In The Name of God The Most Compassionate, The Most Merciful

Electric Machines I

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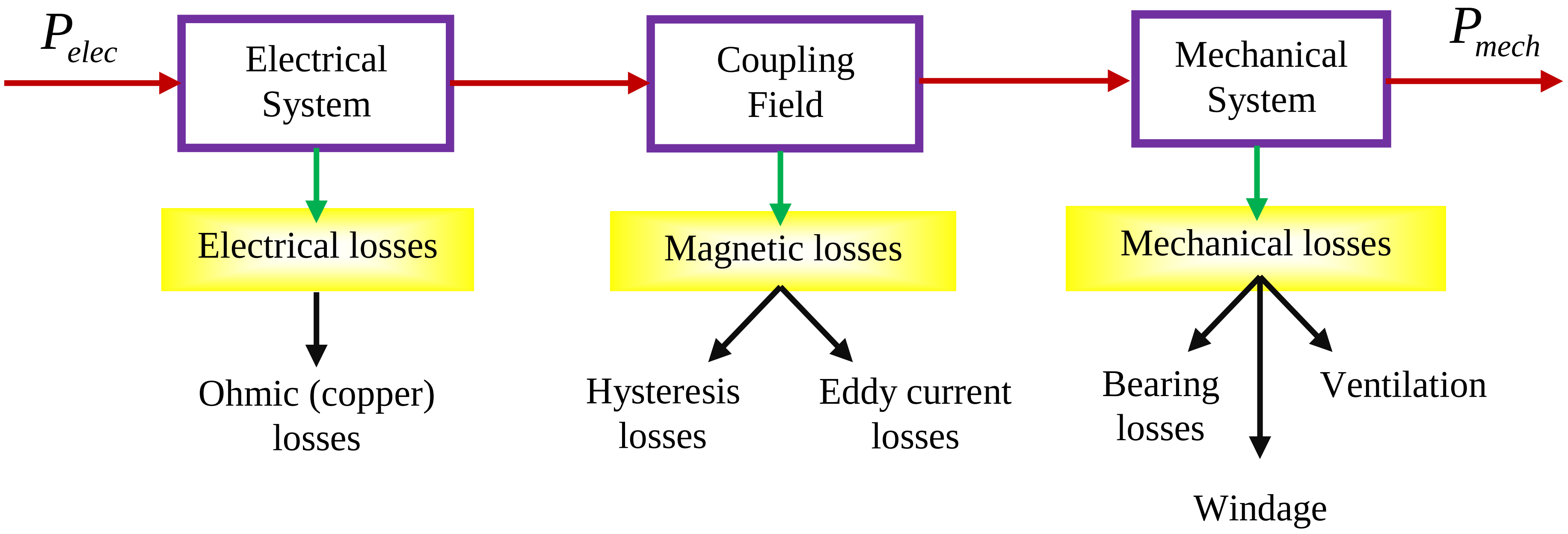
Principle of Electromechanical Energy Conversion

Electromechanical Relations

Motoring Case



P_{elec}

Electrical System

Coupling Field

Mechanical System

P_{mech}

Electrical losses

Magnetic losses

Mechanical losses

Ohmic (copper) losses

Hysteresis losses

Eddy current losses

Bearing losses

Ventilation

Windage

$$W_{elec} = W_{field} + W_{mech}$$

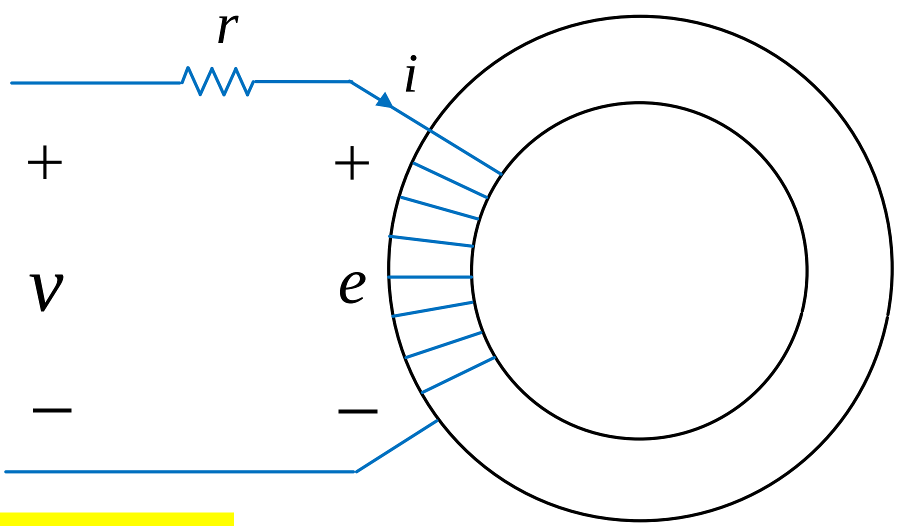
$$dW_{elec} = dW_{field} + dW_{mech}$$

Magnetic System with Single Excitation

$$e = v - r\,i$$

$$ei = vi - r\,i^2$$

$$eidt = vidt - r\,i^2 dt$$



$$dW_{elec} = dW_{field} + dW_{mech}$$

$$dW_{mech} = 0$$

$$dW_{elec} = dW_{field} = eidt$$

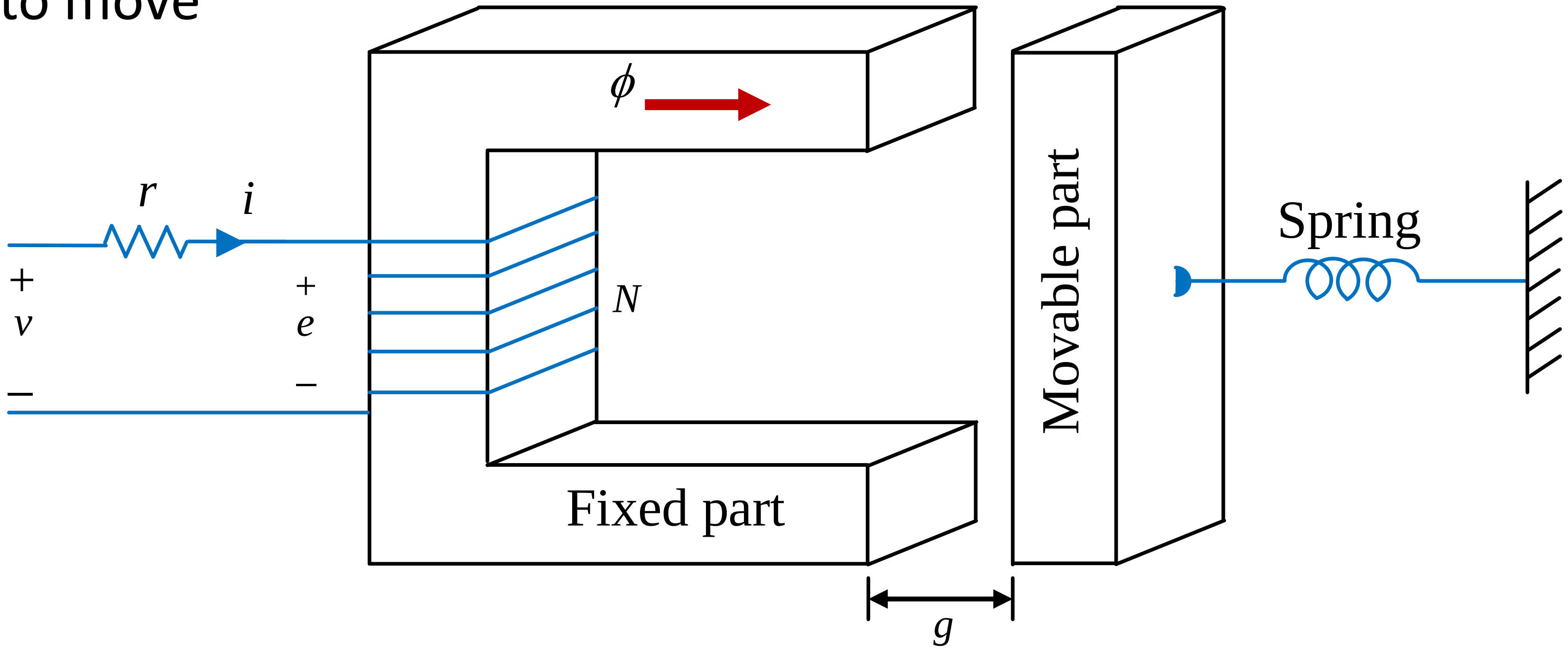
$$e = \frac{d\lambda}{dt}$$

$$\lambda = N\phi$$

$$dW_{field} = id\lambda$$

Magnetic Relay with Single Excitation

Assumption 1: The movable part **cannot move** or is **not allowed** to move

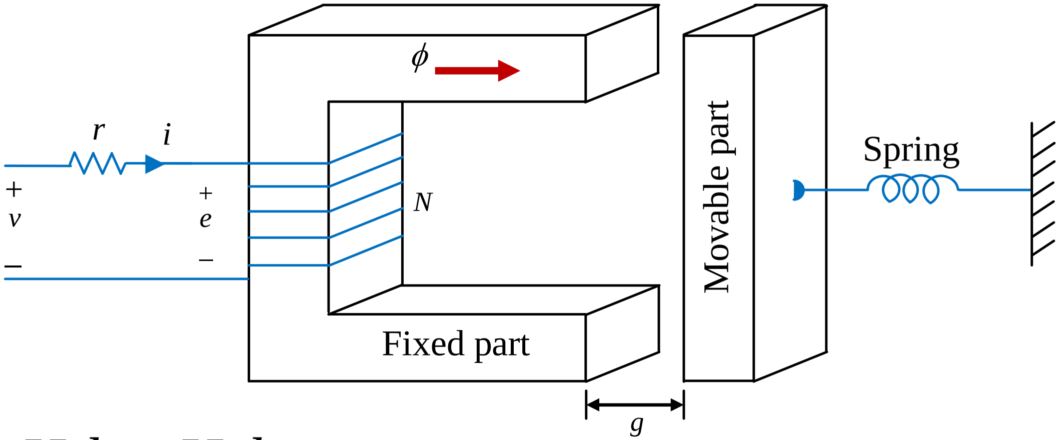


$$dW_{mech} = 0$$

$$dW_{elec} = dW_{field} = id\lambda$$

Magnetic Relay with Single Excitation

Assumption 1: The movable part **cannot move**



$$dW_{mech} = 0$$

$$dW_{elec} = dW_{field} = id\lambda$$

$$\left.\begin{aligned} W_{field} &= \int_0^{\lambda} id\lambda \\ \lambda &= N\phi \\ Ni &= H_c l_c + H_g l_g \end{aligned}\right\} W_{field} = \int_0^B \frac{H_c l_c + H_g l_g}{N} NAdB$$

$$H_g = \frac{B}{\mu_0} \qquad W_{field} = \int_0^B \left(H_c l_c + \frac{B}{\mu_0} l_g \right) AdB$$

Magnetic Relay with Single Excitation

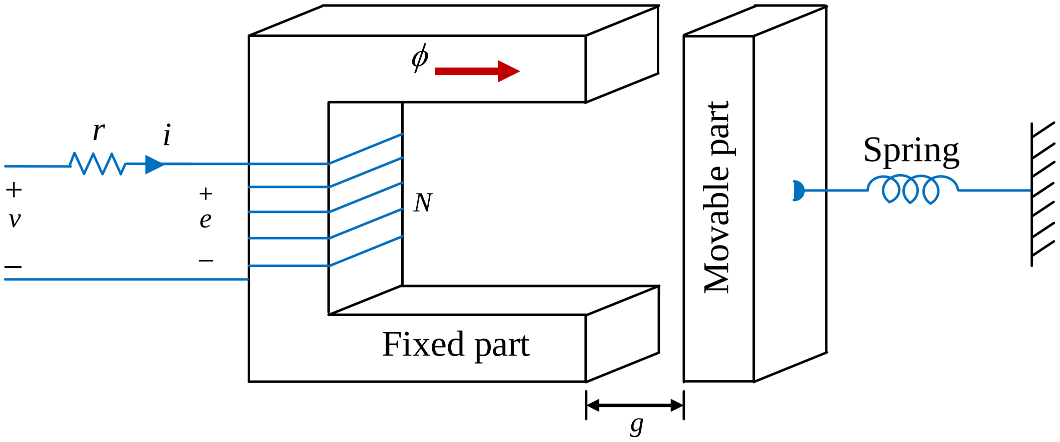
Assumption 1: The movable part **cannot move**

$$W_{field} = \int_0^B \left(H_c l_c + \frac{B}{\mu_0} l_g \right) A dB$$

$$W_{field} = V_c \int_0^B H_c dB + V_g \frac{B^2}{2\mu_0}$$

Stored magnetic energy in the core

Stored magnetic energy in the air-gap



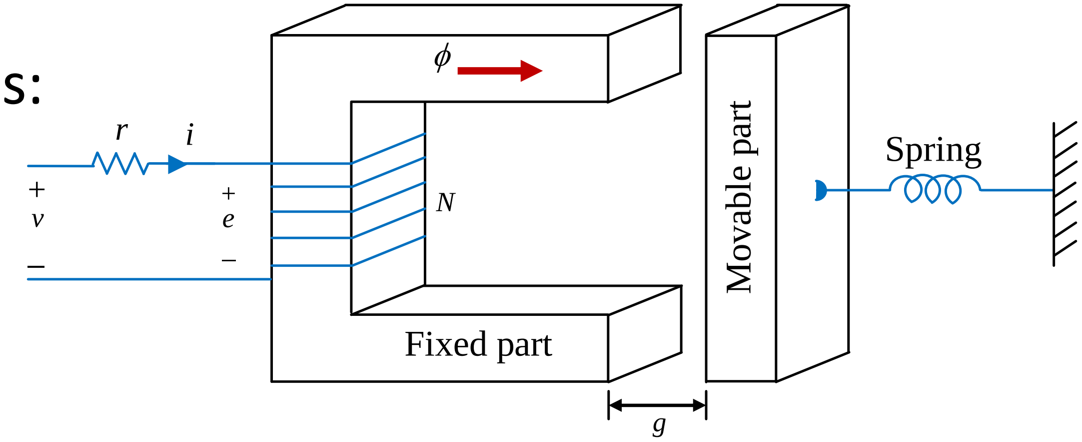
where

V_c is the core volume

V_g is the air-gap volume

Magnetic Relay with Single Excitation

Assumption 1: The movable part **cannot move**



In the case of linear systems:

$\mu_r \rightarrow \text{constant}$

$$H_c = \frac{B}{\mu_0 \mu_r}$$

$$W_{field} = V_c \frac{B^2}{2\mu_0\mu_r} + V_g \frac{B^2}{2\mu_0}$$

Stored magnetic energy in the core

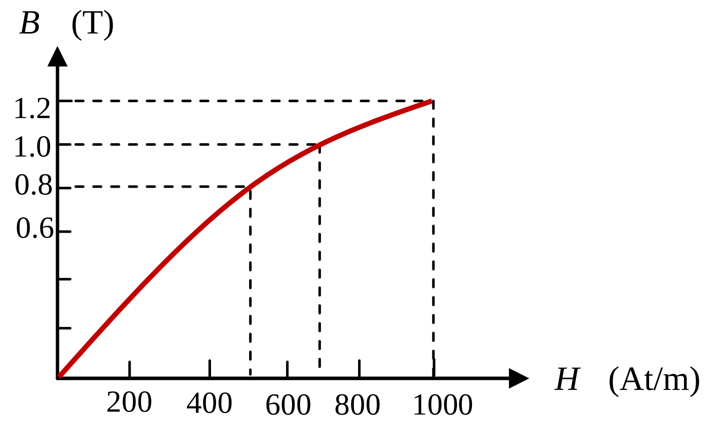
Stored magnetic energy in the air-gap

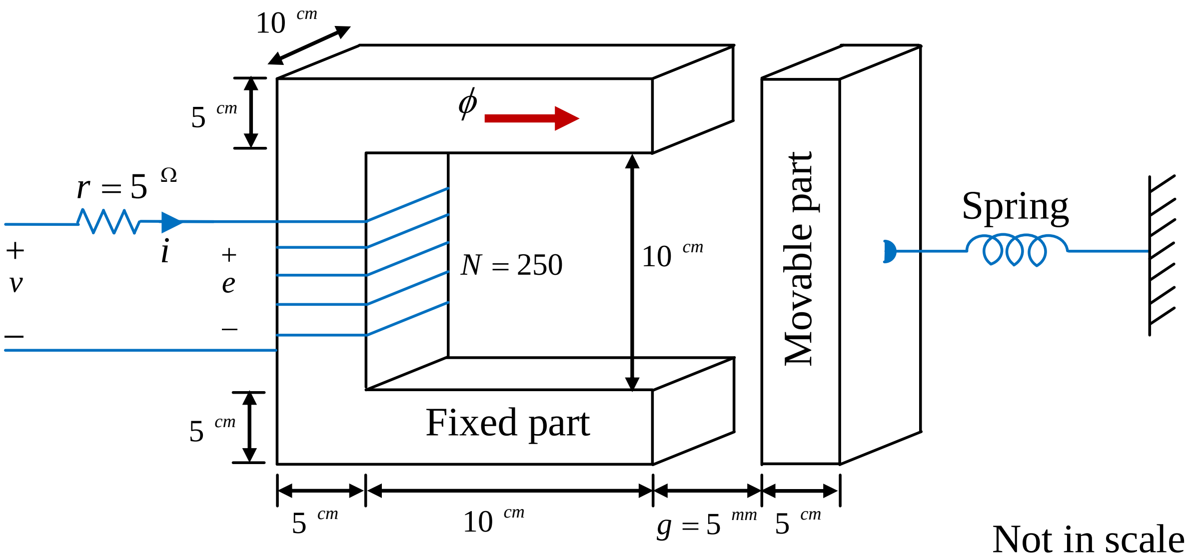
Magnetic Relay with Single Excitation

Example 1: In the following system if the air-gap flux density is 1 T and air-gap length is constant and fringing effect is neglected, calculate:

a) The DC source voltage;

b) Stored magnetic energy.





