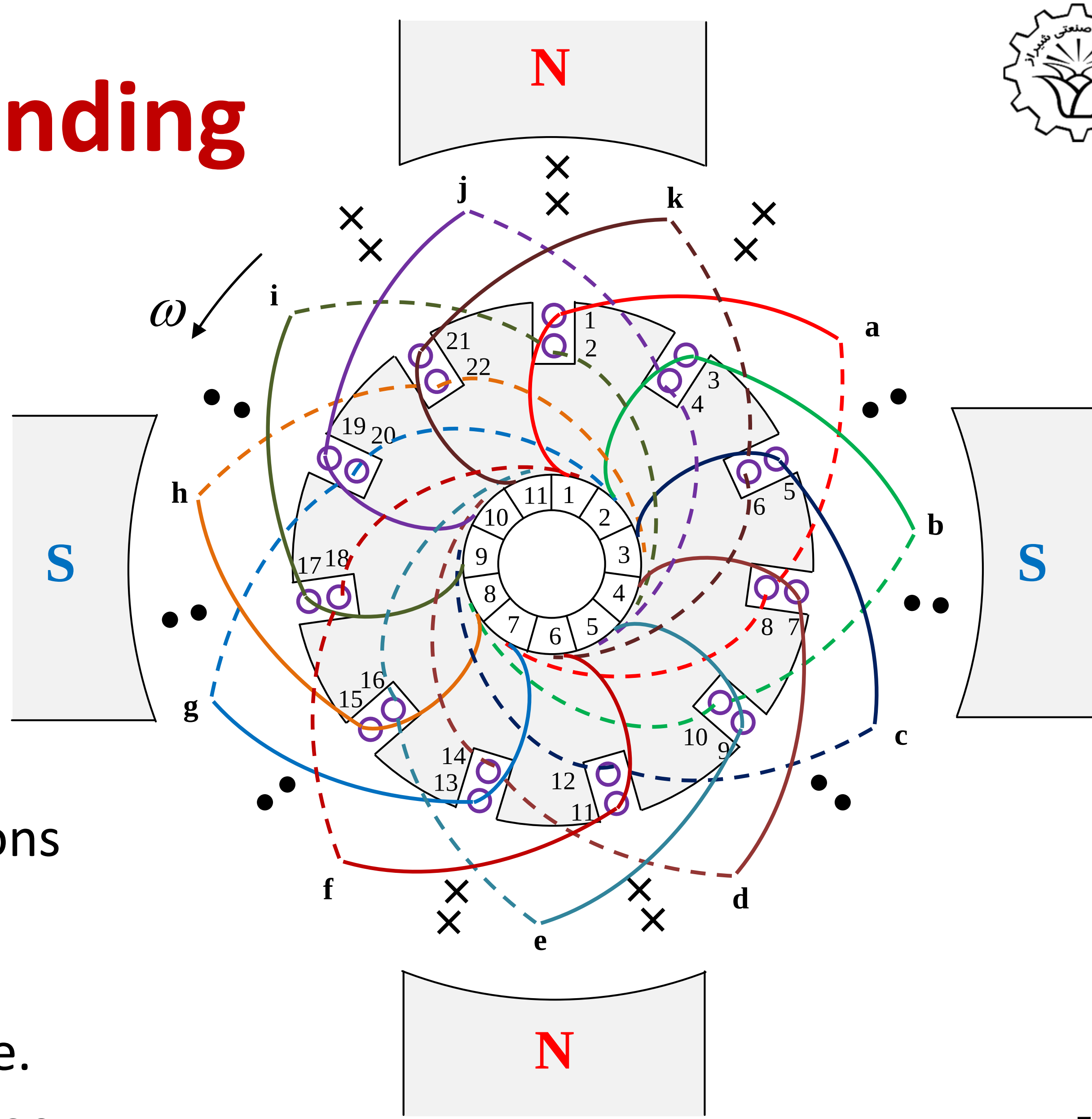
Wave Winding

All coils



Wave Winding

Brushes location:



Find the direction of current in each coil based on their positions under N or S pole:

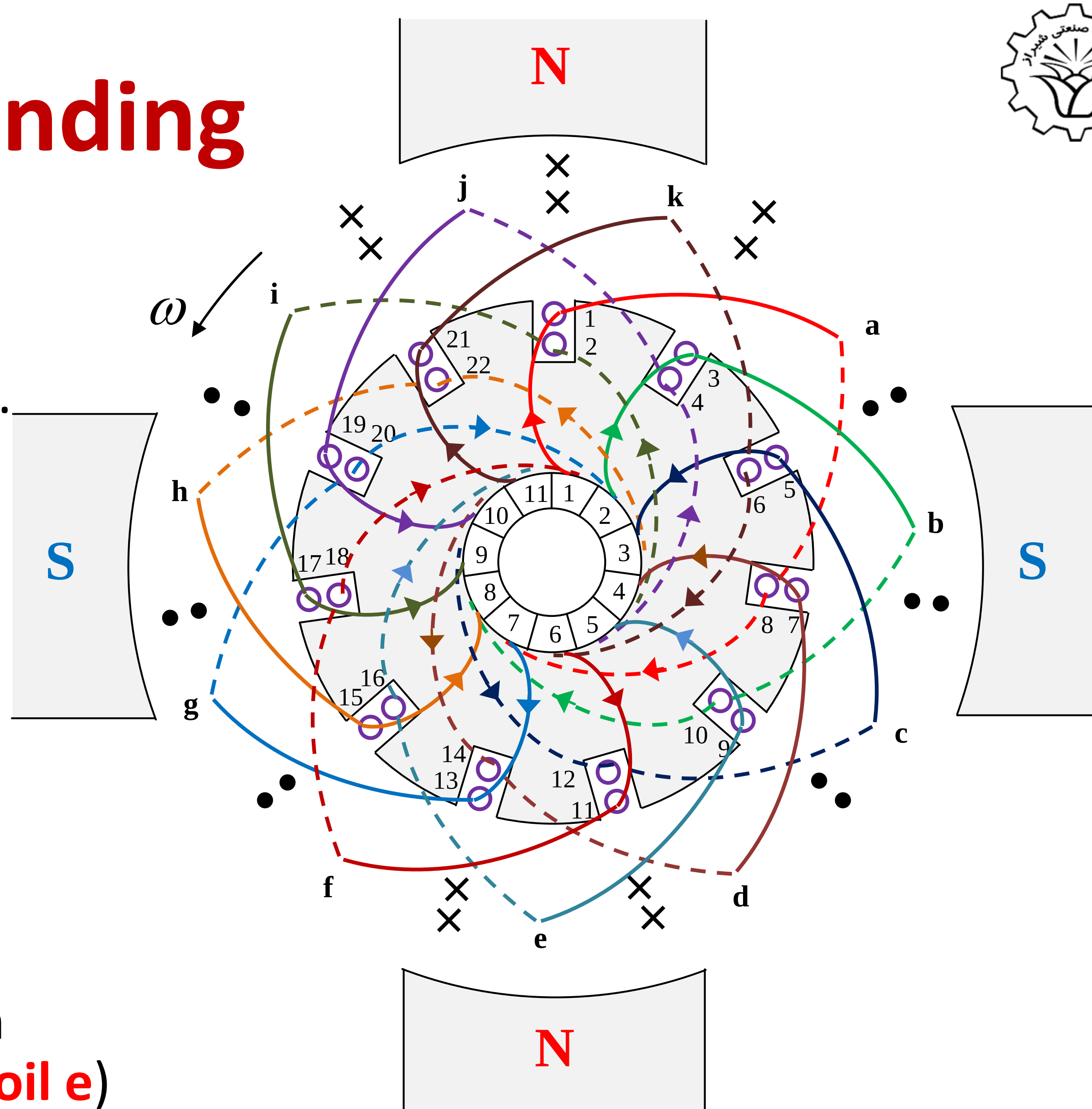
Under N into the page.