Not in scale

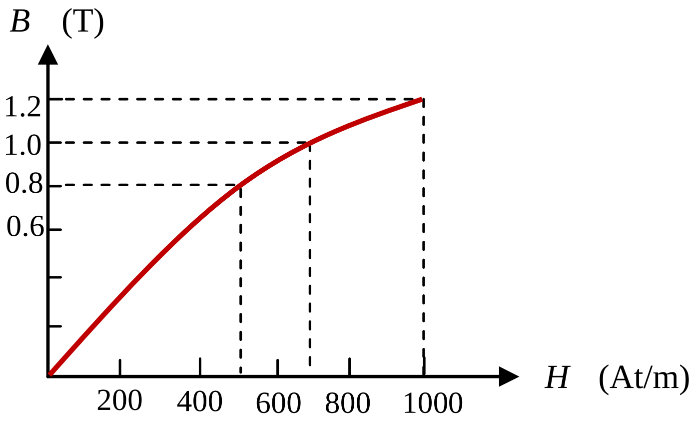
Magnetic Relay with Single Excitation

Solution 1: $B_g = B_c = 1 \text{ T}$

From the curve $H_c = 670 \text{ At/m}$

$$H_g = \frac{B_g}{\mu_0} = \frac{1}{4\pi \times 10^{-7}} = 795 \times 10^3 \text{ At/m}$$

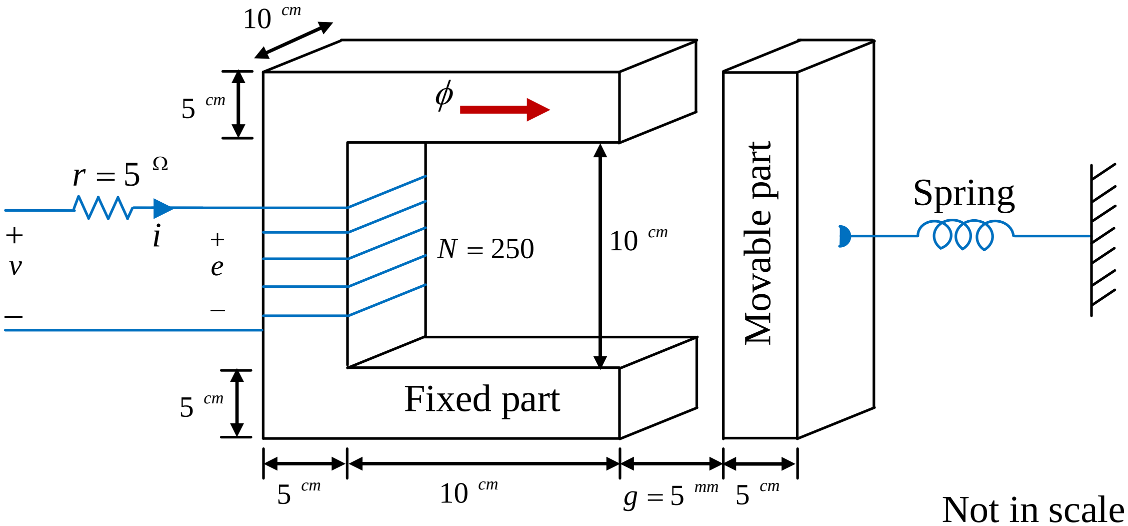
$l_c = 0.6 \text{ m}$ $l_g = 5 \text{ mm}$



$$Ni = H_c l_c + 2H_g l_g$$

$$i = \frac{1}{N}\left(H_c l_c + 2H_g l_g\right) = 33.4 \text{ A}$$

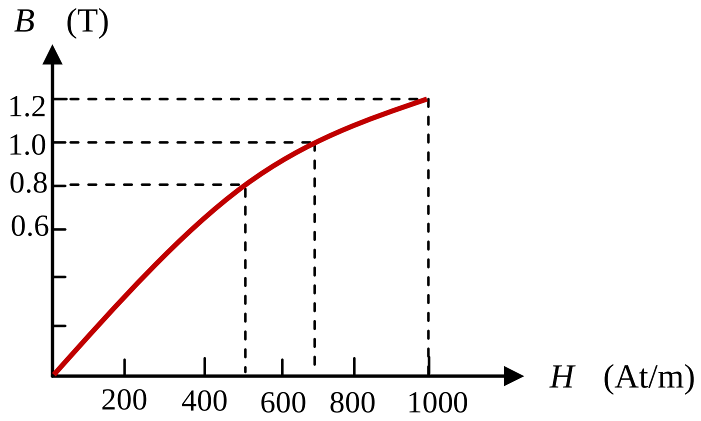
$$v_{dc} = ri = 167 \text{ V}$$



Not in scale

Magnetic Relay with Single Excitation

Solution 1: $B_g = B_c = 1\ \mathrm{T}$



$$W_{field} = V_c \int_0^1 H_c dB + V_g \frac{B^2}{2\mu_0}$$

$$V_g = 2\times 0.005\times 0.1\times 0.05 = 5\times 10^{-5}\ \ \mathrm{m}^3$$

$$V_c = 0.6\times 0.1\times 0.05 = 3\times 10^{-3}\ \ \mathrm{m}^3$$

$$W_{field} = 3\times 10^{-3}\times 335 + 5\times 10^{-5}\times \frac{1^2}{2\times 4\pi\times 10^{-7}}$$

$$W_{field} = 1.005 + 19.895 = 20.9\ \ \mathrm{J}$$

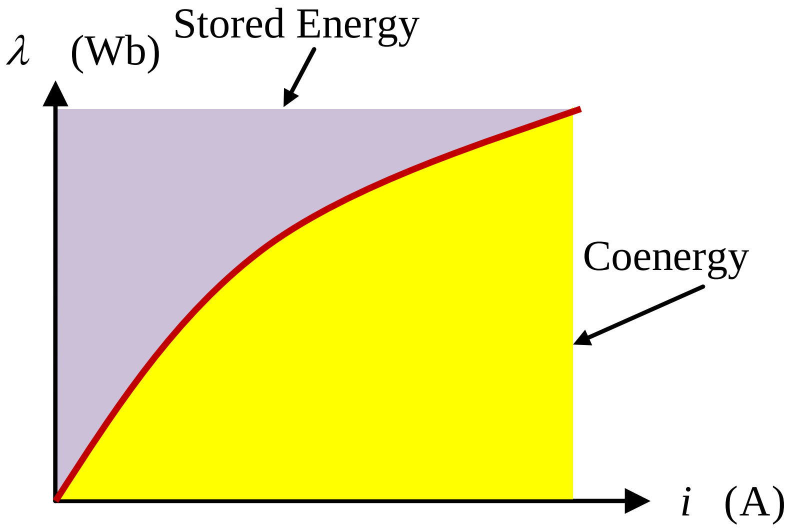
Energy and Coenergy

Energy

$$W_{field} = \int_0^{\lambda} i d\lambda$$

Coenergy

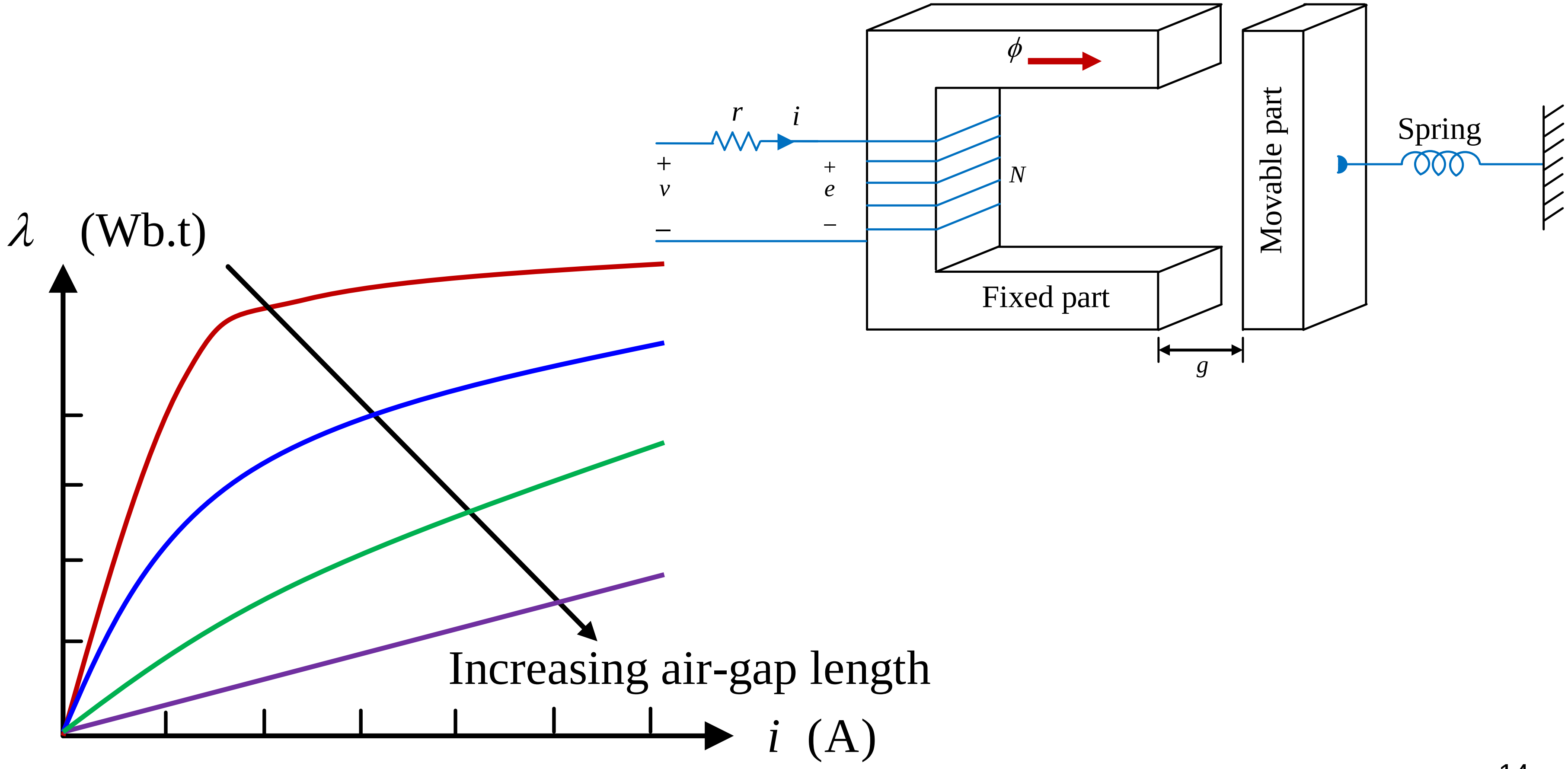
$$W'_{field} = \int_0^{i} \lambda di$$



$$W_{field} + W'_{field} = \lambda i$$

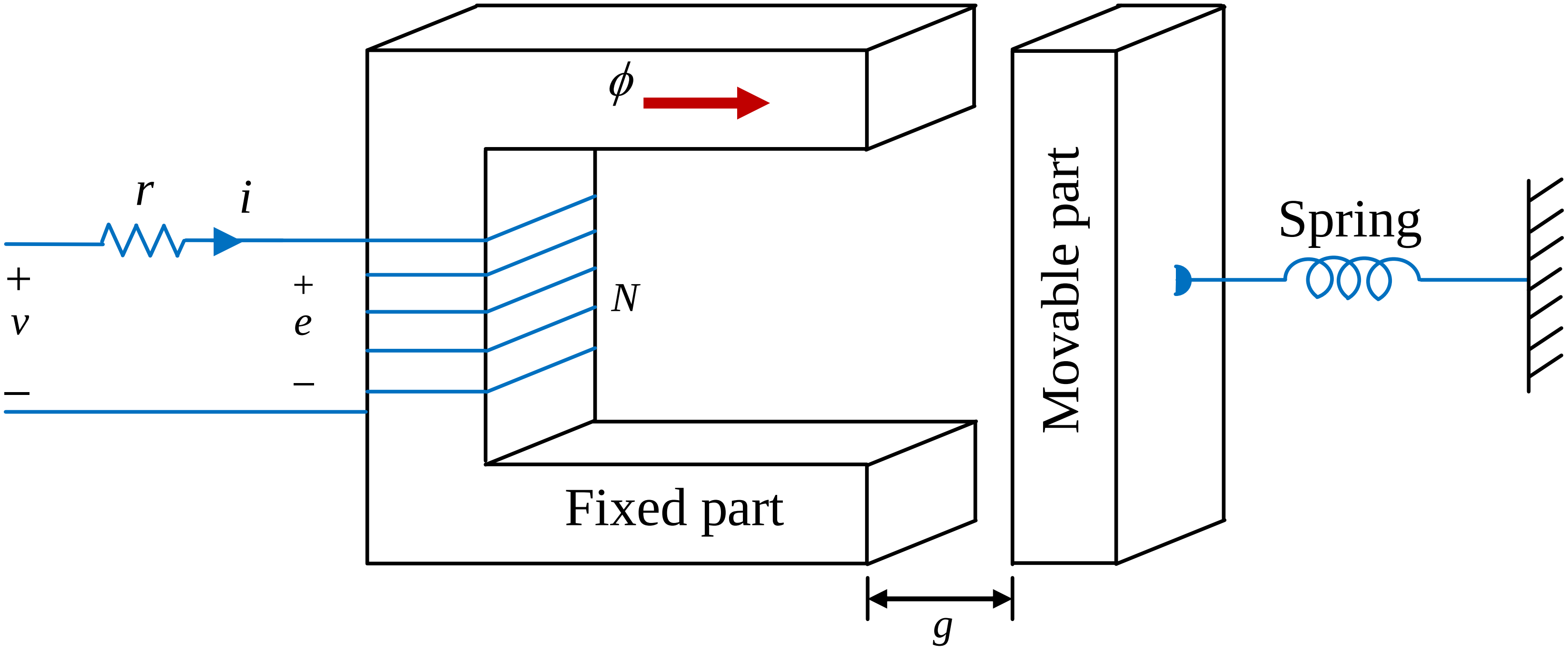
λ-i curve in system with changing air-gap

The λ-i curve varies with the air-gap length



Magnetic Relay with Single Excitation

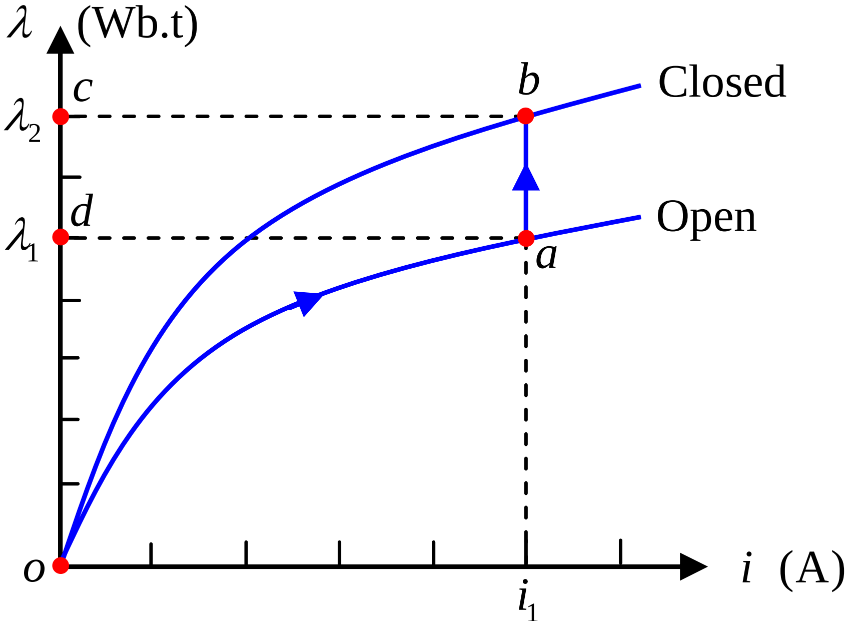
Assumption 2: The movable part can move but **slowly**

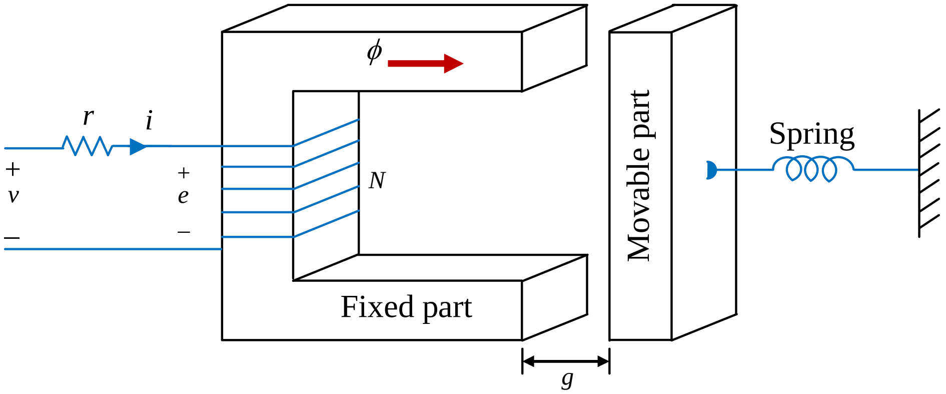


In this case the **current remains constant** during the movement.

Magnetic Relay with Single Excitation

Assumption 2: The movable part can move but **slowly**





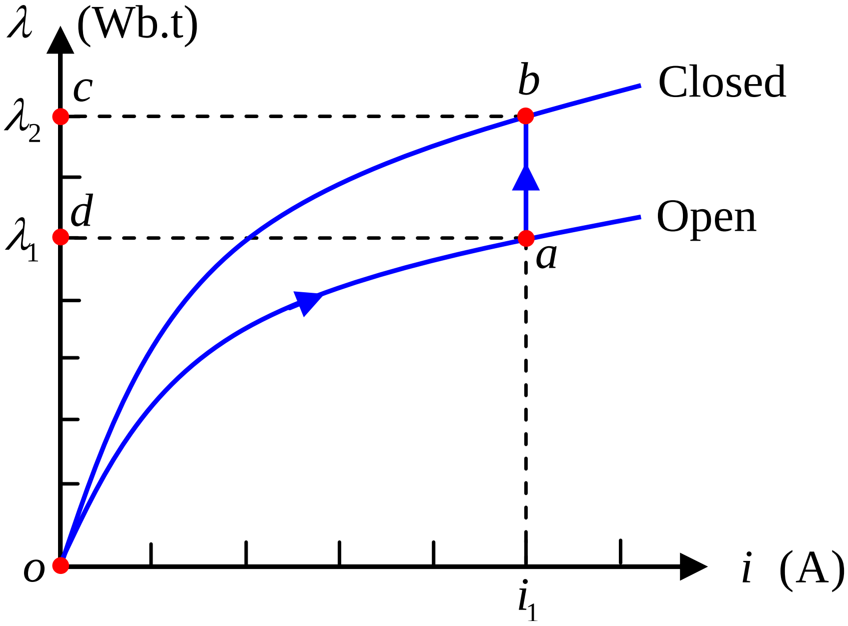
$$dW_{elec} = dW_{field} + dW_{mech}$$

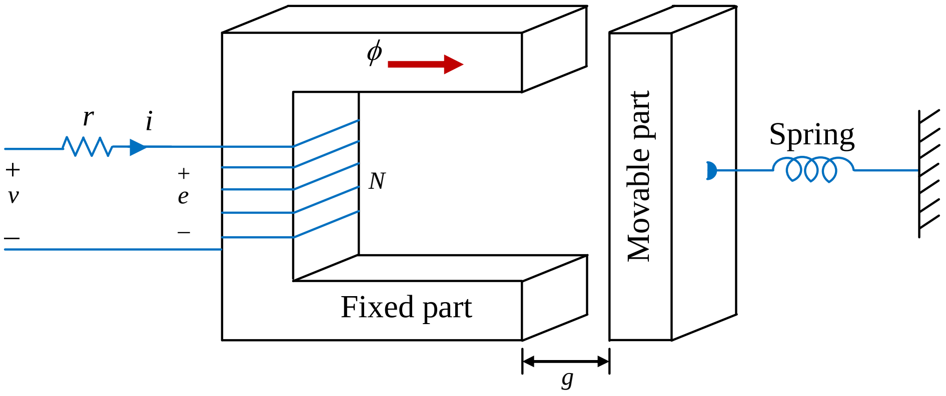
From o to a: No movement yet, therefore $dW_{mech} = 0$

$$dW_{elec} = dW_{field} = id\lambda = A_{oad}$$

Magnetic Relay with Single Excitation

Assumption 2: The movable part can move but **slowly**





$$dW_{elec} = dW_{field} + dW_{mech}$$

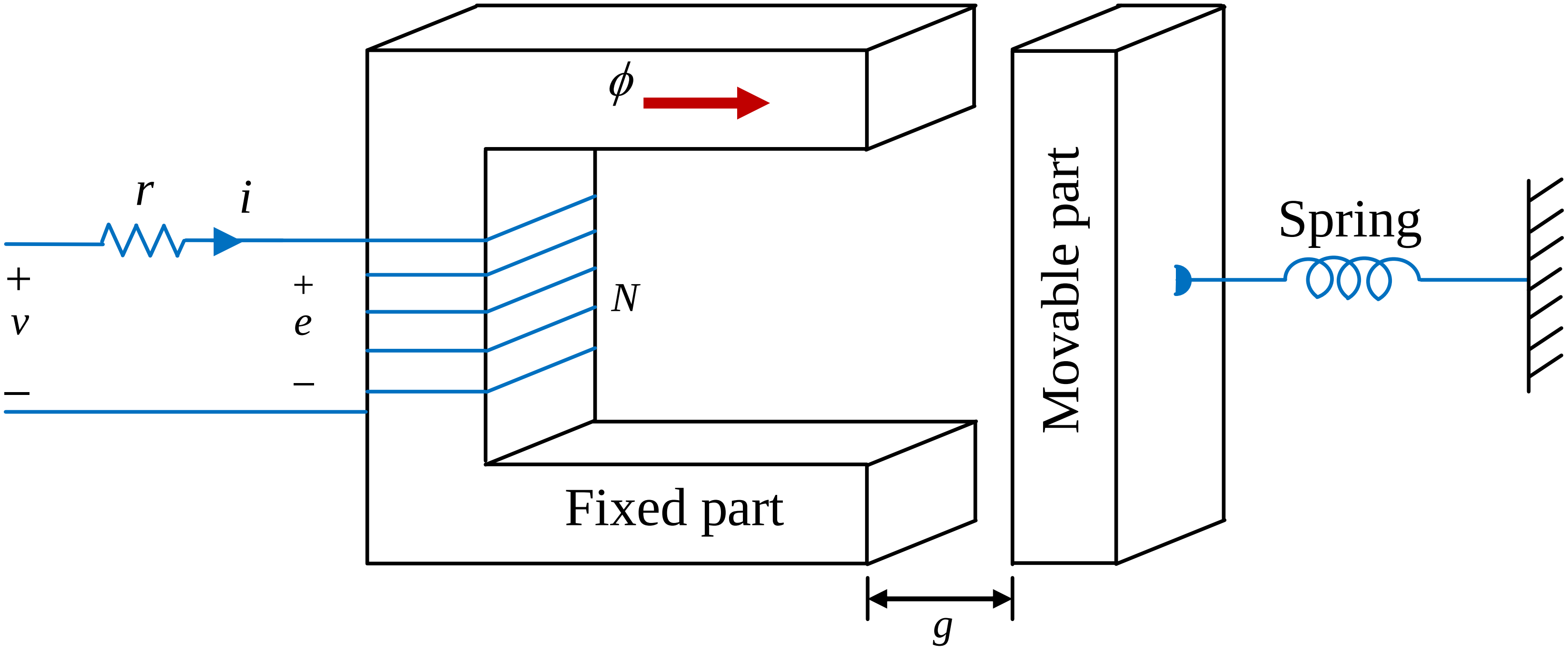
From *a* to *b*: $dW_{elec} = id\lambda = i_1(\lambda_2 - \lambda_1) = A_{abcd}$

$$dW_{field} = W_{field(b)} - W_{field(a)} = A_{obc} - A_{oad}$$

$$dW_{mech} = A_{oab}$$

Magnetic Relay with Single Excitation

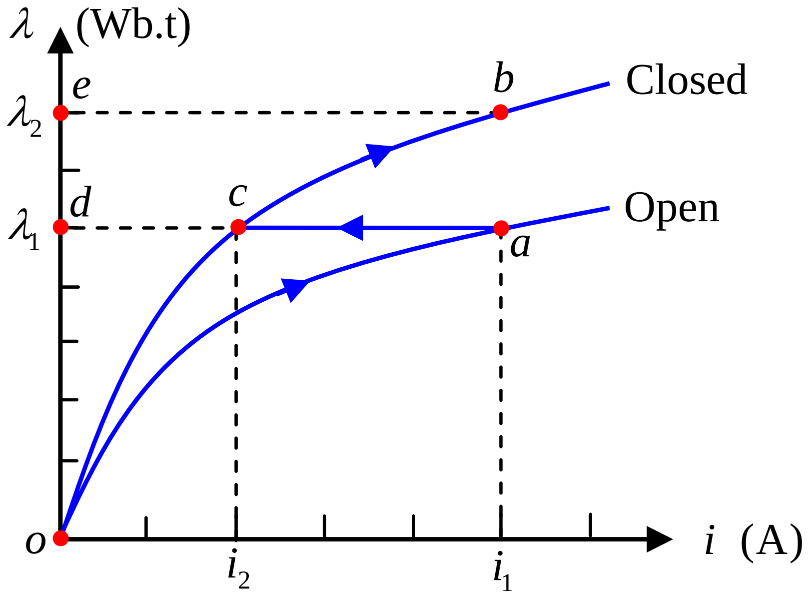
Assumption 3: The movable part can move but **very fast**

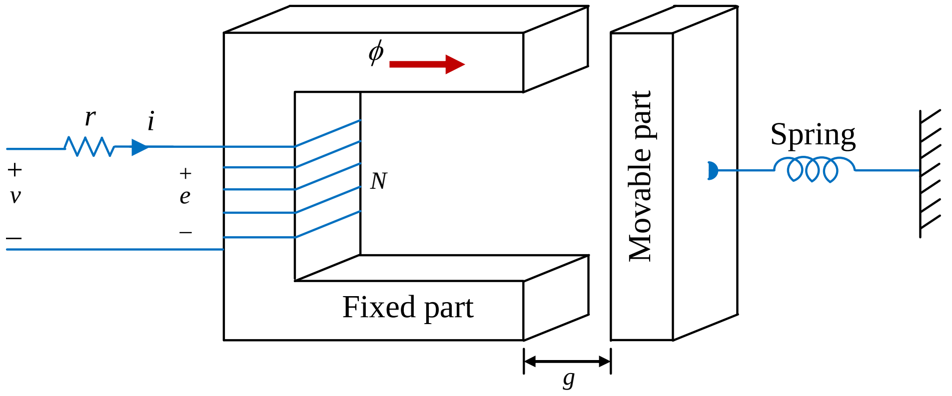


In this case the **flux linkage remains constant** during the movement.

Magnetic Relay with Single Excitation

Assumption 3: The movable part can move but **very fast**





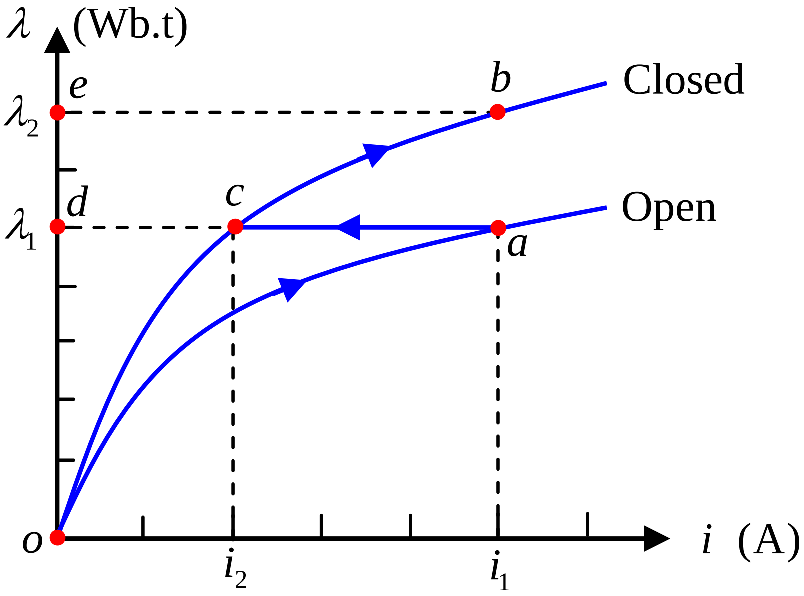
$$dW_{elec} = dW_{field} + dW_{mech}$$

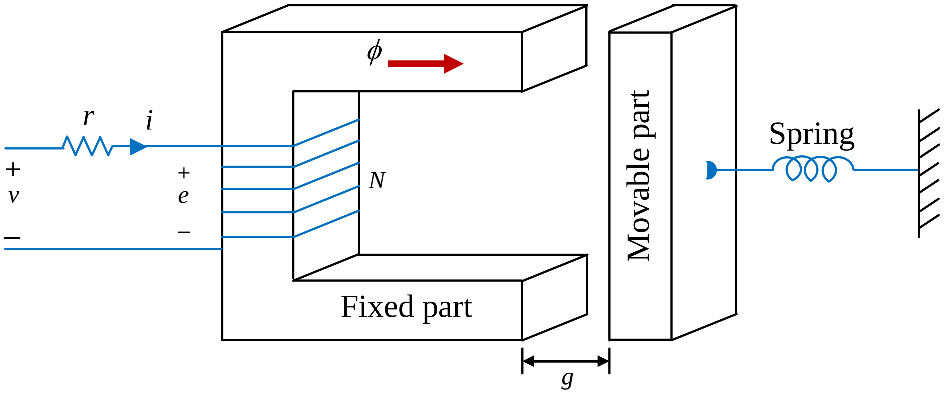
From *o* to *a*: No movement yet, therefore $dW_{mech} = 0$

$$dW_{elec} = dW_{field} = i d\lambda = A_{oad}$$

Magnetic Relay with Single Excitation

Assumption 3: The movable part can move but **very fast**





$$dW_{elec} = dW_{field} + dW_{mech}$$

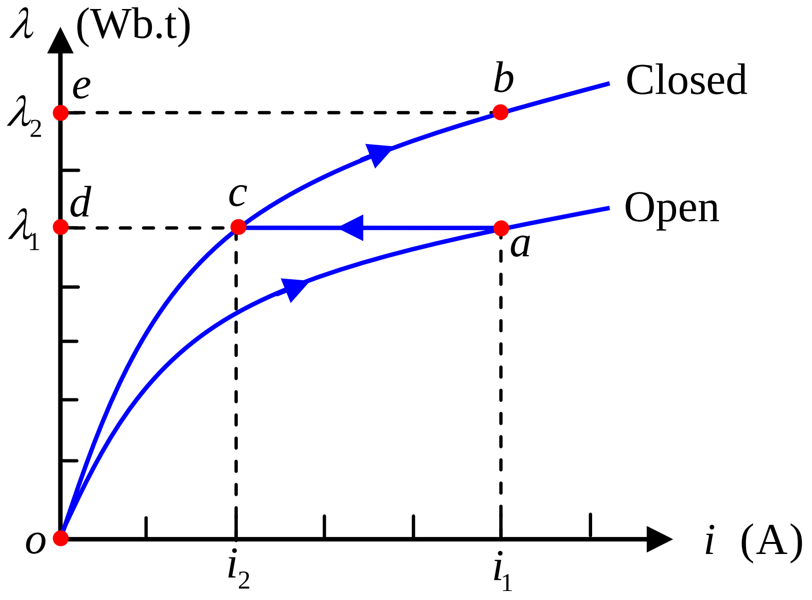
From a to c: $dW_{elec} = id\lambda = 0$

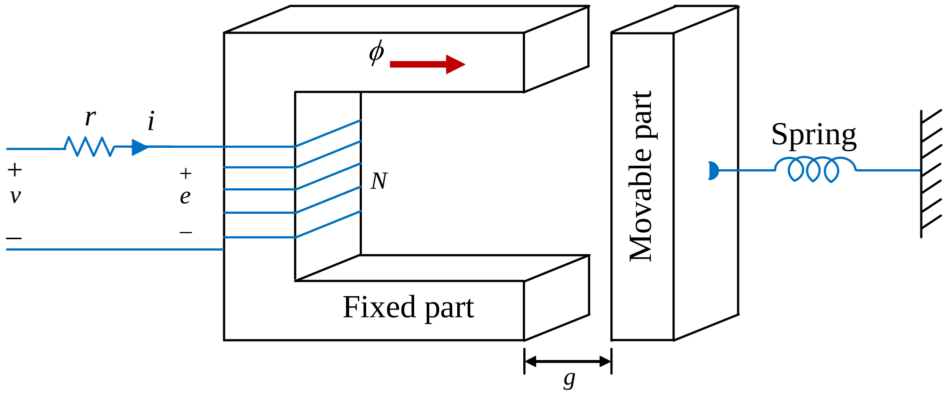
$$dW_{field} = W_{field(c)} - W_{field(a)} = A_{ocd} - A_{oad}$$

$$dW_{mech} = A_{oac}$$

Magnetic Relay with Single Excitation

Assumption 3: The movable part can move but **very fast**





$$dW_{elec} = dW_{field} + dW_{mech}$$

From c to b: No movement, therefore $dW_{mech} = 0$

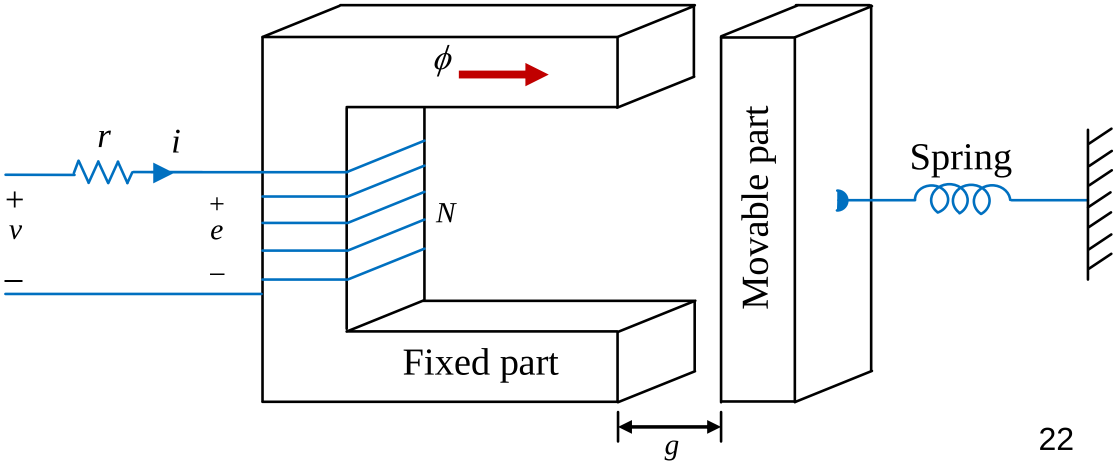
$$dW_{elec} = dW_{field} = id\lambda = A_{cbed}$$

Some Questions (H.W)

1- Why does the current remain constant during the slow movement?

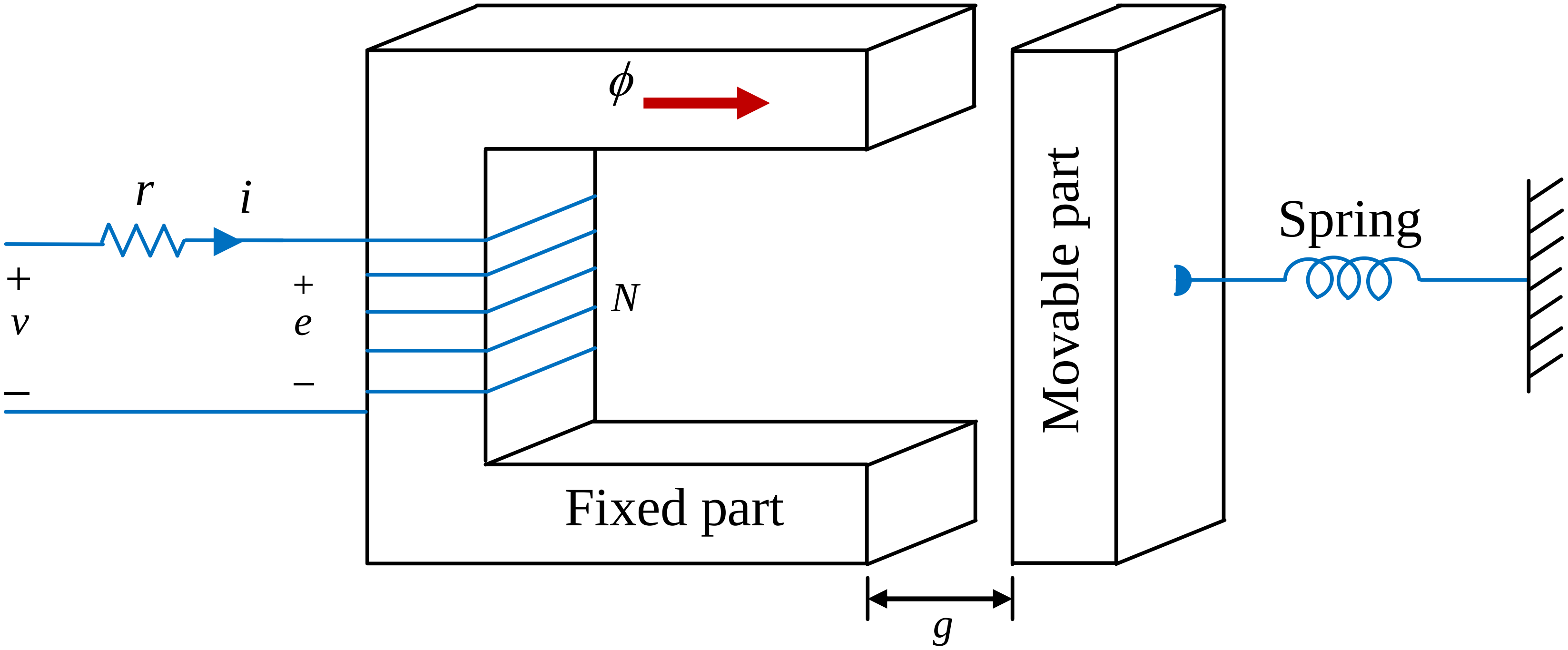
2- Why does the flux linkage remain constant during the fast movement?

3- Why should the currents at the open and closed stages be the same?