Under S out of the page.

Wave Winding

Brushes location:

Draw the current directions on the coils.



Find those slabs in which both currents enter or from which both currents leave.

(**Slab 8**)

Or

Find those coils in which the current is in opposite direction. (**Coil e**)

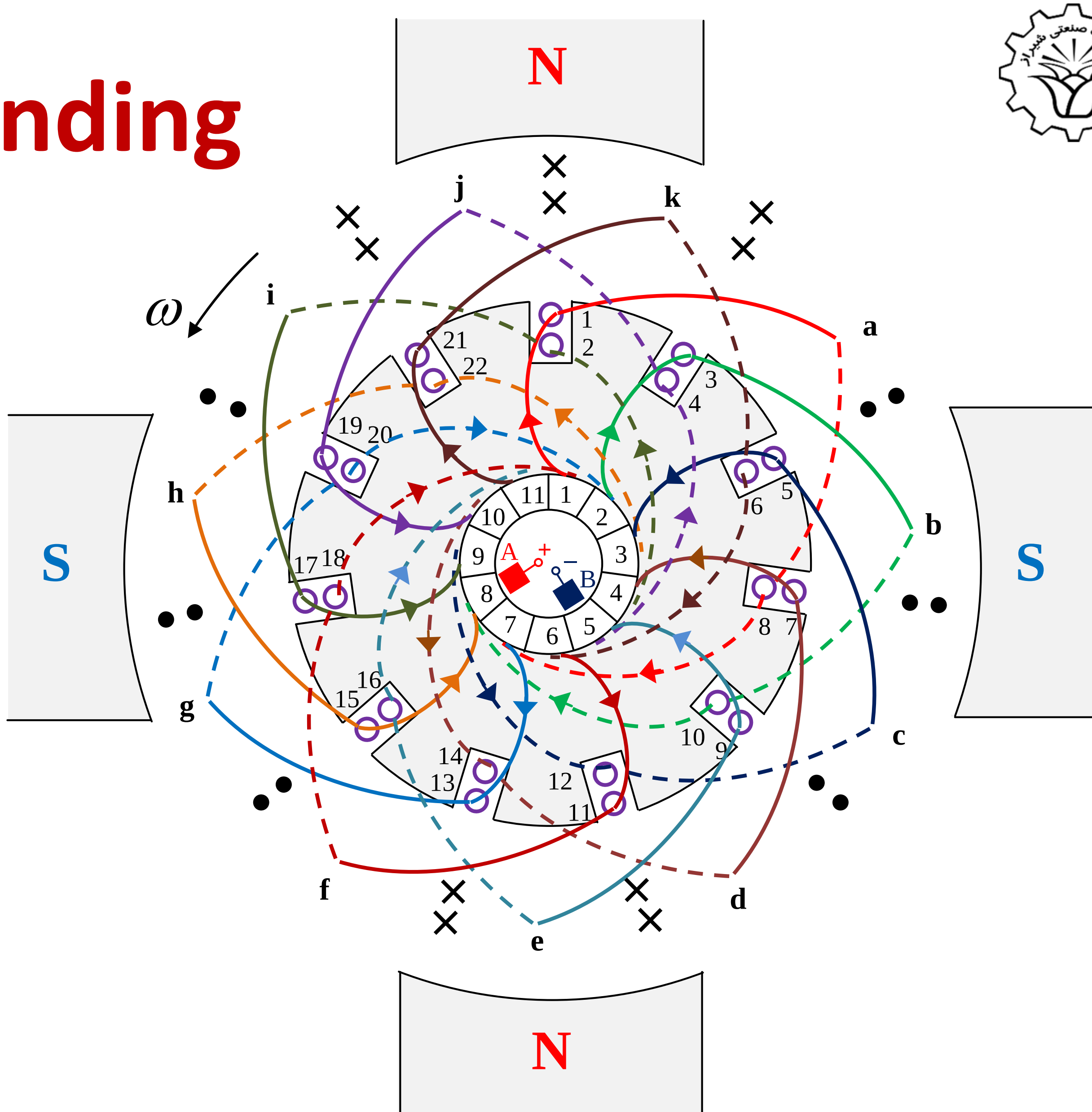
Wave Winding

Brushes location:

Slab 8 is a good place for one brush location.

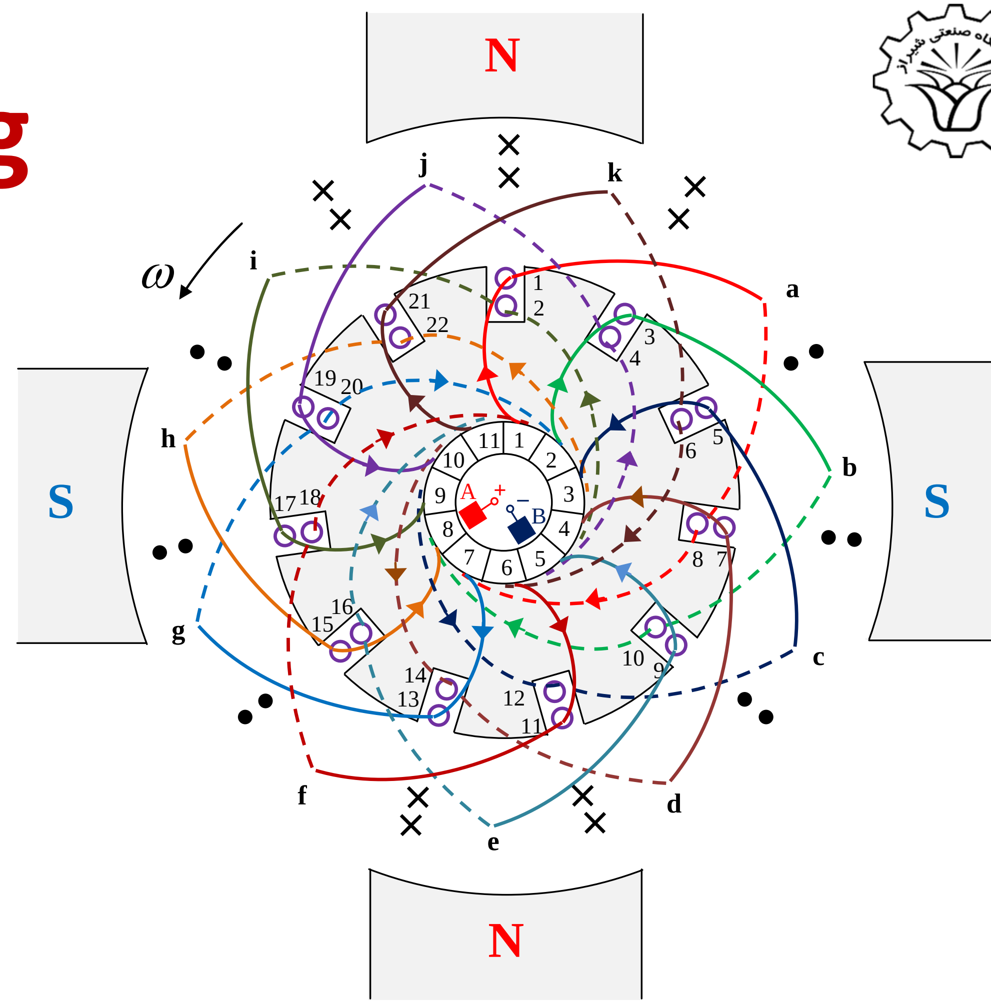
Also the slabs connected to **coil e** (**slabs 5 and 11**) are good candidate for another brush location.