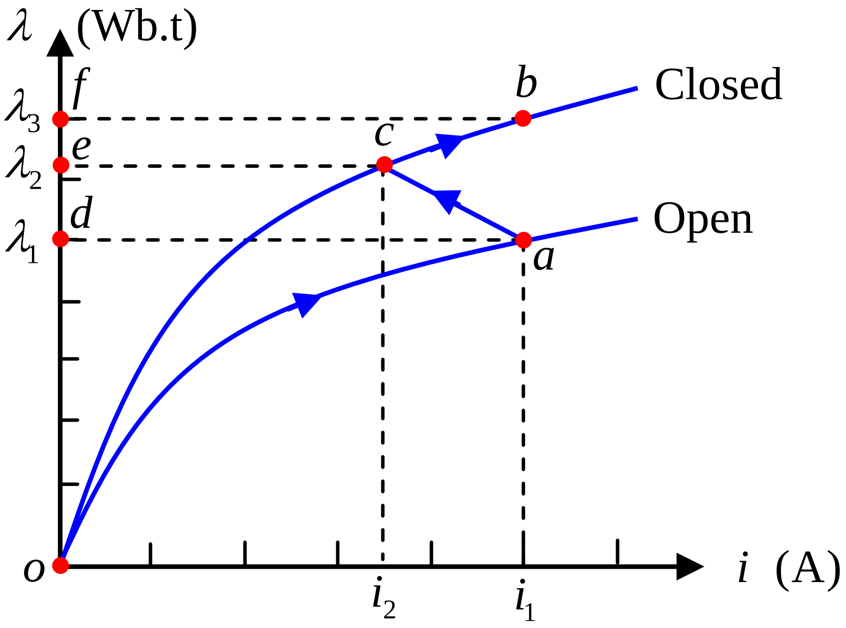
Magnetic Relay with Single Excitation

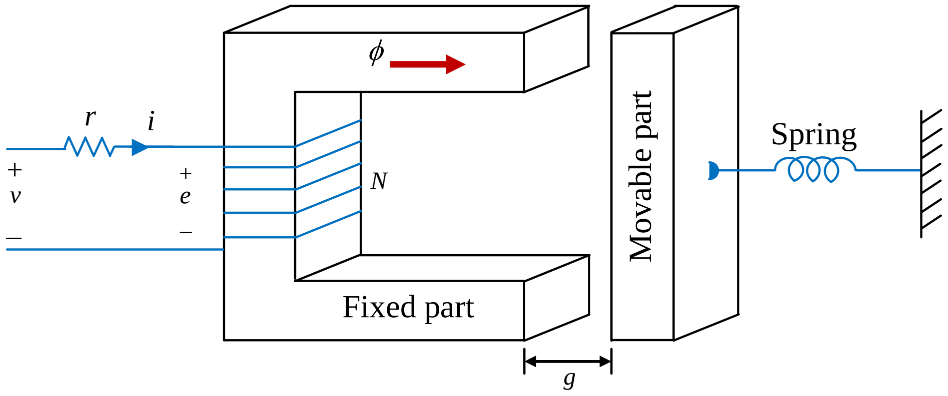
Assumption 4: The movable part moves with **normal speed**



Magnetic Relay with Single Excitation

Assumption 4: The movable part moves with **normal speed**





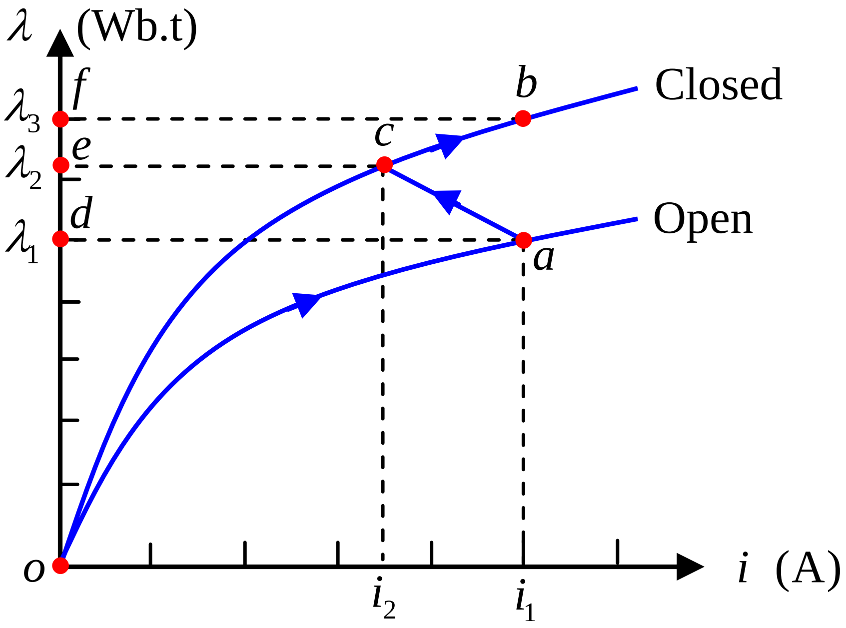
$$dW_{elec} = dW_{field} + dW_{mech}$$

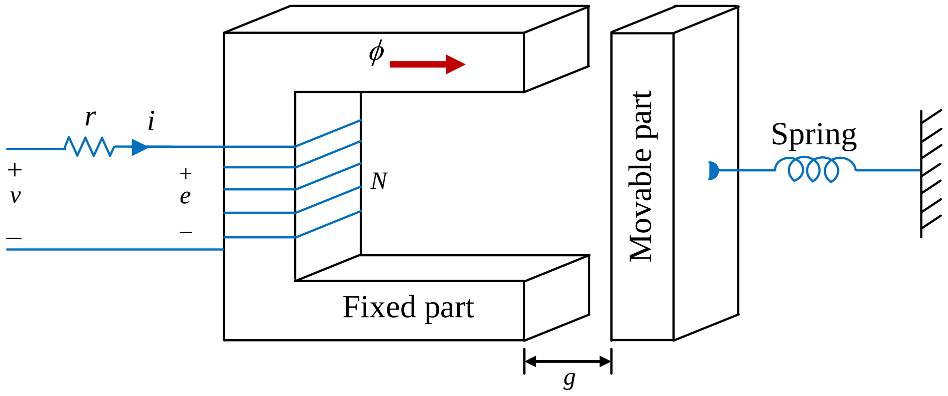
From *o* to *a*: No movement yet, therefore $dW_{mech} = 0$

$$dW_{elec} = dW_{field} = id\lambda = A_{oad}$$

Magnetic Relay with Single Excitation

Assumption 4: The movable part moves with **normal speed**





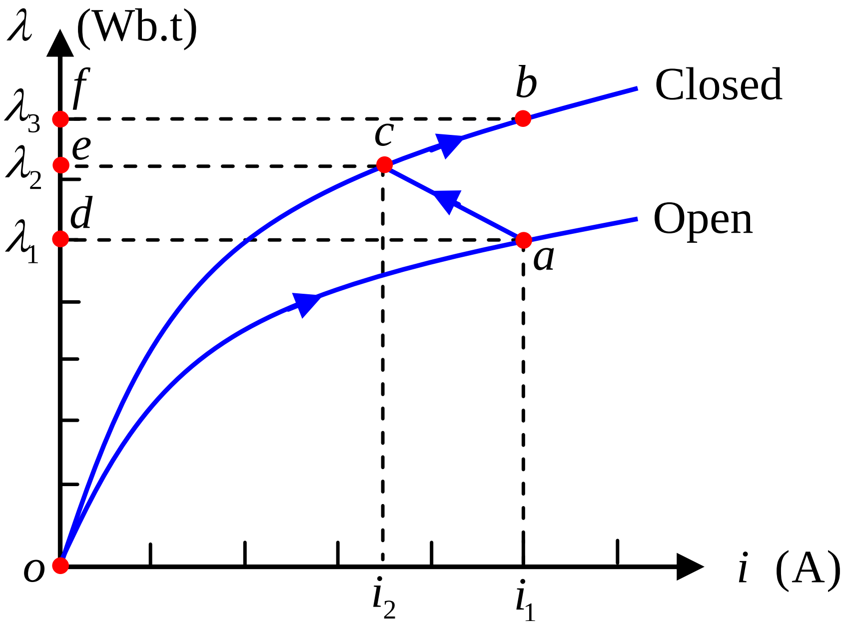
$$dW_{elec} = dW_{field} + dW_{mech}$$

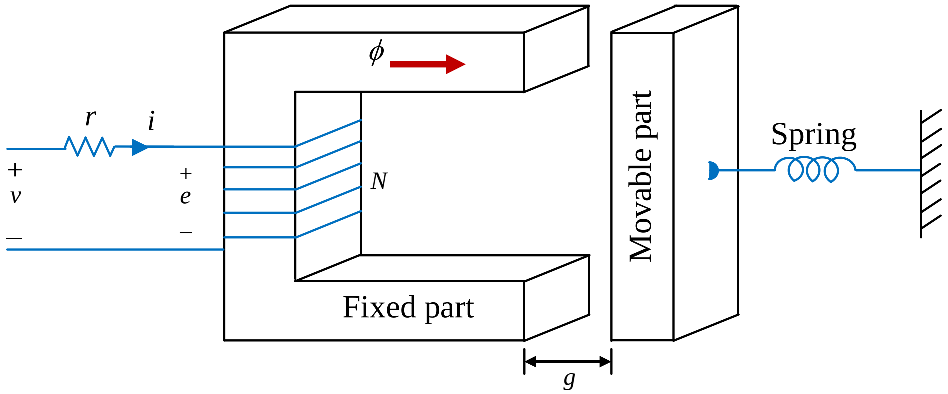
From *a* to *c*:

$$\left.\begin{array}{l} dW_{elec} = i d\lambda = A_{aced} \\ \\ dW_{field} = W_{field(c)} - W_{field(a)} = A_{oce} - A_{oad} \end{array}\right\} \quad \boxed{dW_{mech} = A_{oac}}$$

Magnetic Relay with Single Excitation

Assumption 4: The movable part moves with **normal speed**





$$dW_{elec} = dW_{field} + dW_{mech}$$

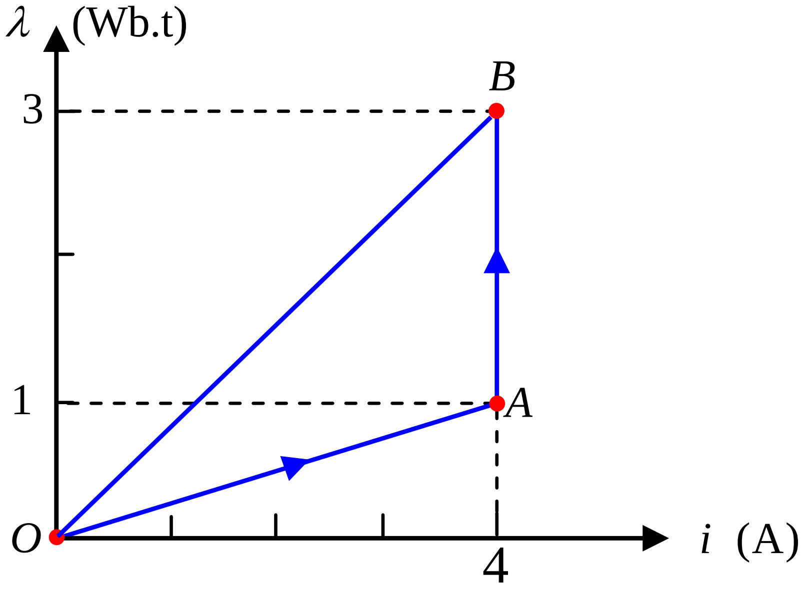
From c to b: No Movement, therefore $dW_{mech} = 0$

$$dW_{elec} = dW_{field} = id\lambda = A_{cbfe}$$

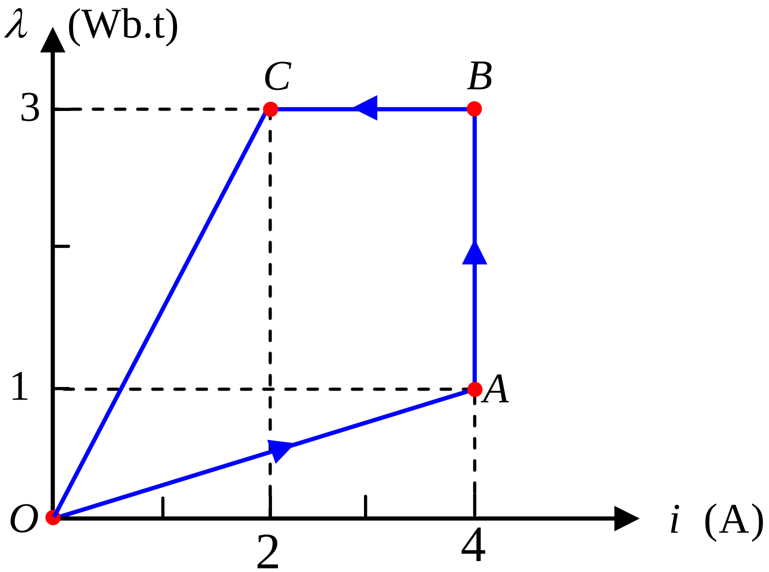
Electromechanical Energy Conversion

Example 2: The energy conversion cycles of two machines are OABO and OABCO curves shown below. If the energy conversion efficiency is defined as follows, calculate R_1 and R_2

$$R = \frac{\text{Converted Energy}}{\text{Input Electrical Energy}}$$



(1)



(2)

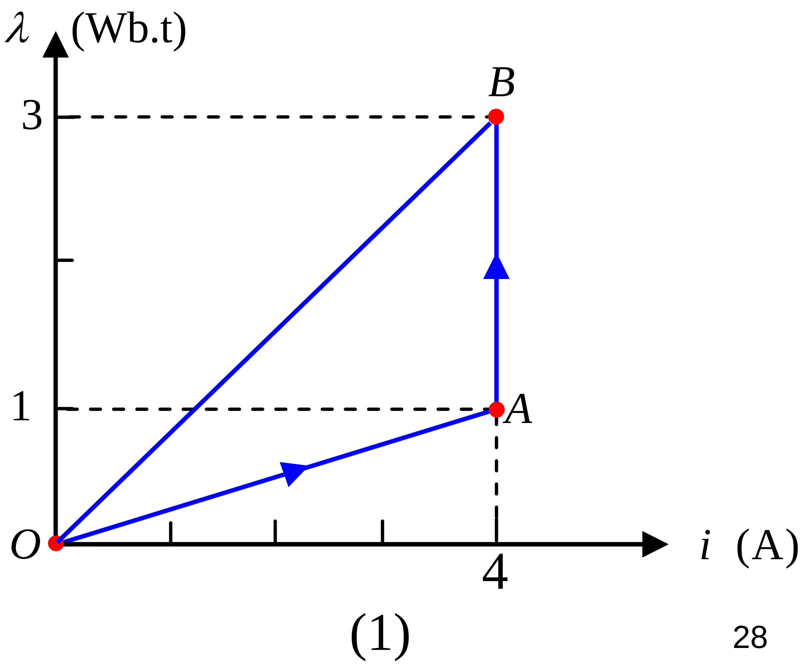
Electromechanical Energy Conversion

Solution 2: Part (1)

$$W_{elec1} = \int_{\lambda_O}^{\lambda_A} i d\lambda + \int_{\lambda_A}^{\lambda_B} i d\lambda = \int_0^1 4\lambda d\lambda + \int_1^3 4 d\lambda = 10$$

$$W_{mech\,1} = A_{OABO} = 4$$

$$R_1 = \frac{W_{mech\,1}}{W_{elec1}} = \frac{4}{10} = 0.4$$



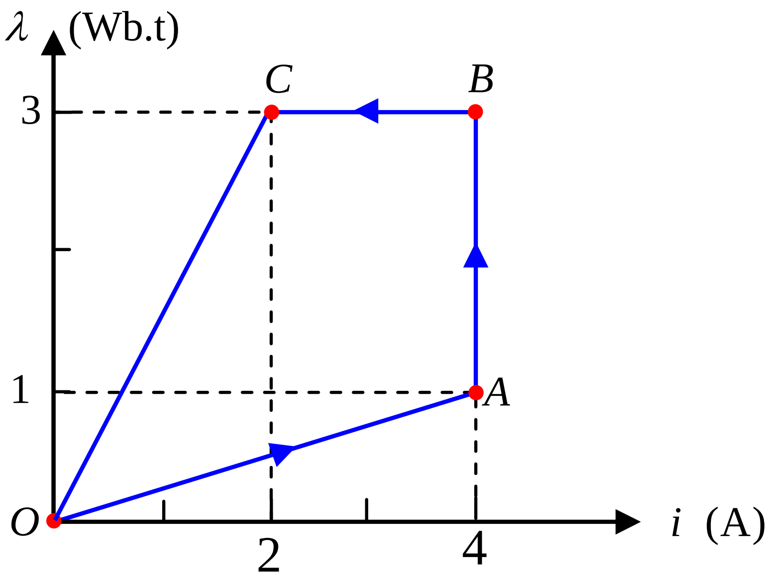
Electromechanical Energy Conversion

Solution 2: Part (2)

$$W_{elec2} = \int_{\lambda_O}^{\lambda_A} i d\lambda + \int_{\lambda_A}^{\lambda_B} i d\lambda + \int_{\lambda_B}^{\lambda_C} i d\lambda = \int_0^1 4\lambda d\lambda + \int_1^3 4 d\lambda \int_3^3 i d\lambda = 10$$

$$W_{mech\ 2} = A_{OABCO} = 7$$

$$R_2 = \frac{W_{mech\ 2}}{W_{elec2}} = \frac{7}{10} = 0.7$$

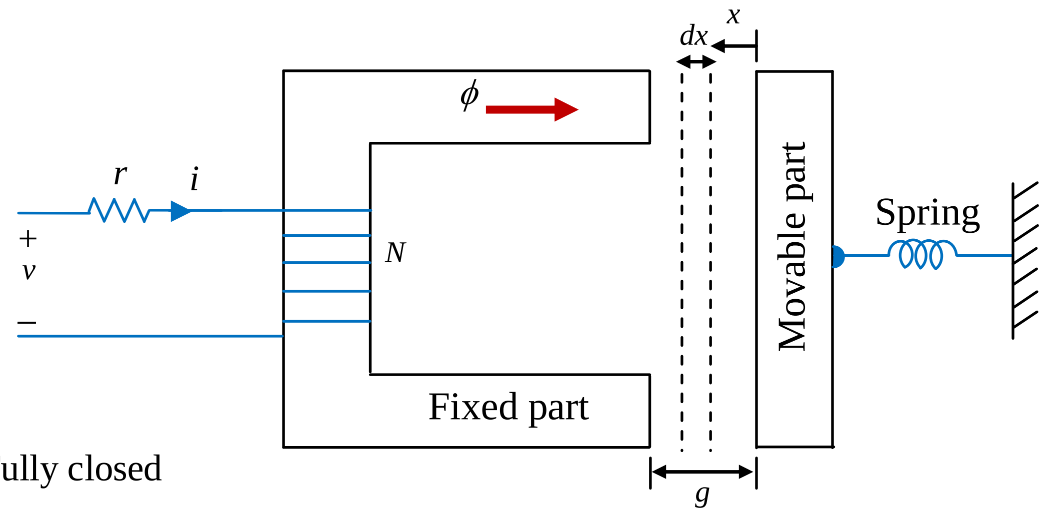


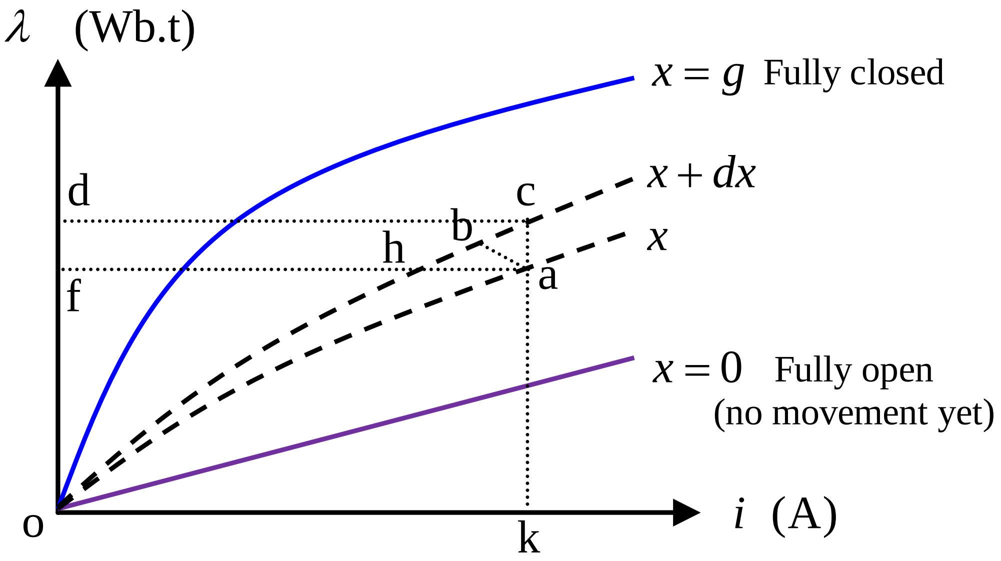
(2)

Mechanical Force

The average force is calculated as the mechanical work divided by the displacement

$$F_{ave} = \frac{\text{Mechanical Work}}{\text{Displacement}}$$





Mechanical Force

$$dW_{mech} = A_{oab}$$

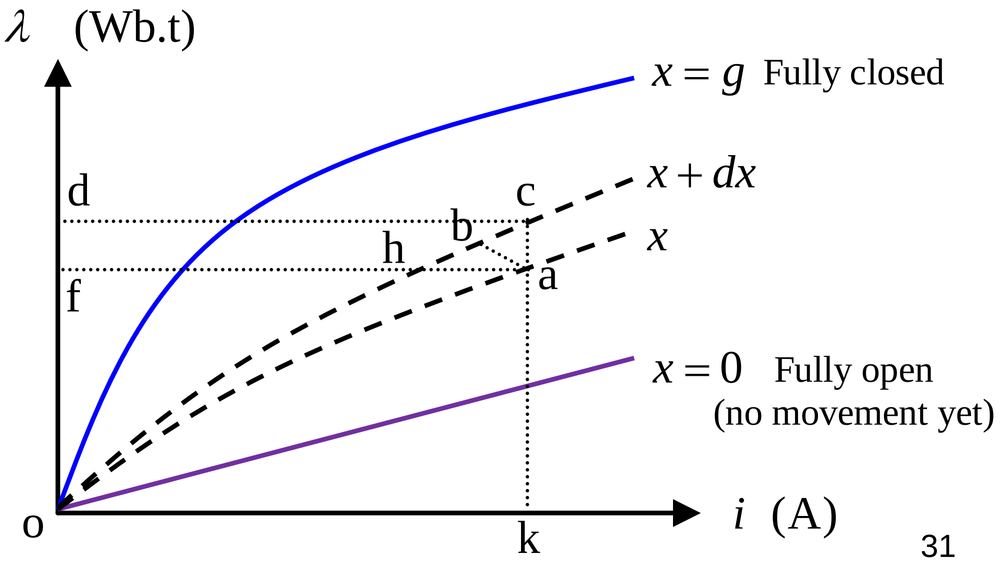
$$dW_{elec} = dW_{field} + dW_{mech}$$

- If the flux linkage is constant (fast movement),

$$\left.\begin{aligned} dW_{mech} &= A_{oah} \\ dW_{elec} &= 0 \end{aligned}\right\} \quad dW_{mech} = -dW_{field}$$

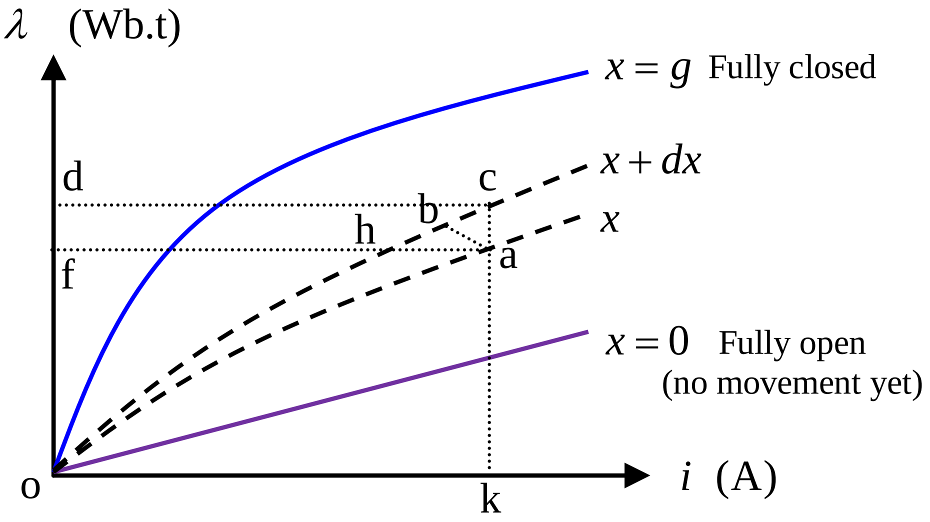
$$\Rightarrow \quad Fdx = -dW_{field}$$

$$\Rightarrow \quad \boxed{F = -\left.\frac{\partial W_{field}(\lambda, x)}{\partial x}\right|_{\lambda=cte}}$$



Mechanical Force

$$dW_{mech} = A_{oab}$$



- If the current is constant (slow movement),

$$dW_{mech} = A_{oac}$$

$$dW_{elec} = A_{acdf} = A_{okcd} - A_{okaf}$$

$$dW_{elec} = \left(W_{field(c)} + W'_{field(c)}\right) - \left(W_{field(a)} + W'_{field(a)}\right) = dW_{field} + dW'_{field}$$

$$\boxed{dW_{elec} = dW_{field} + dW_{mech}} \Rightarrow dW_{mech} = dW'_{field}$$

$$\Rightarrow F\,dx = dW'_{field} \Rightarrow \boxed{F = \left.\frac{\partial W'_{field}(i,x)}{\partial x}\right|_{i=cte}}$$

Force and Torque Calculation

Translational movement Force	Rotational movement Torque
$F = -\left.\frac{\partial W_{field}(\lambda, x)}{\partial x}\right\|_{\lambda=cte}$	$T = -\left.\frac{\partial W_{field}(\lambda, \theta)}{\partial \theta}\right\|_{\lambda=cte}$
$F = \left.\frac{\partial W'_{field}(i, x)}{\partial x}\right\|_{i=cte}$	$T = \left.\frac{\partial W'_{field}(i, \theta)}{\partial \theta}\right\|_{i=cte}$

where θ is the angular position.

Electromechanical Energy Conversion

Example 3: In an electromechanical system $i = (\lambda x/0.09)^2$ for $0 \le i \le 4$ and $3 \le x \le 10^{\text{ cm}}$. Calculate the force exerted on the movable part if the current is 3 A and air-gap length is 5 cm.

Solution: 1st method (Coenergy)

$$W'_{field} = \int_0^i \lambda di$$

$$F = \left.\frac{\partial W'_{field}(i,x)}{\partial x}\right|_{i=cte}$$

2nd method (Energy)

$$W_{field} = \int_0^\lambda i d\lambda$$

$$F = -\left.\frac{\partial W_{field}(\lambda,x)}{\partial x}\right|_{\lambda=cte}$$

Electromechanical Energy Conversion

$i = (\lambda x / 0.09)^2 \qquad 0 \le i \le 4 \qquad 3 \le x \le 10 \ ^{\text{cm}}$

$i = 3\ \text{A} \qquad x = 5\ \text{cm} \qquad F = ?$

Solution 3: 1st method (Coenergy)

$$\lambda = \frac{0.09\sqrt{i}}{x} \Rightarrow W'_{field} = \int_0^i \lambda di = \int_0^i \frac{0.09\sqrt{i}}{x} di = \frac{0.09}{x}\frac{2}{3} i^{3/2}$$

$$F = \left.\frac{\partial W'_{field}(i,x)}{\partial x}\right|_{i=cte} = -\frac{0.09}{x^2}\frac{2}{3} i^{3/2}$$

For $i = 3\ \text{A}$ and $x = 5\ \text{cm}$ $\Rightarrow$ $F = -\frac{0.09}{x^2}\frac{2}{3} i^{3/2} = -124\ \ \text{N}$

Electromechanical Energy Conversion

$i = (\lambda x / 0.09)^2$ $\qquad 0 \le i \le 4 \qquad 3 \le x \le 10$ cm

$i = 3$ A $\qquad x = 5$ cm $\qquad F = ?$

Solution 3: 2nd method (Energy)

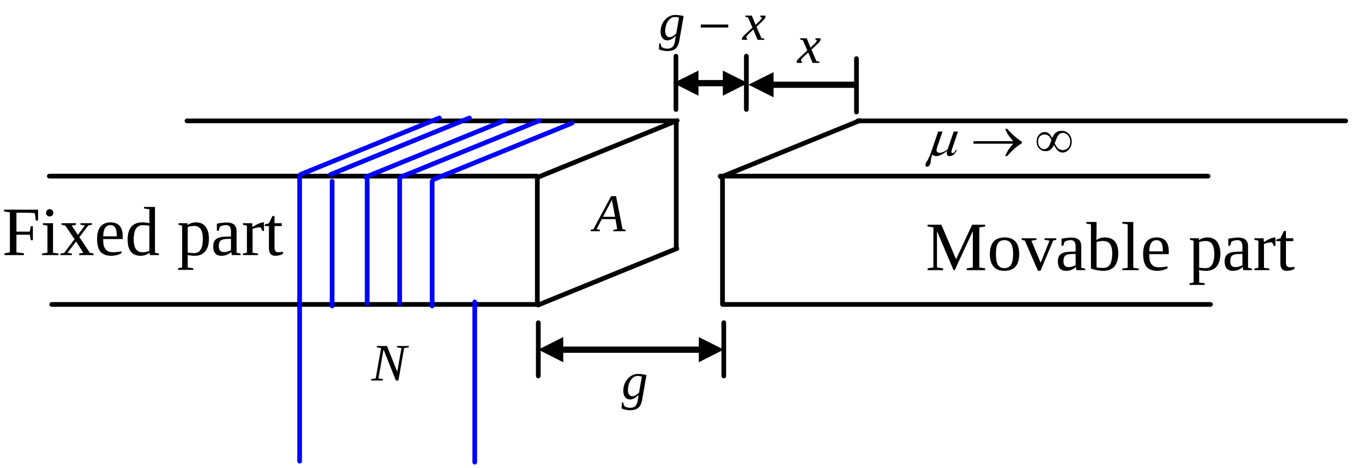
$$W_{field} = \int_0^\lambda i d\lambda = \int_0^\lambda \left(\frac{\lambda x}{0.09} \right)^2 d\lambda = \frac{x^2}{0.09^2} \frac{\lambda^3}{3}$$

$$F = -\frac{\partial W_{field}(\lambda, x)}{\partial x}\bigg|_{\lambda = cte} = -\frac{2x}{0.09^2}\frac{\lambda^3}{3} = -\frac{2x}{0.09^2}\frac{1}{3}\left(\frac{0.09\sqrt{i}}{x}\right)^3$$

For $i = 3$ A and $x = 5$ cm → $F = -\frac{2x}{0.09^2}\frac{1}{3}\left(\frac{0.09\sqrt{i}}{x}\right)^3 = -124.7$ N

Electromechanical Energy Conversion

Example 4: In the following system assume flux is constant during the movement, calculate the average force.

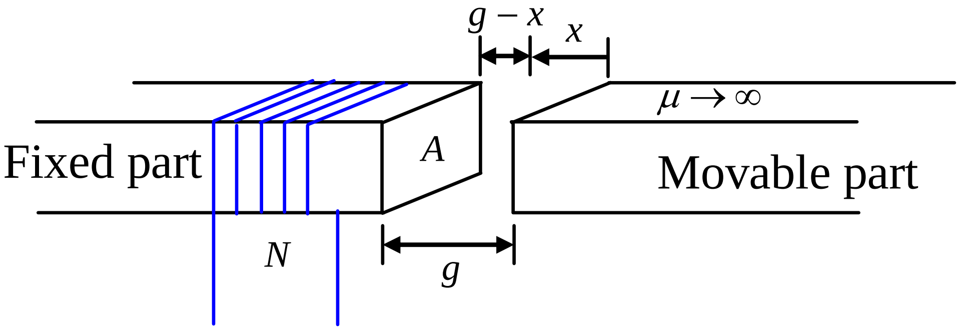


Electromechanical Energy Conversion

Solution 4: Since the permeability of the core goes to infinity, the system is linear.

$\lambda = Li$ $\lambda = N\phi$

$$L = \frac{N^2}{\Re}$$



$$W_{field} = \tfrac{1}{2}Li^2 = \tfrac{1}{2}\lambda i = \tfrac{1}{2}\frac{\lambda^2}{L} = \tfrac{1}{2}\Re\phi^2 \Rightarrow W_{field} = \tfrac{1}{2}\left(\frac{g-x}{\mu_0 A}\right)\phi^2$$

$$F = -\left.\frac{\partial W_{field}(\lambda, x)}{\partial x}\right|_{\lambda=cte} = \tfrac{1}{2}\phi^2\frac{1}{\mu_0 A}$$

Electromechanical Energy Conversion

Example 5: In an electromechanical system the $\lambda - i$ characteristics is defined as $i = \lambda^{3/2} + 2.5\lambda(x-1)^2$ for $0 < x < 1^{\mathrm{m}}$, calculate the average force when $x = 0.6^{\mathrm{m}}$.

$$W_{field} = \int_0^{\lambda} i d\lambda = \int_0^{\lambda} \left(\lambda^{3/2} + 2.5\lambda(x-1)^2\right) d\lambda$$

$$W_{field} = \tfrac{2}{5}\lambda^{5/2} + \tfrac{5}{4}\lambda^2(x-1)^2$$

$$F = -\frac{\partial W_{field}(\lambda, x)}{\partial x}\bigg|_{\lambda = cte} = -\frac{5}{4} 2\lambda^2(x-1)$$

$$F(x = 0.6) = \lambda^2$$

Developed Torque in Doubly Excited Systems

Consider the following electric motor with double excitation.

$$dW_{elec} = i_s d\lambda_s + i_r d\lambda_r$$

$$\begin{cases} \lambda_s = L_s i_s + M_{sr} i_r \\ \lambda_r = M_{sr} i_s + L_r i_r \end{cases}$$

where

i_s stator current

i_r rotor current

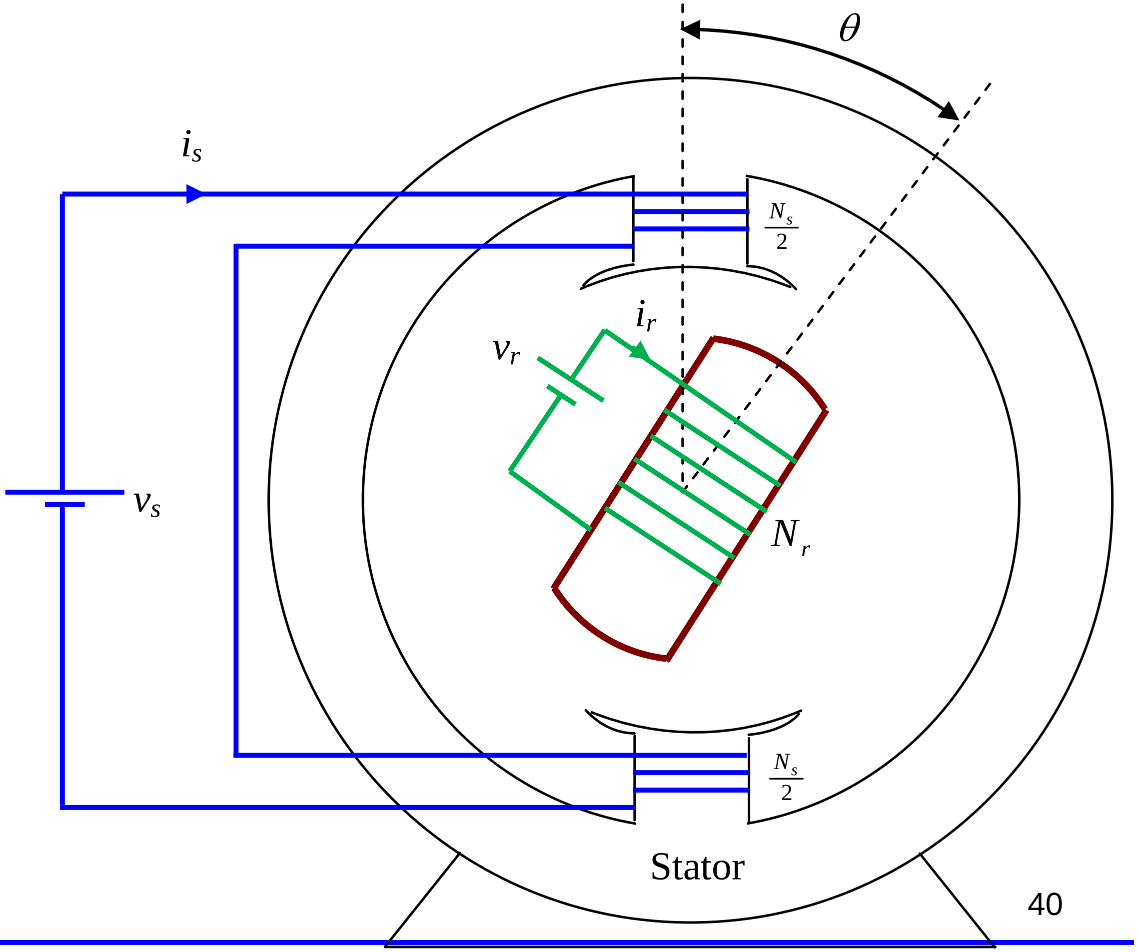
λ_s stator flux linkage

λ_r rotor flux linkage

L_s stator self-inductance

L_r rotor self-inductance

M_{sr} mutual inductance

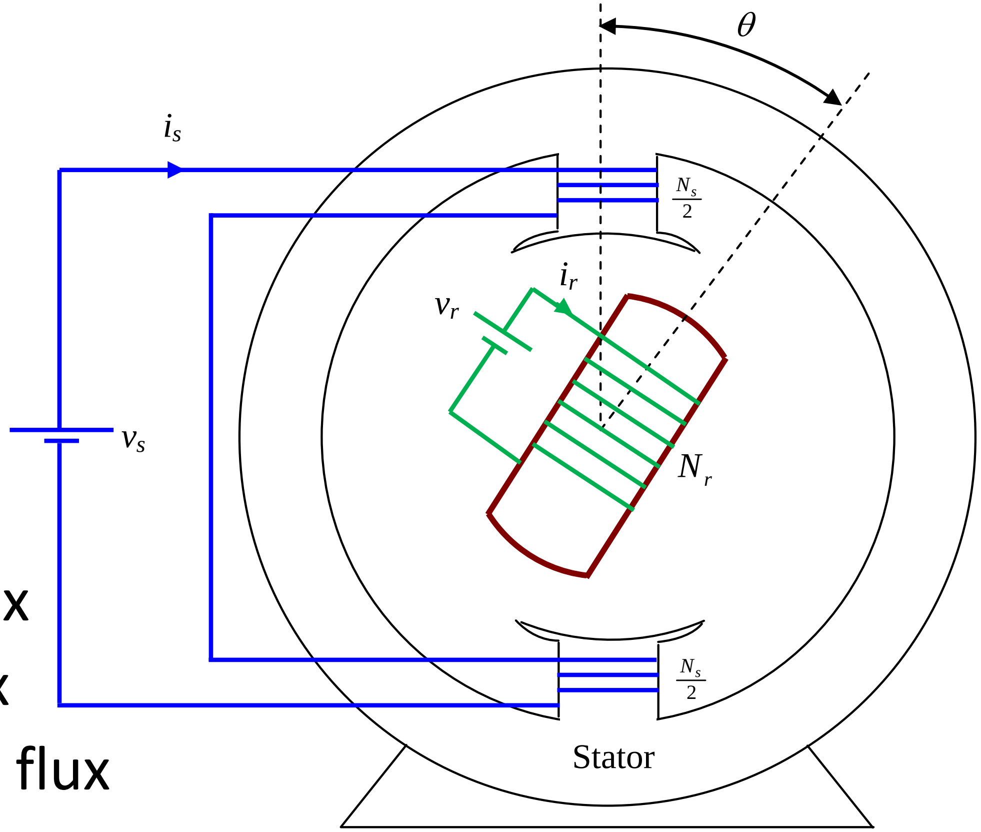


Developed Torque in Doubly Excited Systems

The inductances are defined as follows

$$L_s = \frac{N_s^2}{\Re_s} \qquad M_{sr} = \frac{N_s N_r}{\Re_{sr}}$$

$$L_r = \frac{N_r^2}{\Re_r}$$



where

N_s stator number of turns

N_r rotor number of turns

$\Re_s$ reluctance seen by stator flux

$\Re_r$ reluctance seen by rotor flux

$\Re_{sr}$ reluctance seen by resultant flux

Developed Torque in Doubly Excited Systems

Assumption 1: The rotor cannot rotate

$$dW_{elec} = i_s d\lambda_s + i_r d\lambda_r$$

$$\begin{cases} \lambda_s = L_s i_s + M_{sr} i_r \\ \lambda_r = M_{sr} i_s + L_r i_r \end{cases}$$

$$dW_{elec} = i_s d\left(L_s i_s + M_{sr} i_r\right) + i_r d\left(M_{sr} i_s + L_r i_r\right)$$