So **slabs 8 and 5** are selected.



Wave Winding

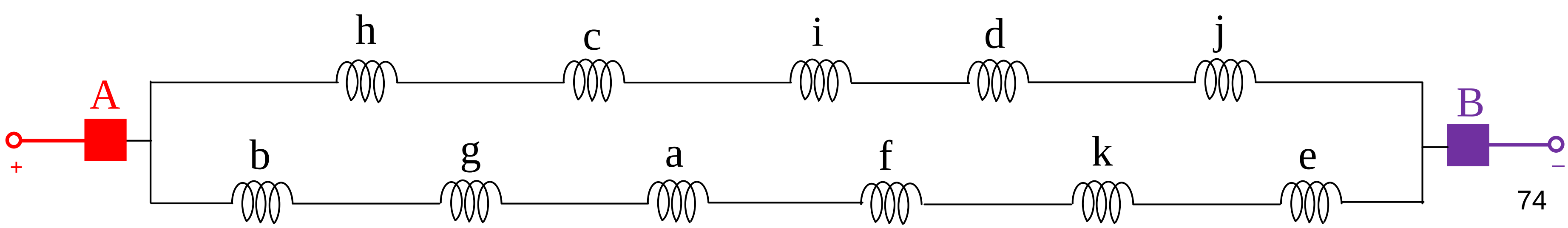
Coil connection



In simplex wave winding:

The number of parallel paths is always 2.

The number of brushes is always 2.



Lap & Wave Winding

Example: Consider a DC Machine with **6 poles** and **12 slots** with **2-layer**. Each coil has **20 turns**. The magnetic flux and rotational velocity are so that the induced voltage in each conductor is **2.2 V**. Each coil has the capacity to pass **100 A** current.

a) Find voltage, current and power of the machine if the winding is simplex lap winding.

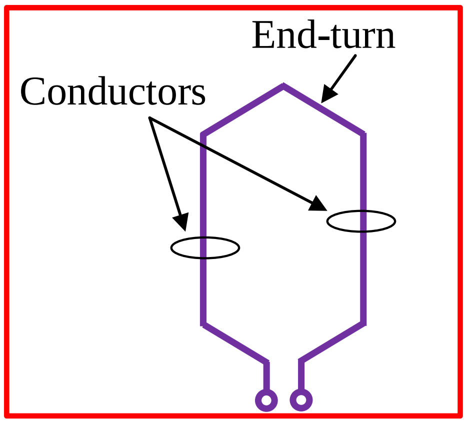
b) Find voltage, current and power of the machine if the winding is simplex wave winding.

$p = 6$ $\quad n_s = 12$ $\quad n_l = 2$ $\Rightarrow$ $n_c = \frac{1}{2} n_l n_s = 12$

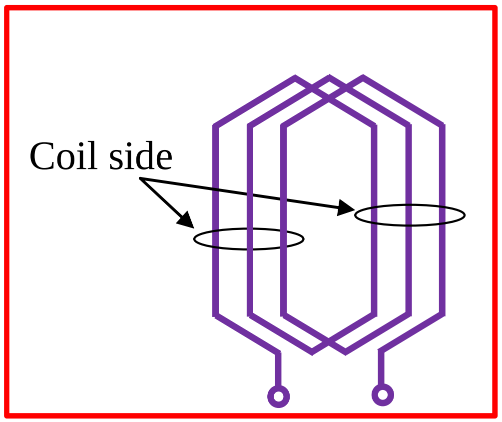
$N_{turns}^{coil} = 20$ $\quad E_z = 2.2\ \text{V}$ $\quad I_{coil} = 100\ \text{A}$

Definition Revisited

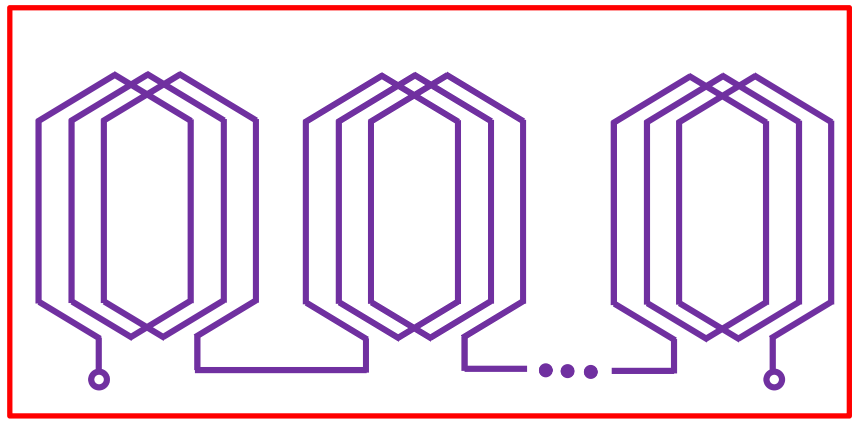
- **Turn:** consists of two conductors connected by an end-turn.
- **Coil**: consists of a number of turns connected in series.
- **Winding**: consists of several coils connected in series (or parallel)



Turn



coil



Winding

Lap & Wave Winding

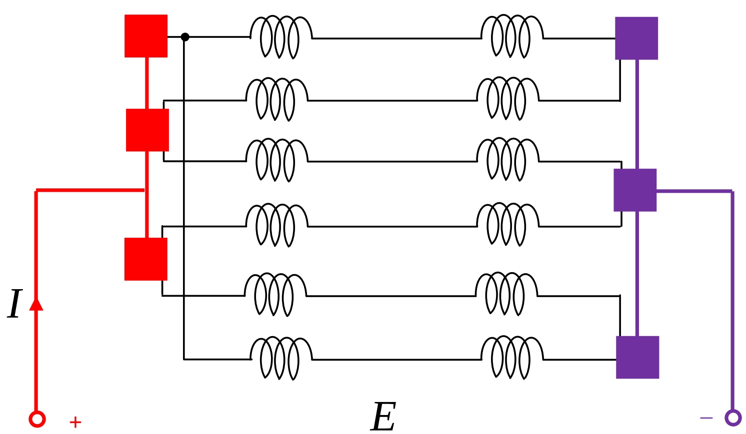
Solution part a) Find voltage, current and power of the machine if the winding is simplex lap winding.