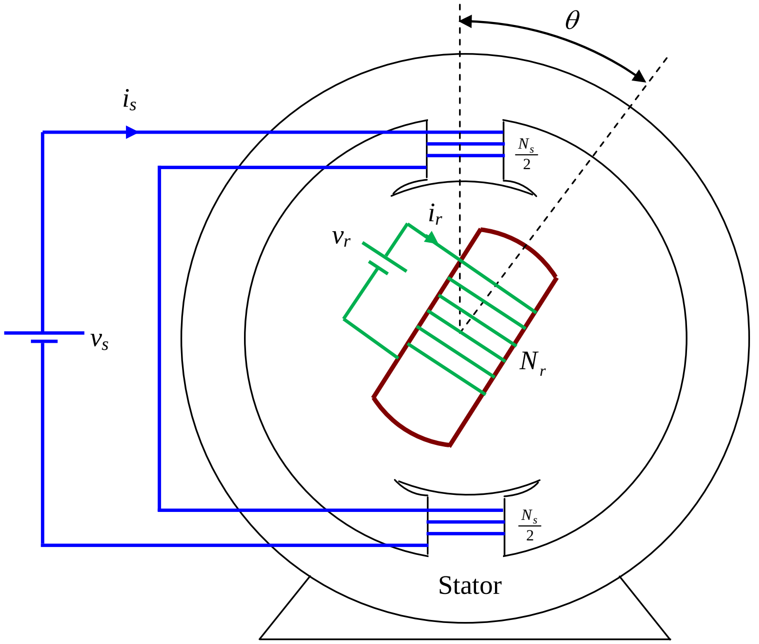
$$dW_{elec} = L_s i_s di_s + M_{sr} i_s di_r + M_{sr} i_r di_s + L_r i_r di_r$$

$$dW_{elec} = dW_{field} + \underset{0}{dW_{mech}}$$

$$dW_{field} = L_s i_s di_s + M_{sr} d\left(i_s i_r\right) + L_r i_r di_r$$

$$W_{field} = L_s \int_0^{i_s} i_s di_s + M_{sr} \int_0^{i_s i_r} d\left(i_s i_r\right) + L_r \int_0^{i_r} i_r di_r$$

$$\boxed{W_{field} = \tfrac{1}{2} L_s i_s^2 + M_{sr} i_s i_r + \tfrac{1}{2} L_r i_r^2} \quad \textbf{(1)}$$



Developed Torque in Doubly Excited Systems

Assumption 2: The rotor can rotate

$$dW_{elec} = i_s d\lambda_s + i_r d\lambda_r$$

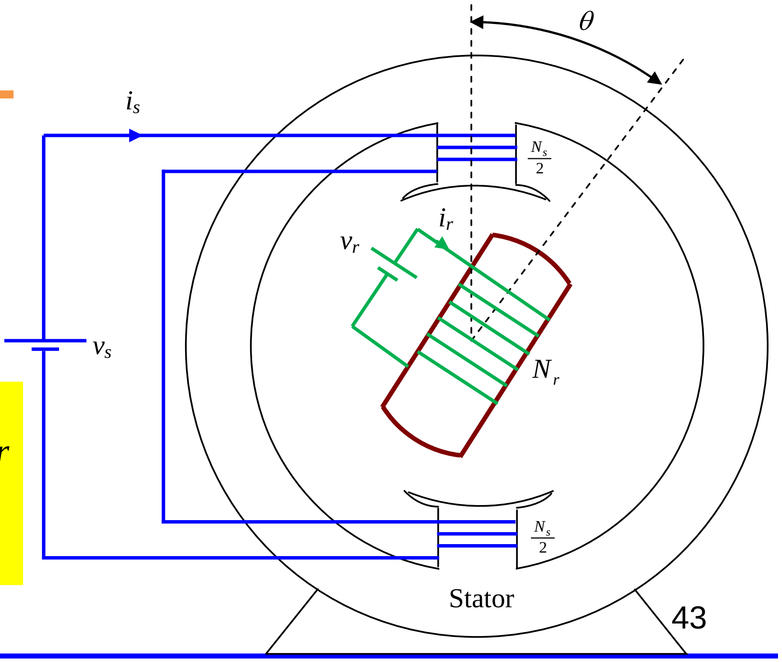
$$\begin{cases} \lambda_s = L_s i_s + M_{sr} i_r \\ \lambda_r = M_{sr} i_s + L_r i_r \end{cases}$$

$$dW_{elec} = i_s d\left(L_s i_s + M_{sr} i_r\right) + i_r d\left(M_{sr} i_s + L_r i_r\right)$$

$$\begin{aligned} dW_{elec} = {} & L_s i_s di_s + i_s^2 dL_s + M_{sr} i_s di_r + i_r i_s dM_{sr} \\ & + M_{sr} i_r di_s + i_s i_r dM_{sr} + L_r i_r di_r + i_r^2 dL_r \end{aligned}$$

Taking differentiation from the relation $W_{field} = \frac{1}{2} L_s i_s^2 + M_{sr} i_s i_r + \frac{1}{2} L_r i_r^2$ yields

$$\begin{aligned} dW_{field} = {} & L_s i_s di_s + \tfrac{1}{2} i_s^2 dL_s + M_{sr} i_s di_r + i_r i_s dM_{sr} \\ & + M_{sr} i_r di_s + L_r i_r di_r + \tfrac{1}{2} i_r^2 dL_r \end{aligned}$$



Developed Torque in Doubly Excited Systems

Assumption 2: The rotor can rotate

$$dW_{elec} = i_s d\lambda_s + i_r d\lambda_r$$

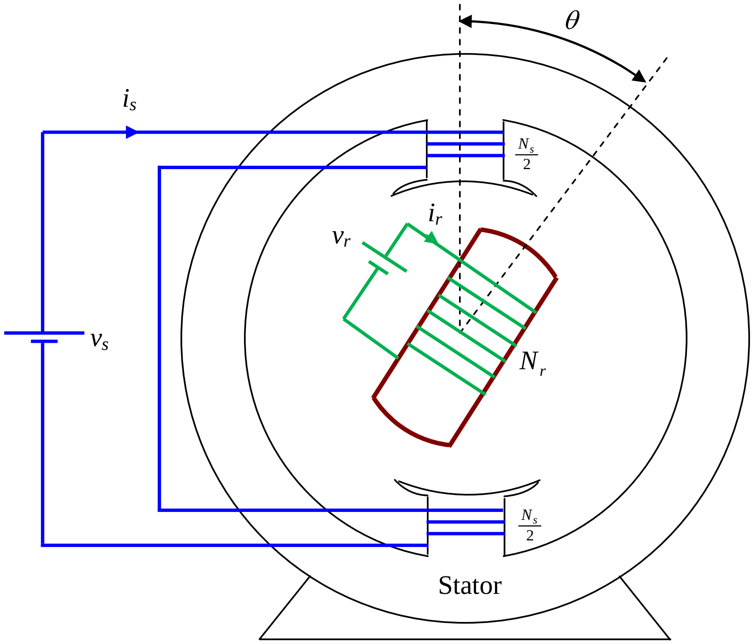
$$\begin{cases} \lambda_s = L_s i_s + M_{sr} i_r \\ \lambda_r = M_{sr} i_s + L_r i_r \end{cases}$$

$$\begin{aligned} dW_{elec} = {} & L_s i_s di_s + i_s^2 dL_s + M_{sr} i_s di_r + i_r i_s dM_{sr} \\ & + M_{sr} i_r di_s + i_s i_r dM_{sr} + L_r i_r di_r + i_r^2 dL_r \end{aligned}$$

$$\begin{aligned} dW_{field} = {} & L_s i_s di_s + \tfrac{1}{2} i_s^2 dL_s + M_{sr} i_s di_r + i_r i_s dM_{sr} \\ & + M_{sr} i_r di_s + L_r i_r di_r + \tfrac{1}{2} i_r^2 dL_r \end{aligned}$$

$$dW_{elec} = dW_{field} + dW_{mech}$$

$$\boxed{dW_{mech} = \tfrac{1}{2} i_s^2 dL_s + i_s i_r dM_{sr} + \tfrac{1}{2} i_r^2 dL_r}$$



Developed Torque in Doubly Excited Systems

Assumption 2: The rotor can rotate

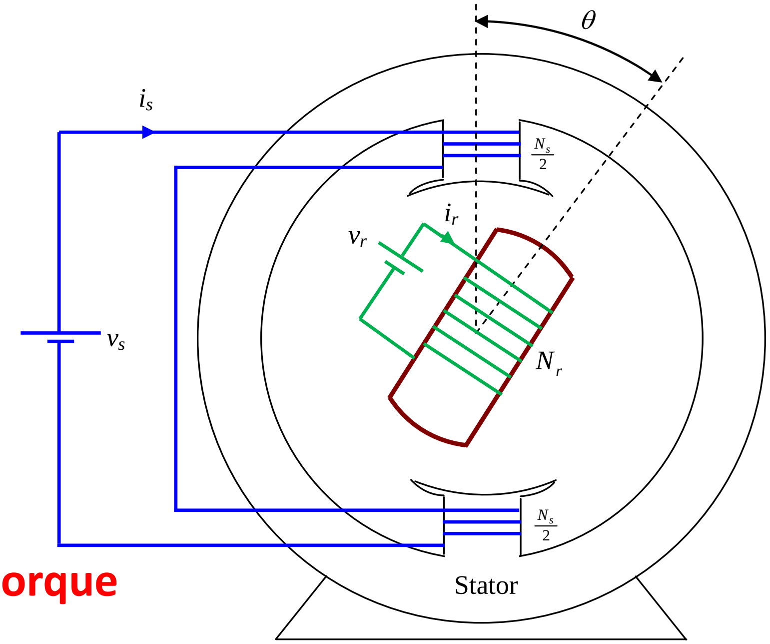
$$dW_{mech} = \tfrac{1}{2} i_s^2 dL_s + i_s i_r dM_{sr} + \tfrac{1}{2} i_r^2 dL_r$$

Since the torque is defined as

$$T = \frac{dW_{mech}}{d\theta}$$

it yields

$$T = \underbrace{\tfrac{1}{2} i_s^2 \frac{dL_s}{d\theta}}_{\text{Reluctance torque}} + \underbrace{i_s i_r \frac{dM_{sr}}{d\theta}}_{\text{Electromagnetic torque}} + \underbrace{\tfrac{1}{2} i_r^2 \frac{dL_r}{d\theta}}_{\text{Reluctance torque}}$$



Developed Torque in Doubly Excited Systems

Case 1: Both rotor and stator have salient structures:

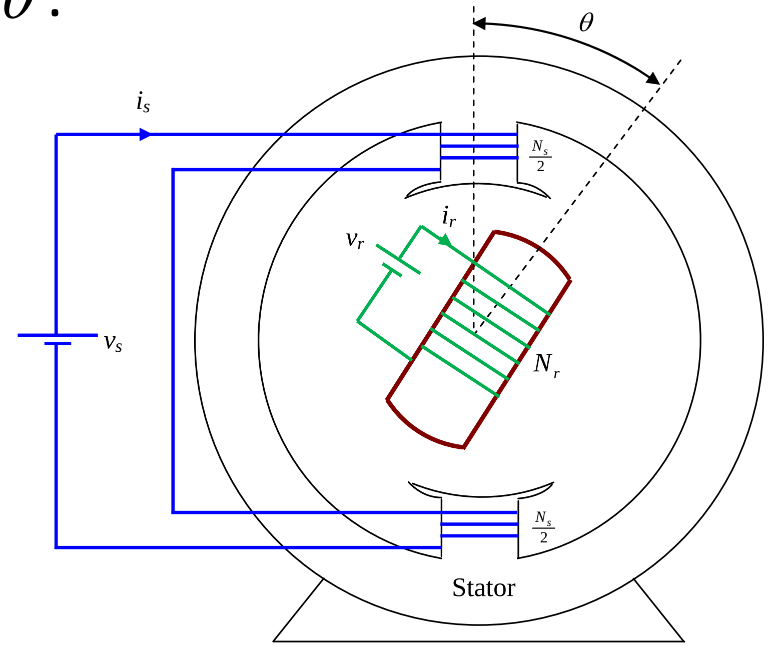
L_s , L_r and M_{sr} are function of θ .

$$T = \underbrace{\frac{1}{2} i_s^2 \frac{dL_s}{d\theta}}_{\text{Reluctance torque due to rotor saliency}} + \underbrace{i_s i_r \frac{dM_{sr}}{d\theta}}_{\text{Electromagnetic torque}} + \underbrace{\frac{1}{2} i_r^2 \frac{dL_r}{d\theta}}_{\text{Reluctance torque due to stator saliency}}$$

Electromagnetic torque

Reluctance torque due to rotor saliency

Reluctance torque due to stator saliency



Reluctance torque is independent of **current direction**.

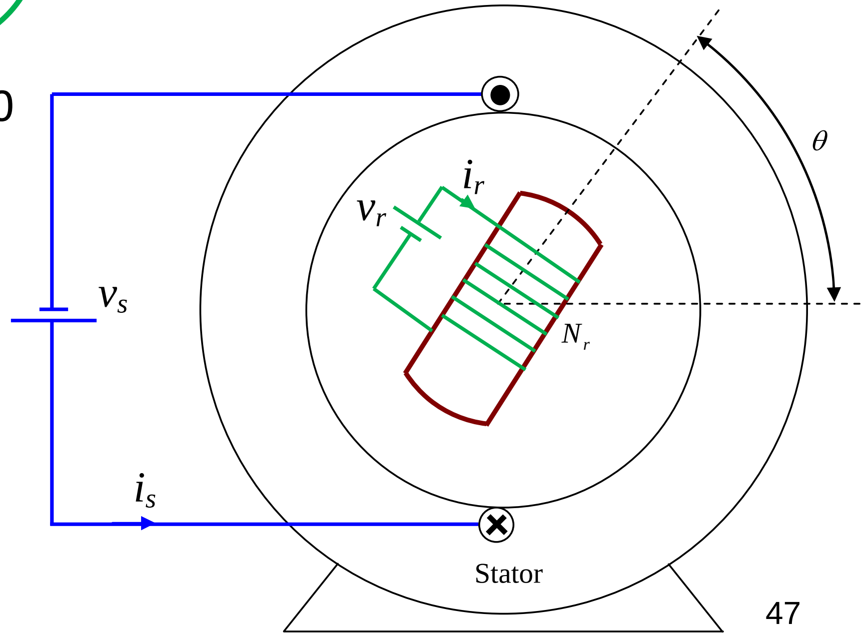
Developed Torque in Doubly Excited Systems

Case 2: Rotor has salient structures but stator is cylindrical:

L_s and M_{sr} are function of θ .

$$T = \underbrace{\tfrac{1}{2} i_s^2 \frac{dL_s}{d\theta}}_{\text{Reluctance torque due to rotor saliency}} + \underbrace{i_s i_r \frac{dM_{sr}}{d\theta}}_{\text{Electromagnetic torque}} + \underbrace{\cancel{\tfrac{1}{2} i_r^2 \frac{dL_r}{d\theta}}}_{0}$$

$$\boxed{T = \tfrac{1}{2} i_s^2 \frac{dL_s}{d\theta} + i_s i_r \frac{dM_{sr}}{d\theta}}$$

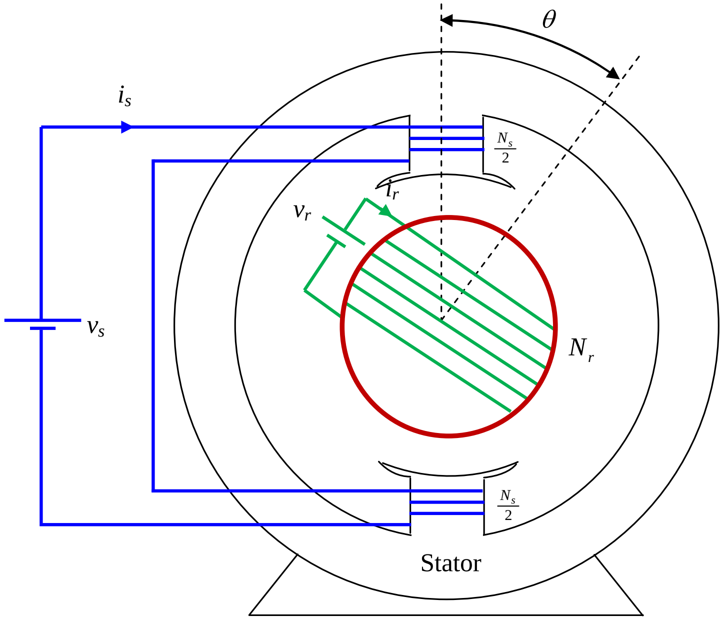


Developed Torque in Doubly Excited Systems

Case 3: Stator has salient structures but rotor is cylindrical:

L_r and M_{sr} are function of θ.

$$T = \underbrace{\cancel{\tfrac{1}{2} i_s^2 \frac{dL_s}{d\theta}}}_{0} + \underbrace{i_s i_r \frac{dM_{sr}}{d\theta}}_{\text{Electromagnetic torque}} + \underbrace{\tfrac{1}{2} i_r^2 \frac{dL_r}{d\theta}}_{\text{Reluctance torque due to rotor saliency}}$$



$$\boxed{T = i_s i_r \frac{dM_{sr}}{d\theta} + \tfrac{1}{2} i_r^2 \frac{dL_r}{d\theta}}$$

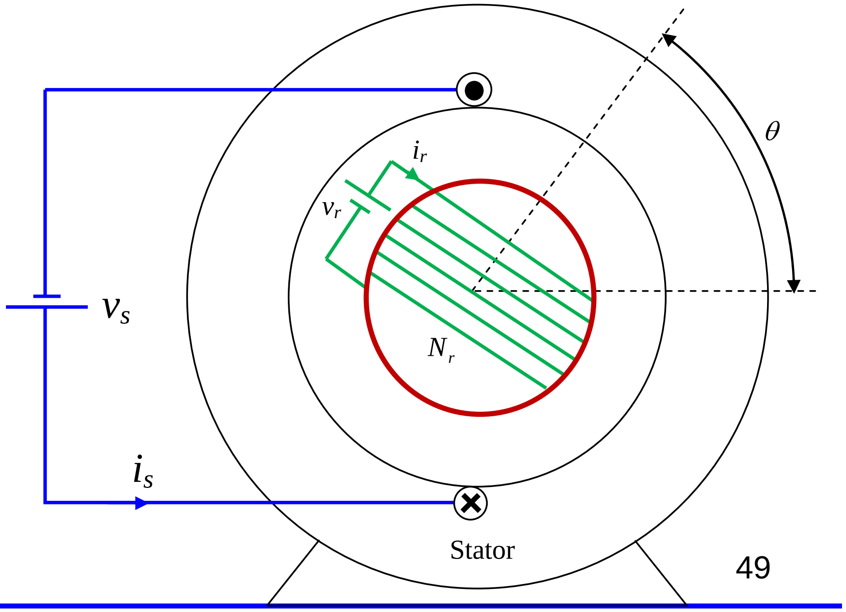
Developed Torque in Doubly Excited Systems