$p = 6$ $n_s = 12$ $n_l = 2$ → $n_c = \frac{1}{2} n_l n_s = 12$

$N_{turns}^{coil} = 20$ $E_z = 2.2 \text{ V}$ $I_{coil} = 100 \text{ A}$

In **simplex lap** winding:

a = number of **parallel paths** = number of **poles** = 6



$$I = aI_{coil} = 6 \times 100 = 600 \text{ A}$$

$$E = 2E_z N_{turns}^{coil} \frac{n_c}{a} = 2 \times 2.2 \times 20 \times \frac{12}{6} = 176 \text{ V}$$

$$P = EI = 176 \times 600 = 105600 \text{ W}$$

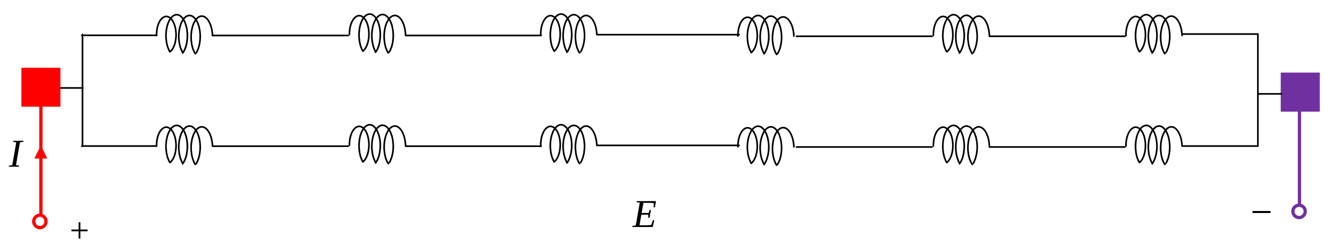
Lap & Wave Winding

Solution part b) Find voltage, current and power of the machine if the winding is simplex wave winding.

$p = 6$ $n_s = 12$ $n_l = 2$ ⇒ $n_c = \frac{1}{2} n_l n_s = 12$

$N_{turns}^{coil} = 20$ $E_z = 2.2\ \text{V}$ $I_{coil} = 100\ \text{A}$

In **simplex wave** winding: a = number of **parallel paths** = 2



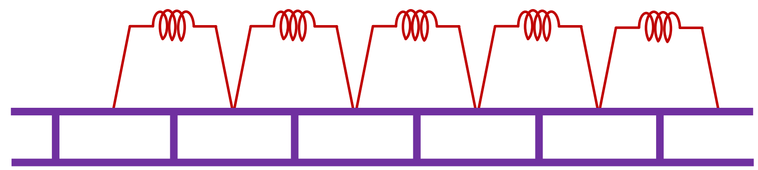
$I = aI_{coil} = 2 \times 100 = 200\ \text{A}$

$$E = 2E_z N_{turns}^{coil} \frac{n_c}{a} = 2 \times 2.2 \times 20 \times \frac{12}{2} = 528\ \text{V}$$

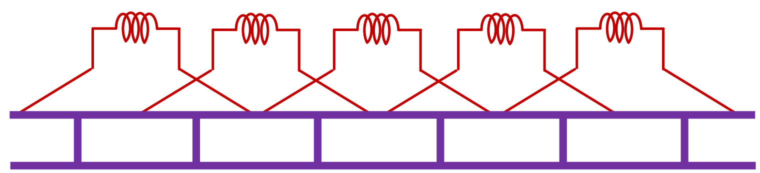
$P = EI = 528 \times 200 = 105600\ \text{W}$

Other Lap windings

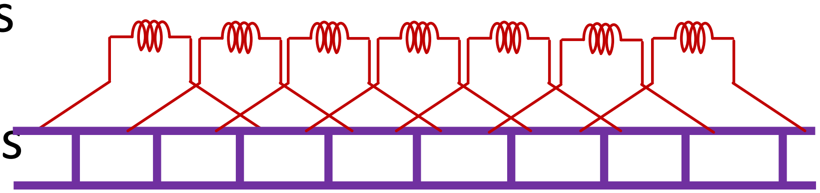
Simplex lap winding: the commutator-pitch is one, i.e. the two ends of a coil are connected to adjacent commutator slabs.



Double lap winding: the commutator-pitch is two in this winding, i.e. the two ends of a coil are connected to two commutator slabs apart from each other by a slab. The number of parallel paths is $2p$.



Triple lap winding: the commutator-pitch is three in this winding, i.e. the two ends of a coil are connected to two commutator slabs apart from each other by two slabs. The number of parallel paths is $3p$.



Commutation Process

Commutation process means the current of a coil becomes zero and it follows by changing in current direction.

In **Figure 1** assume the brush is under slab 1.

Consider coil (1) where its current is I_c **toward right**.

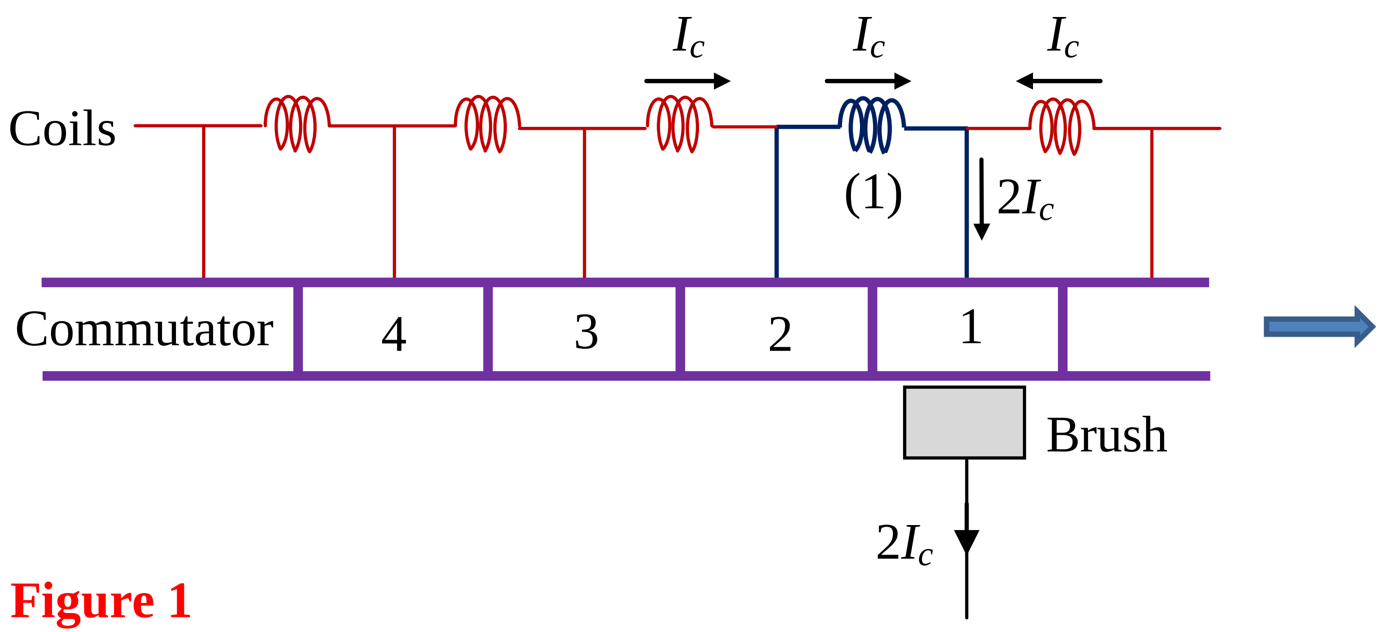


Figure 1

Commutation Process

In **Figure 2** assume the brush is exactly located between slabs 1 and 2.