Case 4: Both rotor and stator are cylindrical (non-salient):

only M_{sr} is a function of θ .

$$T = \underbrace{\cancel{\frac{1}{2} i_s^2 \frac{dL_s}{d\theta}}}_{0} + \underbrace{i_s i_r \frac{dM_{sr}}{d\theta}}_{\text{Electromagnetic torque}} + \underbrace{\cancel{\frac{1}{2} i_r^2 \frac{dL_r}{d\theta}}}_{0}$$

$$\boxed{T = i_s i_r \frac{dM_{sr}}{d\theta}}$$



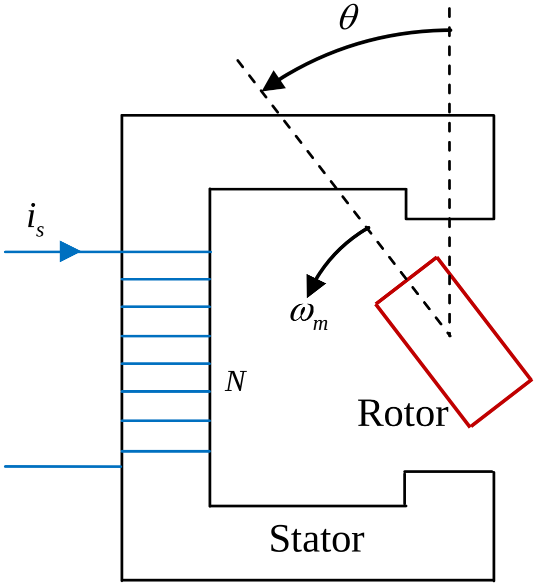
Developed Torque in Doubly Excited Systems

Example 6: In the following figure the rotor has no winding and the self-inductance is assumed to be $L_s = L_0 + L_1 \cos 2\theta$ where θ is the rotor position. If stator current is $i_s = I_m \sin \omega t$.

a) Calculate the **torque** exerted on the rotor.

b) Find the **conditions** in which the average torque is not zero if $\theta = \omega_m t + \delta$ where ω_m is the angular velocity of the rotor and δ is the initial position of the rotor.

$$T = \tfrac{1}{2} i_s^2 \frac{dL_s}{d\theta} + i_s i_r \frac{dM_{sr}}{d\theta} + \tfrac{1}{2} i_r^2 \frac{dL_r}{d\theta}$$



Developed Torque in Doubly Excited Systems

Solution 6: $L_s = L_0 + L_1 \cos 2\theta$ $i_s = I_m \sin\omega\, t$ $i_r = 0$

Part a)

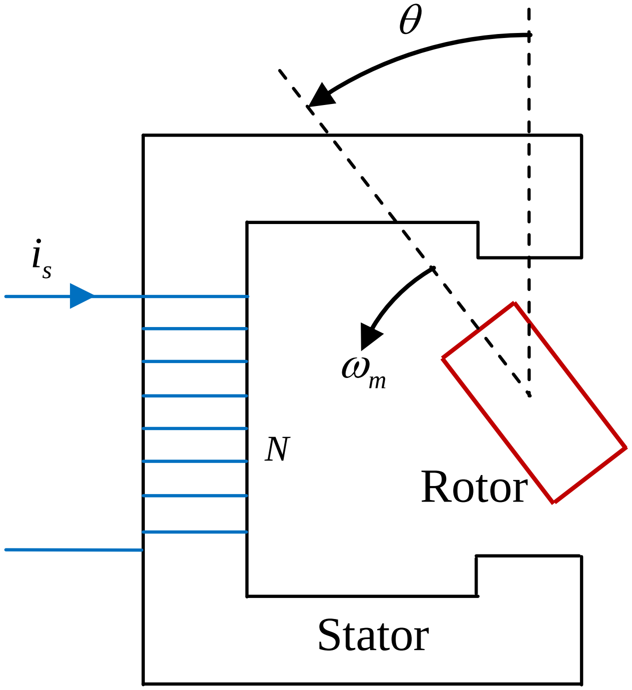
$$T = \tfrac{1}{2} i_s^2 \frac{dL_s}{d\theta} + i_s i_r \frac{dM_{sr}}{d\theta} + \tfrac{1}{2} i_r^2 \frac{dL_r}{d\theta}$$

Since rotor has no winding:

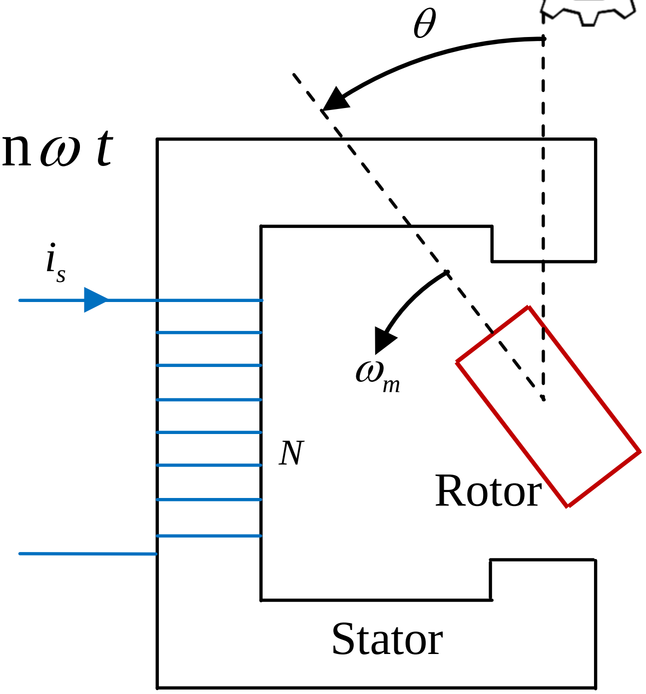
$$T = \tfrac{1}{2} i_s^2 \frac{dL_s}{d\theta}$$

$$T = \tfrac{1}{2}\left(I_m \sin\omega t\right)^2 \frac{d}{d\theta}\left(L_0 + L_1 \cos 2\theta\right)$$

$$T = -I_m^2 L_1 \sin 2\theta \sin^2 \omega t$$



Developed Torque in Doubly Excited Systems



Solution 6: $L_s = L_0 + L_1 \cos 2\theta$ $i_s = I_m \sin \omega t$

Part b) $\theta = \omega_m t + \delta$ $i_r = 0$

$$T = -I_m^2 L_1 \sin 2\theta \sin^2 \omega t$$

$$T = -I_m^2 L_1 \sin 2(\omega_m t + \delta) \frac{1 - \cos 2\omega t}{2}$$

$$T = -\tfrac{1}{2} I_m^2 L_1 \{\sin 2(\omega_m t + \delta) - \tfrac{1}{2} \sin 2[(\omega_m + \omega)t + \delta] - \tfrac{1}{2} \sin 2[(\omega_m - \omega)t + \delta]\}$$

To have non-zero average torque the coefficient of *t* in one of the above sin terms should be zero:

$\omega_m = 0$ or $\omega_m = -\omega$ or $\omega_m = \omega$

Developed Torque in Doubly Excited Systems

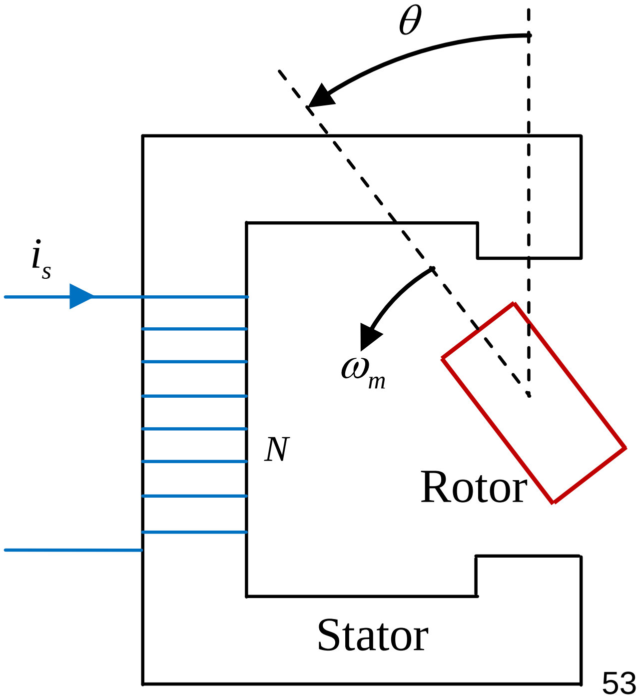
Solution 6:

Part b)

$$T = -\tfrac{1}{2} I_m^2 L_1 \left\{ \sin 2(\omega_m t + \delta) - \tfrac{1}{2} \sin 2[(\omega_m + \omega) t + \delta] - \tfrac{1}{2} \sin 2[(\omega_m - \omega) t + \delta] \right\}$$

1) If $\omega_m = 0$ ⇒ $\boxed{T_{ave} = -\tfrac{1}{2} I_m^2 L_1 \sin 2\delta}$

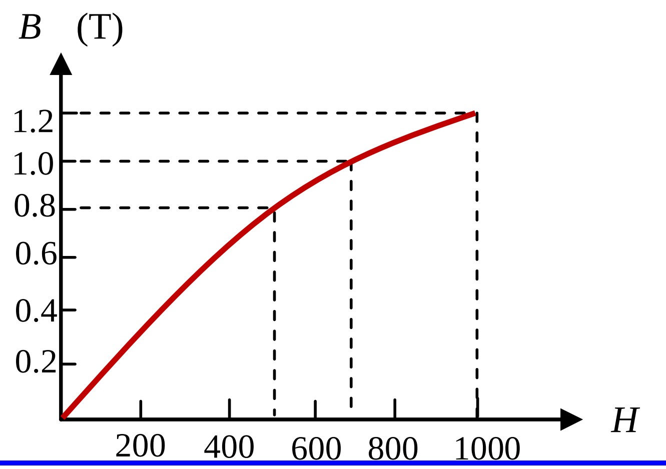
2) If $\omega_m = \pm\omega$ ⇒ $\boxed{T_{ave} = \tfrac{1}{4} I_m^2 L_1 \sin 2\delta}$

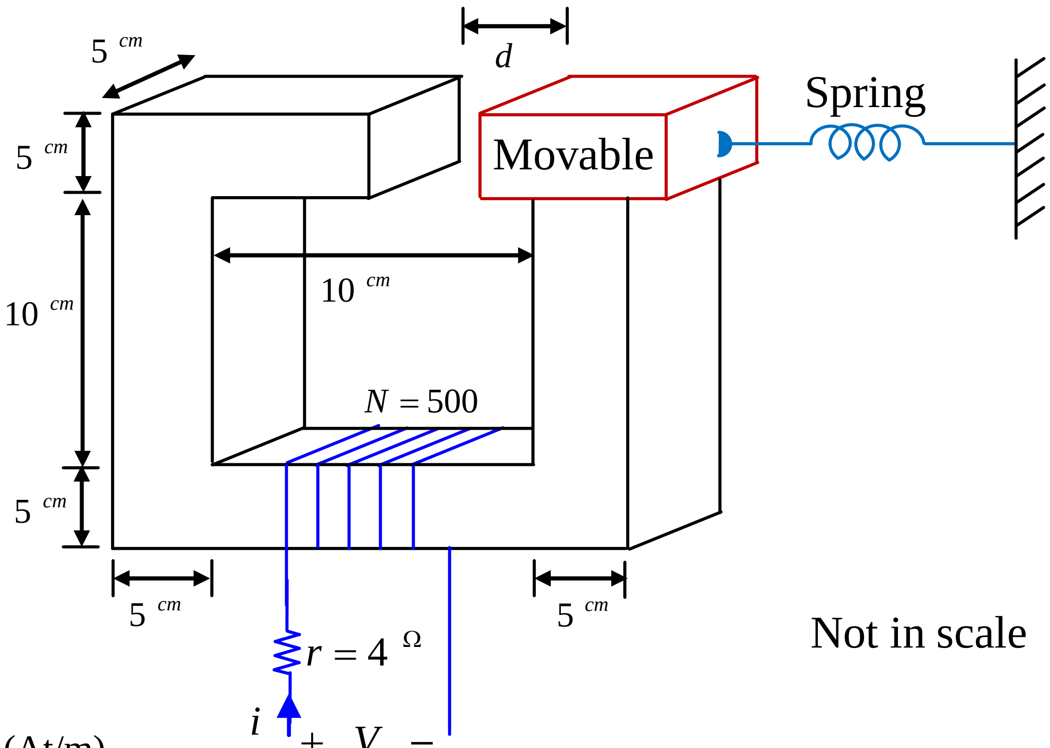


Electromechanical Energy Conversion

Example 7: In the following system assume the movable part is fixed and the air-gap length is $d = 1^{\text{mm}}$.

1) Calculate the current and voltage if flux density in air-gap is 0.5 T.
2) Calculate the stored energy.
3) Obtain the force exerted on the movable part.
4) Compute the inductance of the winding.





Not in scale

Electromechanical Energy Conversion

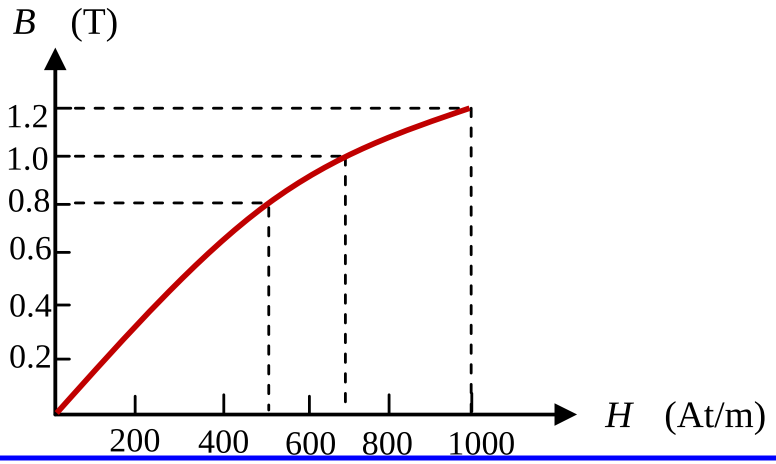
Solution 7: $d = l_g = 1^{\text{ mm}}$ $l_c = 0.6^{\text{ m}}$ $\mu_0 = 4\pi \times 10^{-7}$

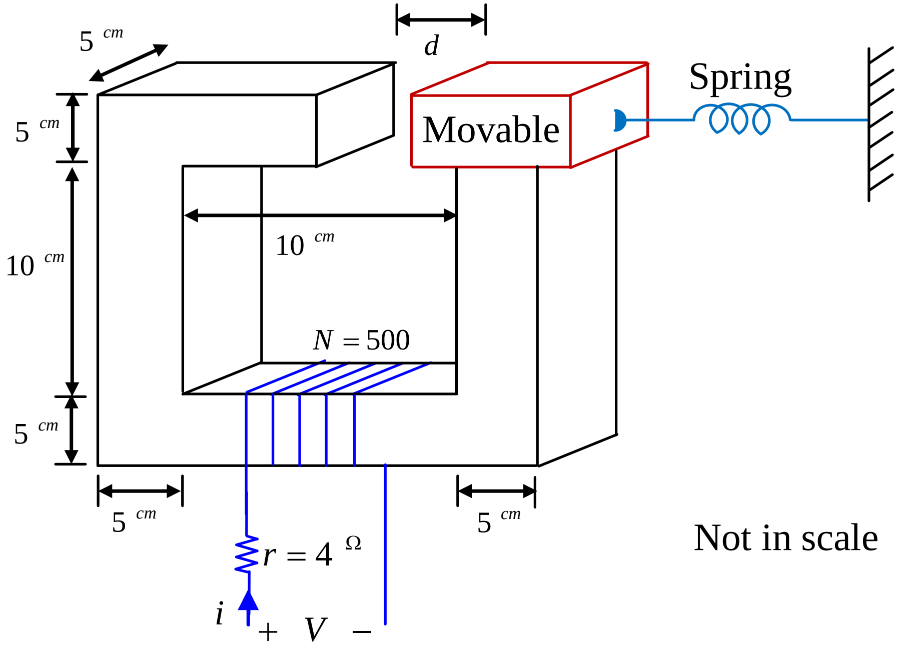
Part 1) $i = ?$ $V = ?$

$B_g = B_c = 0.5 \text{ T}$ From the curve $H_c = 350 \text{ At/m}$

$$i = \frac{H_c l_c + H_g l_g}{N} = \frac{H_c l_c + \frac{B_g}{\mu_0} l_g}{N} \Rightarrow \boxed{i = 1.215 \text{ A}}$$

$V = ri = 4.86 \text{ V}$





Electromechanical Energy Conversion

Solution 7: $d = l_g = 1^{\text{mm}}$ $l_c = 0.6^{\text{m}}$ $\mu_0 = 4\pi\times10^{-7}$

Part 2) $W_{field} = ?$

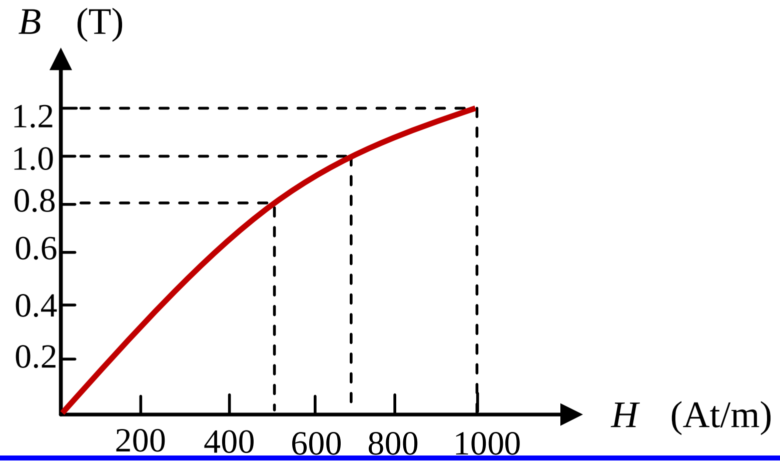
$B = B_g = B_c = 0.5 \text{ T}$ $H_c = 350 \text{ At/m}$