The current of coil (1) is **zero** since the brush causes the coil (1) to be short-circuited.

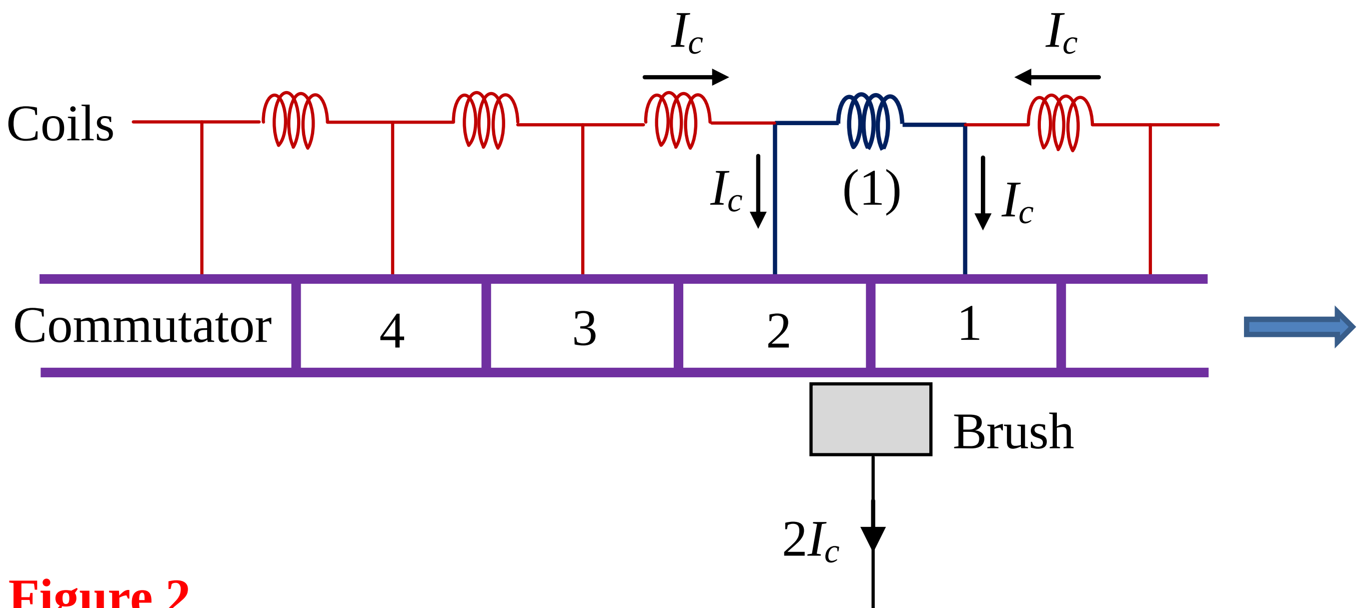


Figure 2

Commutation Process

In **Figure 3** assume the brush is now located under slab 2.

The current of coil (1) is I_c **toward left**.

Coil (1) is said to be under commutation process.

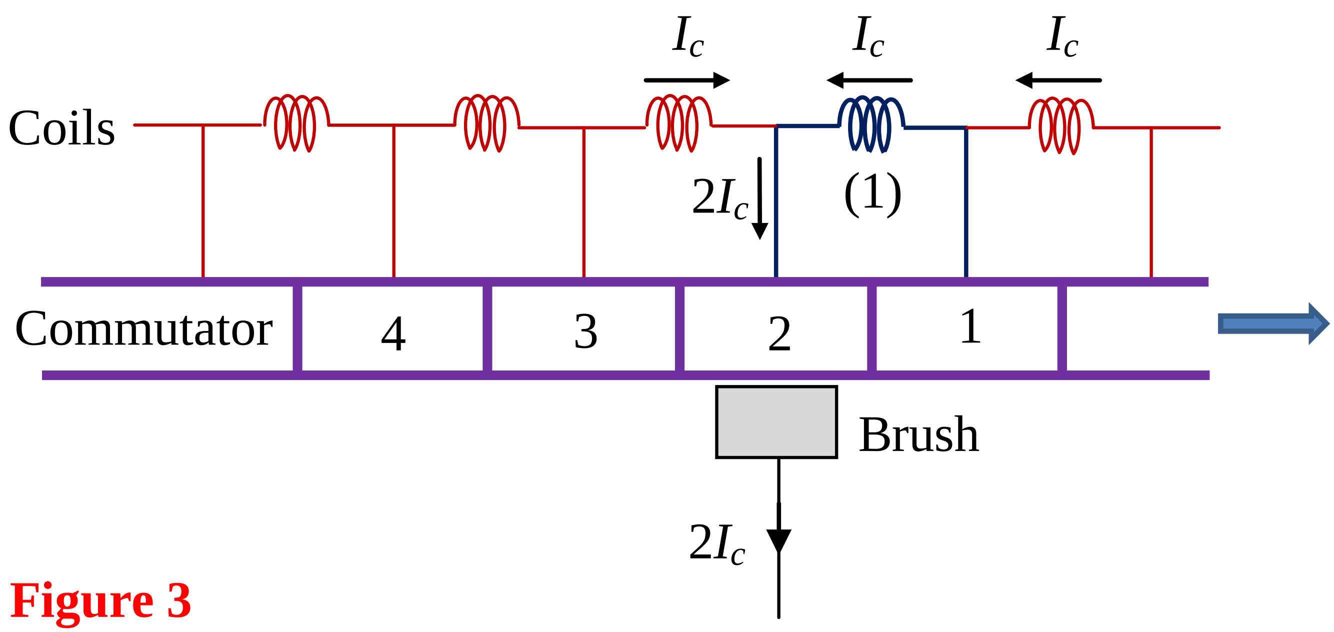


Figure 3

Commutation Process: Points

- Note that the **brushes** are **stationary** and the **coils and commutator slabs** (which are located on the rotor) are **rotating**.

- When the '**coil under commutation**' is short-circuited by the brush, the **induced voltage** in the coil should be **zero** (or at least minimum).

- To **minimize** the **induced voltage** in the 'coil under commutation' while short-circuited, the **sides of the coil** need to be located at the **neutral axis** (between the stator poles where the magnetic flux is very low).

Commutation Process

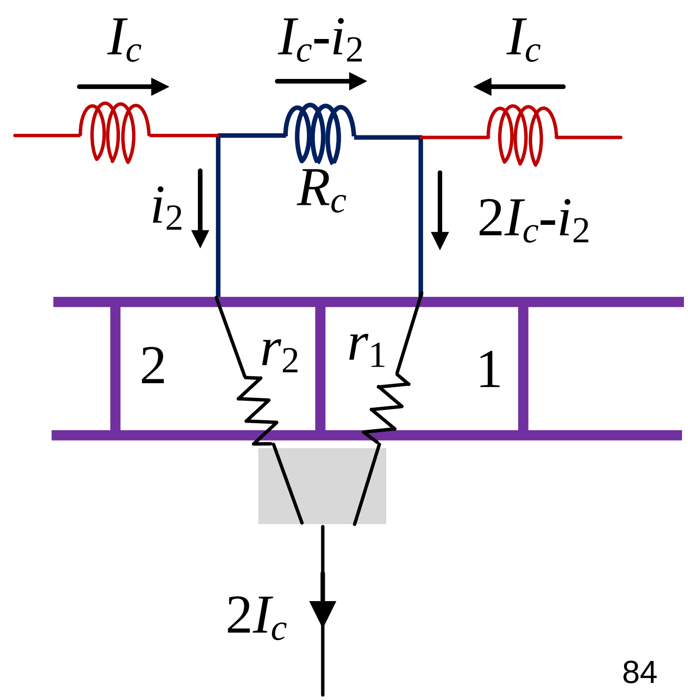
In the following figure assume:

- r_1 is the resistance between the brush and right side of the coil.
- r_2 is the resistance between the brush and left side of the coil.
- R_c is the resistance of the coil.

Writing KVL in the loop yields:

$$r_1(2I_c - i_2) - r_2 i_2 + R_c(I_c - i_2) = 0$$

$$\Rightarrow \quad i_2 = \frac{R_c + 2r_1}{R_c + r_1 + r_2} I_c$$



Commutation Process

$$i_2 = \frac{R_c + 2r_1}{R_c + r_1 + r_2} I_c$$

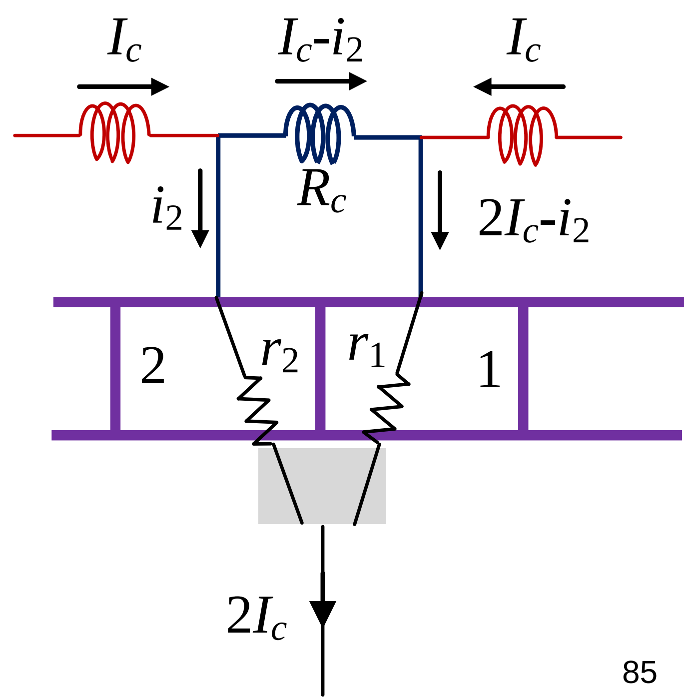
If the current of the coil under commutation is called i_{cc}

$$i_{cc} = I_c - i_2 = I_c - \frac{R_c + 2r_1}{R_c + r_1 + r_2} I_c$$

$$\Rightarrow \quad i_{cc} = I_c \left(1 - \frac{R_c + 2r_1}{R_c + r_1 + r_2} \right)$$

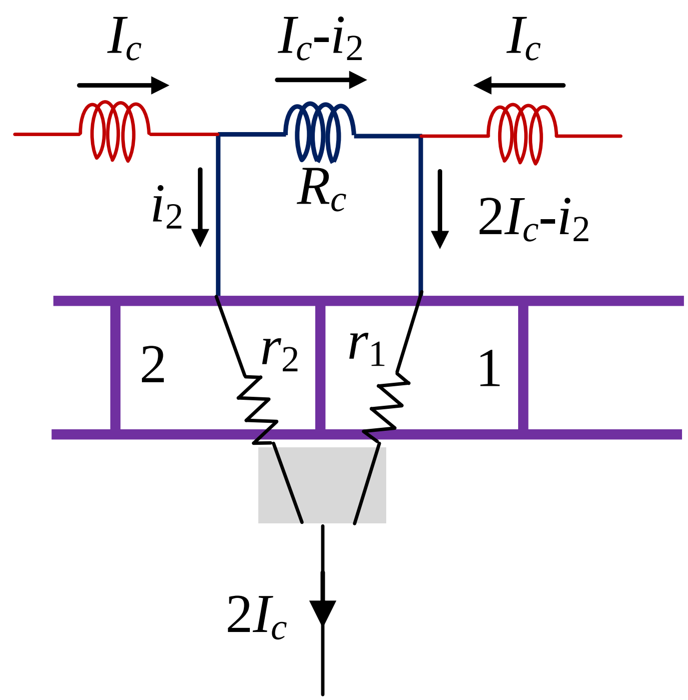
$$\Rightarrow \quad i_{cc} = I_c \left(\frac{r_1 + r_2 - 2r_1}{R_c + r_1 + r_2} \right)$$

$$\Rightarrow \quad i_{cc} = I_c \left(\frac{1 - 2r_1/(r_1 + r_2)}{1 + R_c/(r_1 + r_2)} \right)$$



Commutation Process

$$i_{cc} = I_c\left(\frac{1-2r_1/(r_1+r_2)}{1+R_c/(r_1+r_2)}\right)$$



Since $R_c << r_1 + r_2$

$$\Rightarrow \quad i_{cc} = I_c\left(1-\frac{2r_1}{r_1+r_2}\right)$$

Assume $r_1 \propto \frac{1}{A_1}$ and $r_2 \propto \frac{1}{A_2}$

Where A_1 is the contact area between the brush and slab 1 and A_2 is the contact area between the brush and slab 2.

$$\Rightarrow \quad i_{cc} = I_c\left(1-\frac{2A_2}{A_1+A_2}\right)$$

Commutation Process

$$i_{cc} = I_c\left(1 - \frac{2A_2}{A_1 + A_2}\right)$$

If T is the commutation period of the coil under commutation

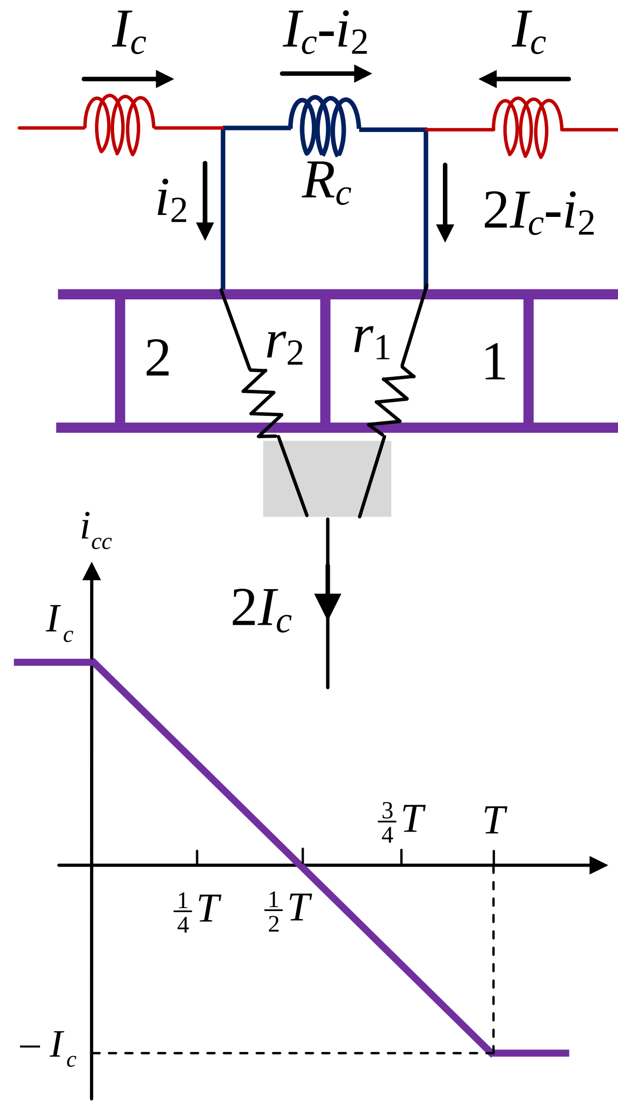
$$t = 0 \quad \Rightarrow \quad A_2 = 0 \quad \Rightarrow \quad i_{cc} = I_c$$

$$t = T/4 \quad \Rightarrow \quad A_1 = 3A_2 \quad \Rightarrow \quad i_{cc} = \tfrac{1}{2} I_c$$

$$t = T/2 \quad \Rightarrow \quad A_1 = A_2 \quad \Rightarrow \quad i_{cc} = 0$$

$$t = 3T/4 \quad \Rightarrow \quad A_2 = 3A_1 \quad \Rightarrow \quad i_{cc} = -\tfrac{1}{2} I_c$$

$$t = T \quad \Rightarrow \quad A_1 = 0 \quad \Rightarrow \quad i_{cc} = -I_c$$



Commutation Process

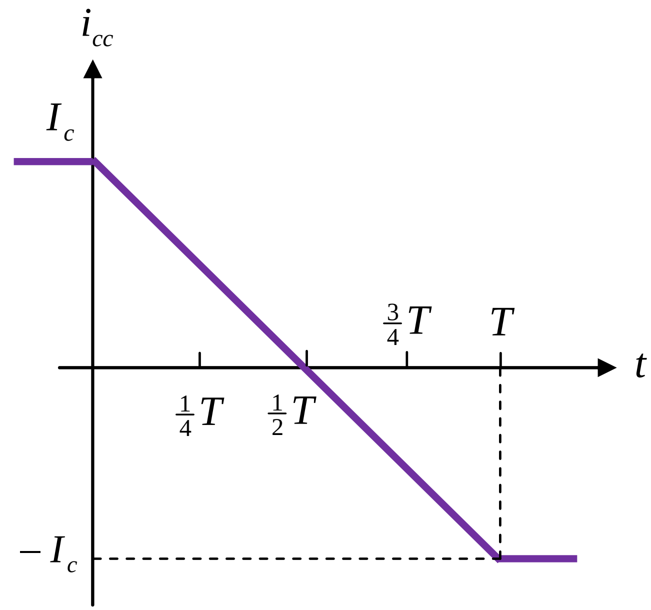
$$t = 0 \quad \Rightarrow \quad A_2 = 0 \quad \Rightarrow \quad i_{cc} = I_c$$

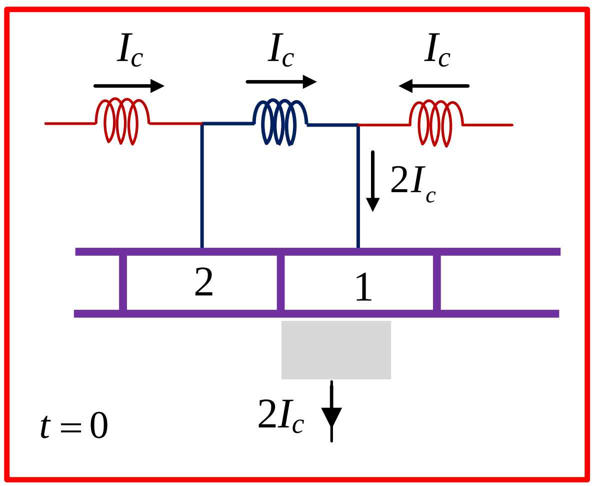
$$t = T/4 \quad \Rightarrow \quad A_1 = 3A_2 \quad \Rightarrow \quad i_{cc} = \tfrac{1}{2} I_c$$

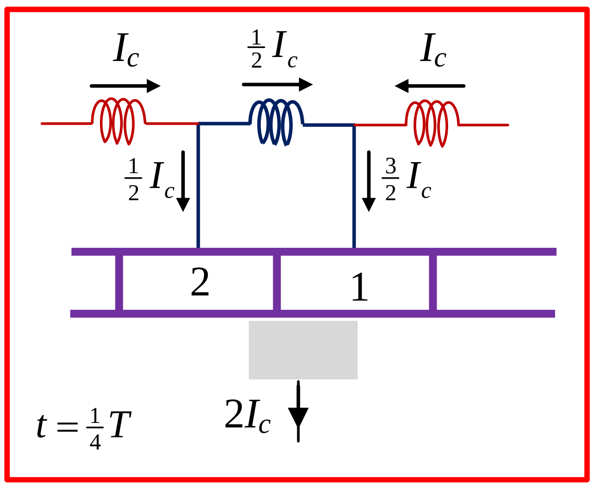
$$t = T/2 \quad \Rightarrow \quad A_1 = A_2 \quad \Rightarrow \quad i_{cc} = 0$$

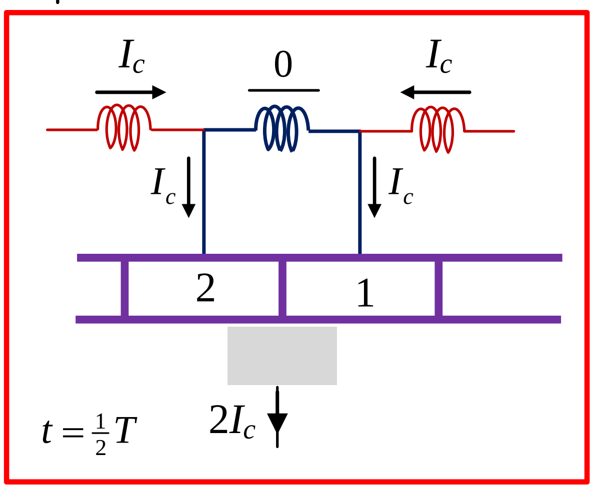
$$t = 3T/4 \quad \Rightarrow \quad A_2 = 3A_1 \quad \Rightarrow \quad i_{cc} = -\tfrac{1}{2} I_c$$

$$t = T \quad \Rightarrow \quad A_1 = 0 \quad \Rightarrow \quad i_{cc} = -I_c$$









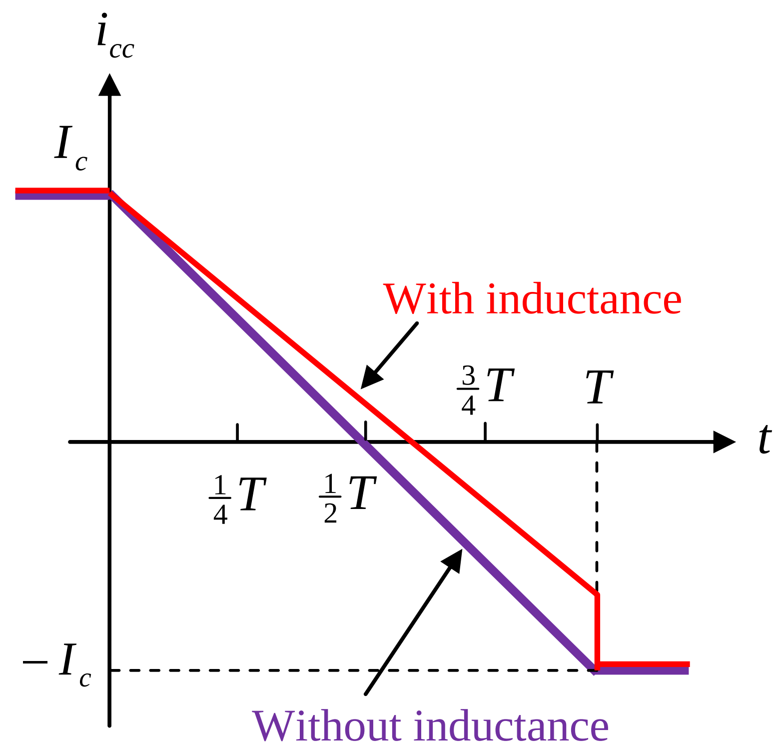
Commutation Process

- In the above discussion, the coil inductance has been neglected.

- Considering the coil inductance will change the commutation process.

$$L\frac{di}{dt}$$

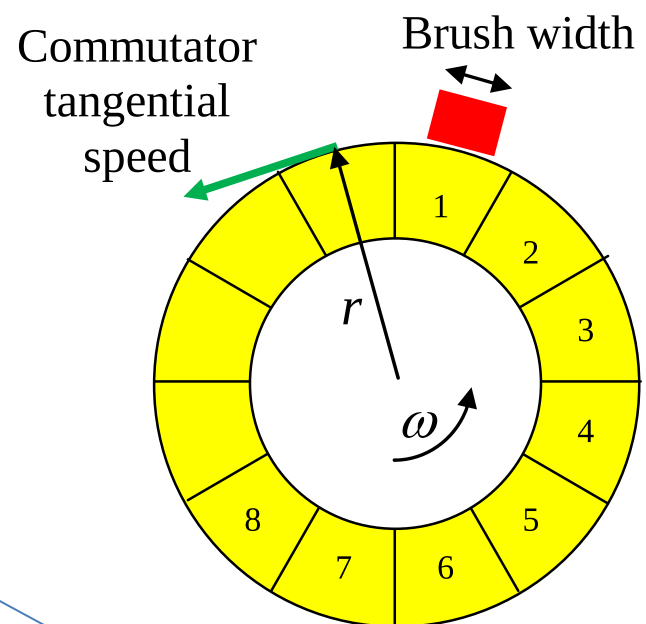
- The change in commutation process will cause sparks.



Commutation Time

- **Commutation time** is the time between the instance when the brush is under slab 1 and the instance when the brush is under slab 2:

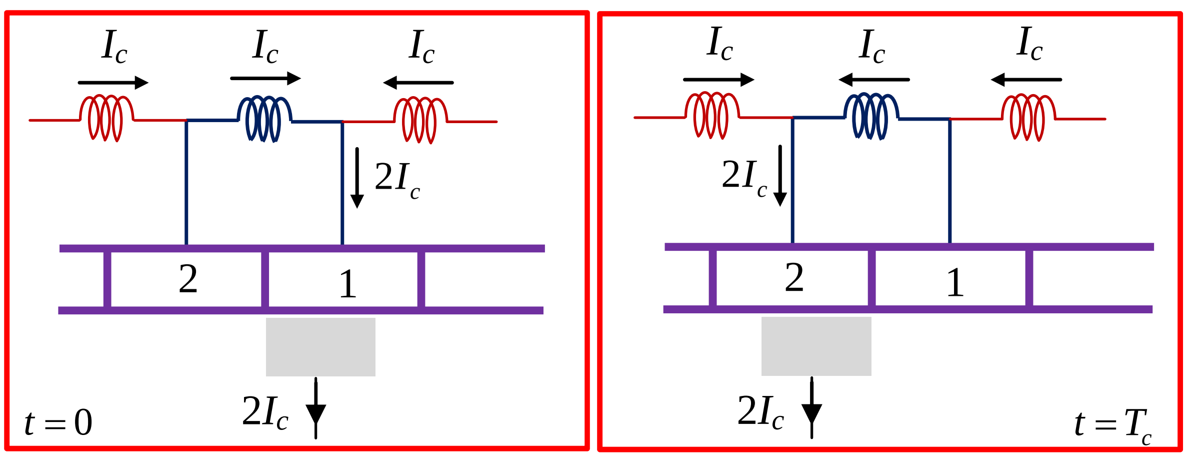
$$T_c = \frac{\text{Brush width}}{\text{Commutator tangential speed}}$$



$$v = r\omega$$

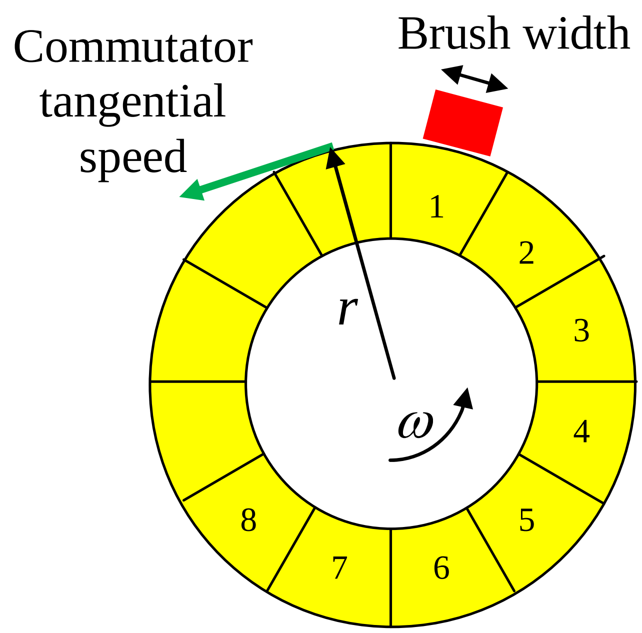
Commutator radius

Rotational speed



Commutation Time

- **Example:** In a DC machine the brush width is 1.5 cm, the commutator diameter is 50 cm. If the rotor rotational speed is 1000 rpm, calculate the commutation time.



$$T_c = \frac{\text{Brush width}}{\text{Commutator tangential speed}}$$

$$v = r\omega = \left(\tfrac{1}{2} 50\times10^{-2}\right)\times\left(1000\times\frac{2\pi}{60}\right) = \frac{25\pi}{3}$$

$$T_c = \frac{1.5\times10^{-2}}{25\pi/3} = 0.57 \text{ ms}$$