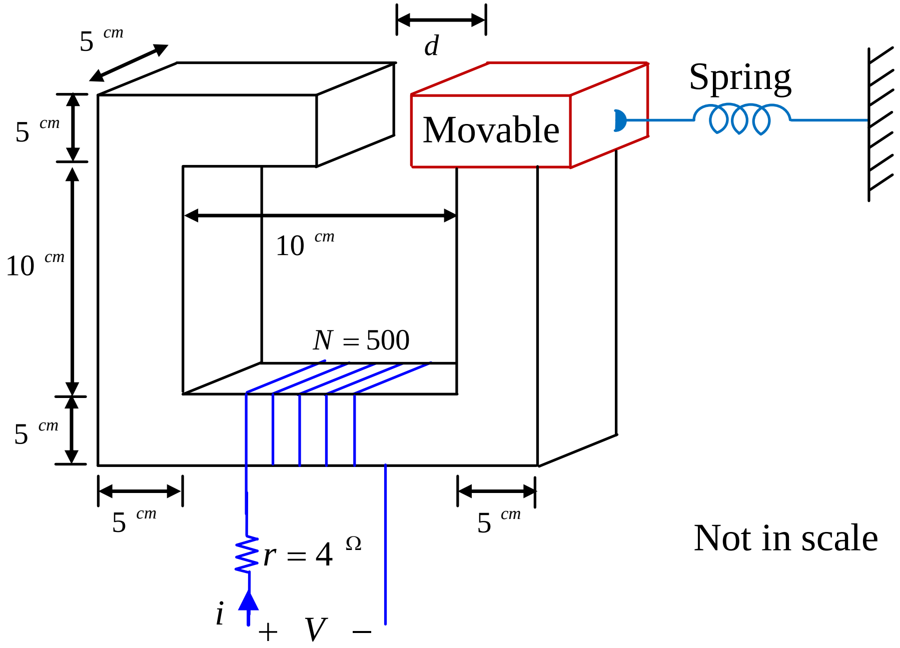
$$V_c = 0.6\times0.05\times0.05 = 1.5\times10^{-3} \text{ m}^3$$

$$V_g = 0.001\times0.05\times0.05 = 2.5\times10^{-6} \text{ m}^3$$

$$W_{field} = V_c\int_0^B H_c dB + V_g \frac{B^2}{2\mu_0}$$

$$W_{field} = 0.38 \text{ J}$$





Electromechanical Energy Conversion

Solution 7: $d = l_g = 1^{\text{ mm}}$ $\quad l_c = 0.6^{\text{ m}}$ $\quad \mu_0 = 4\pi \times 10^{-7}$

Part 3) $F = ?$

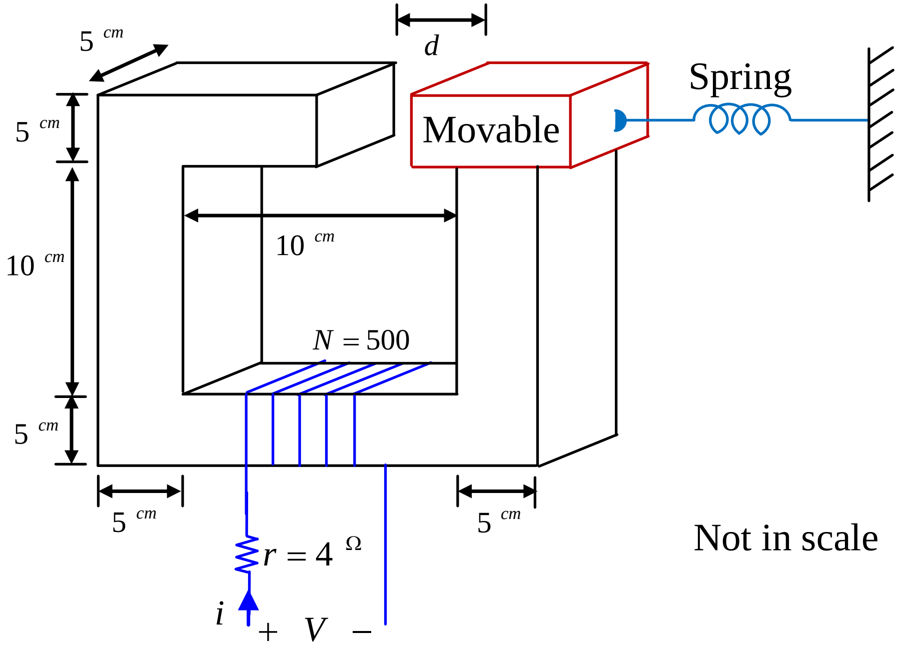
$$B = B_g = B_c = 0.5 \text{ T} \qquad A_g = 0.05 \times 0.05 = 2.5 \times 10^{-3} \ \text{ m}^2$$

$$W_{field} = l_c A_c \int_0^B H_c dB + l_g A_g \frac{B^2}{2\mu_0}$$

$$l_g \to x$$

$$F = -\left.\frac{\partial W_{field}}{\partial x}\right|_{\lambda = cte} = -A_g \frac{B^2}{2\mu_0}$$

$$F = -248.7 \ \text{ N}$$



Electromechanical Energy Conversion

Solution 7: $d = l_g = 1 \text{ mm}$

Part 3) $L = ?$

$l_c = 0.6 \text{ m}$

$\mu_0 = 4\pi \times 10^{-7}$

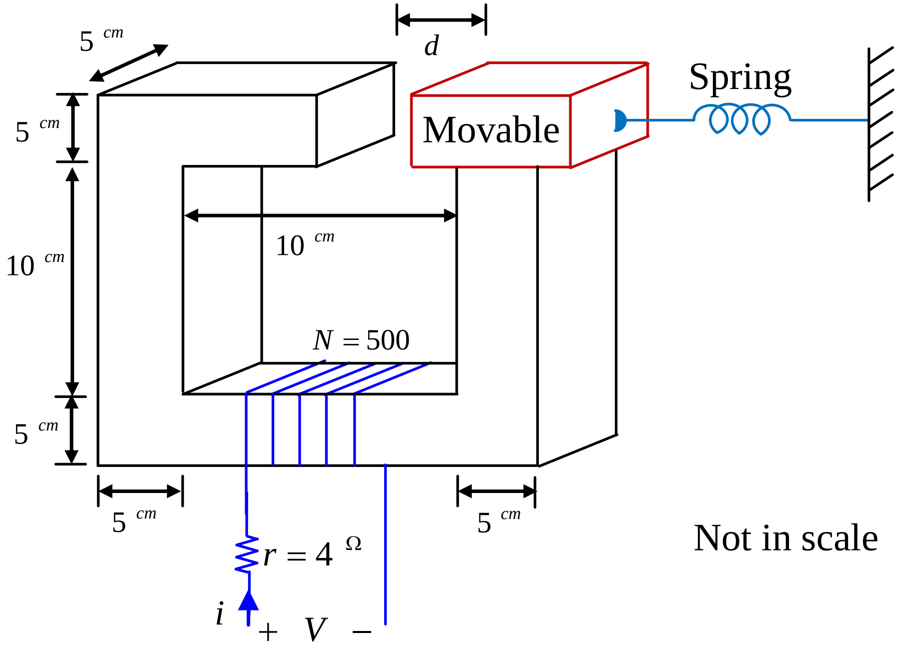
$B = B_g = B_c = 0.5 \text{ T}$

$A_g = 0.05 \times 0.05 = 2.5 \times 10^{-3} \ \text{m}^2$

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{NAB}{i}$$

$$L = \frac{500 \times 2.5 \times 10^{-3} \times 0.5}{1.215}$$

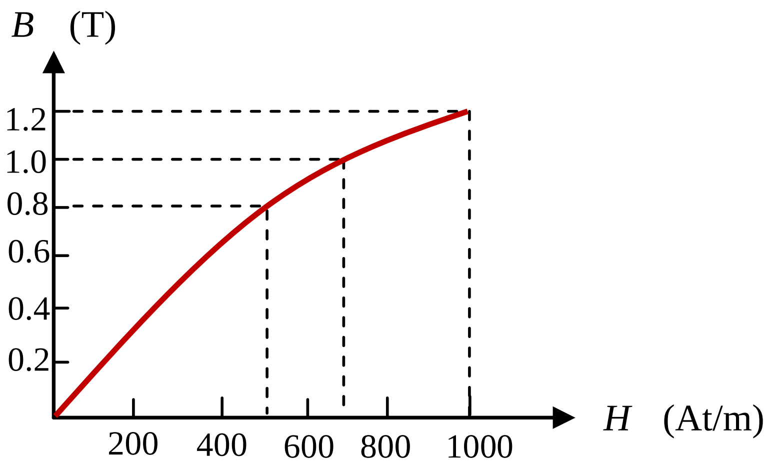
$$L = 0.514 \ \text{H}$$

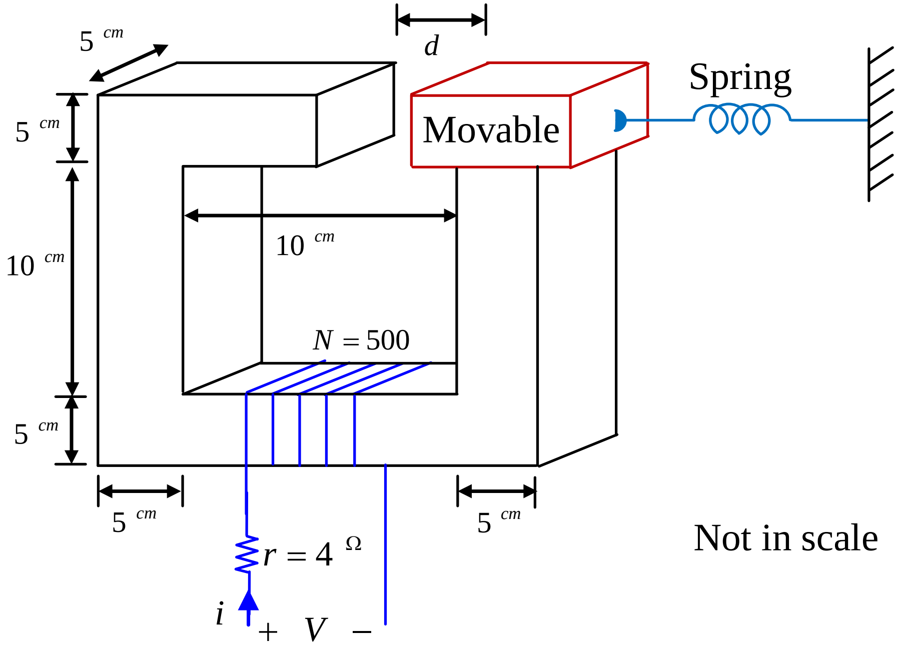


Electromechanical Energy Conversion

Example 8: In the following system assume the current is 1.215 A and the air-gap is fully closed. $d = 0$

1) Calculate the flux density.
2) Calculate the force exerted on spring.
3) Calculate the stored energy





Electromechanical Energy Conversion

Solution 8: $d = l_g = 0$

$l_c = 0.6\ \mathrm{m}$

$\mu_0 = 4\pi\times10^{-7}$

Part 1) $B = ?$

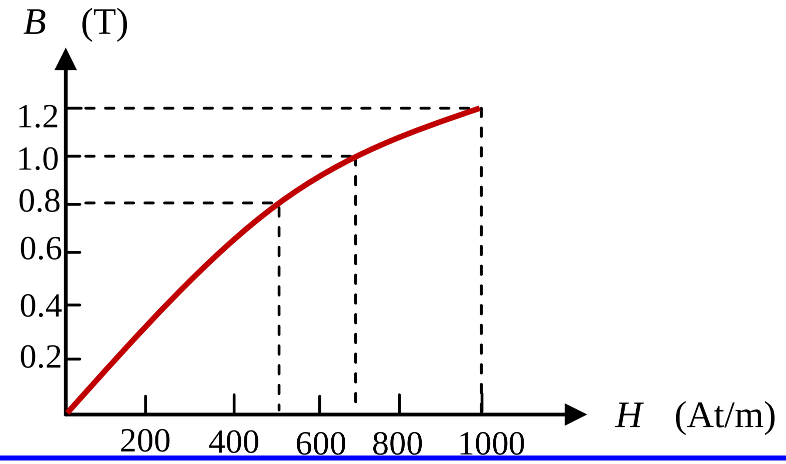
$i = 1.215\ \mathrm{A}$

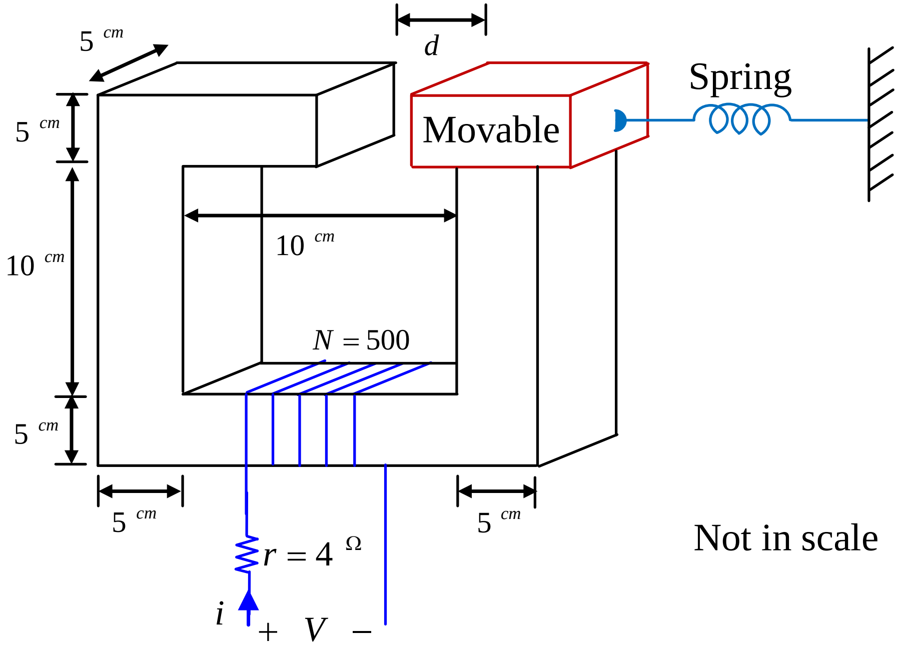
$$Ni = H_c l_c + H_g l_g$$

$$Ni = H_c l_c$$

$$H_c = \frac{Ni}{l_c} = 1012\ \mathrm{At/m}$$

From the curve $B_c = 1.21\ \mathrm{T}$





Electromechanical Energy Conversion

Solution 8: $d = l_g = 0$

Part 2) $W_{field} = ?$

$l_c = 0.6\ \text{m}$

$\mu_0 = 4\pi \times 10^{-7}$

$B_c = 1.21\ \text{T}$

$H_c = 1012\ \text{At/m}$

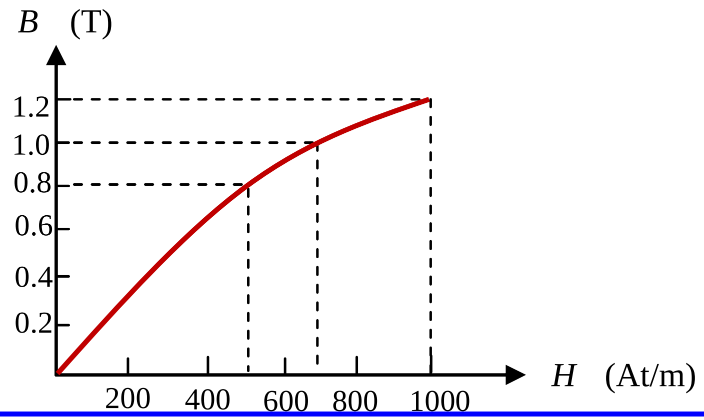
$V_c = 0.6 \times 0.05 \times 0.05 = 1.5 \times 10^{-3}\ \ \text{m}^3$

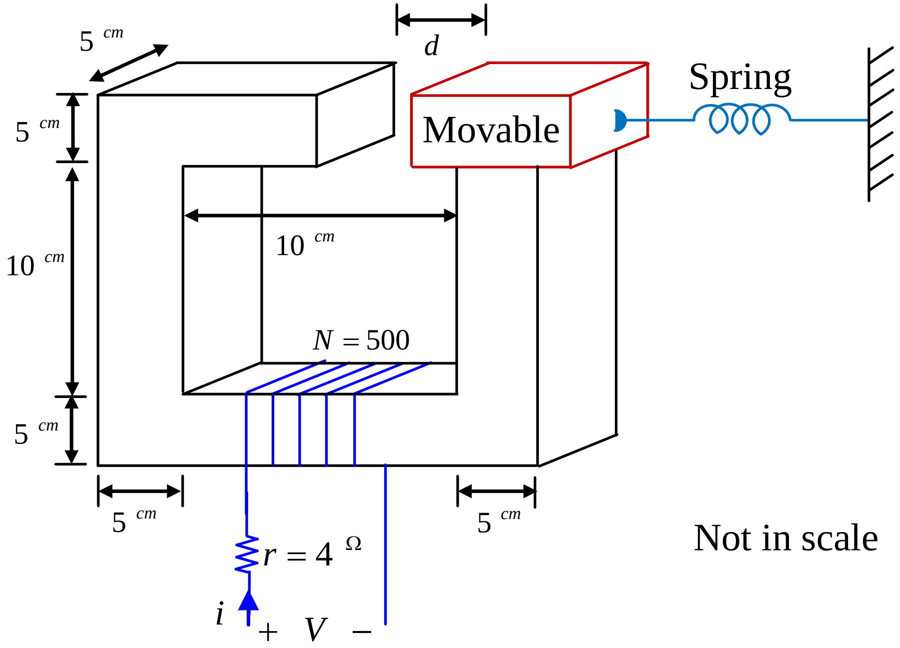
$V_g = 0$

$$W_{field} = V_c \int_0^B H_c dB + V_g \frac{B^2}{2\mu_0}$$

$$W_{field} = V_c \int_0^B H_c dB$$

$$W_{field} = 0.92\ \ \text{J}$$





Electromechanical Energy Conversion

Solution 8: $d = l_g = 0$ $l_c = 0.6\ \text{m}$ $\mu_0 = 4\pi \times 10^{-7}$

Part 3) $F = ?$

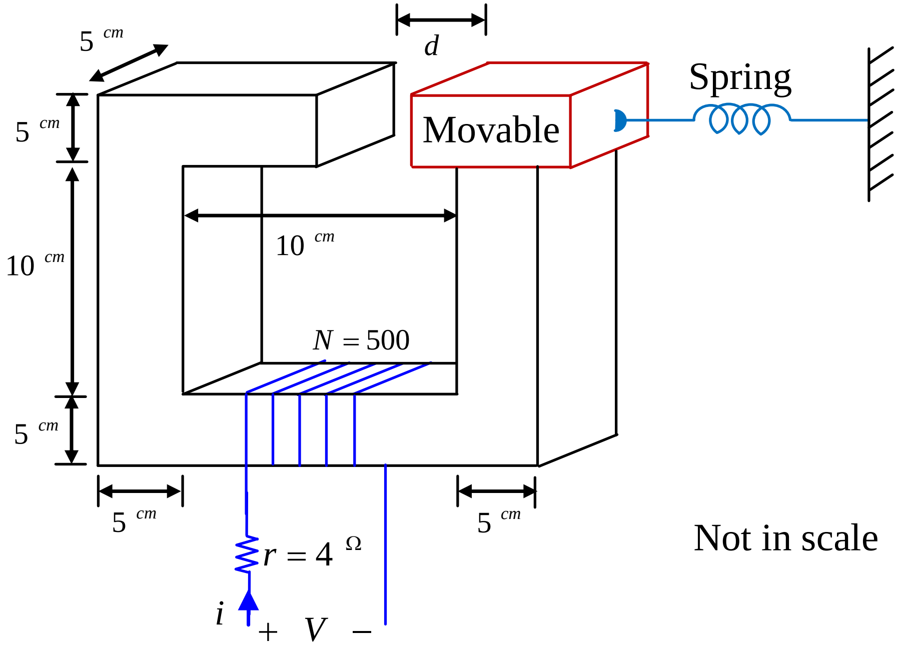
$$B = B_g = B_c = 1.21\ \text{T} \qquad A_g = 0.05 \times 0.05 = 2.5 \times 10^{-3}\ \ \text{m}^2$$

$$W_{field} = l_c A_c \int_0^B H_c dB + l_g A_g \frac{B^2}{2\mu_0}$$

$$l_g \rightarrow x$$

$$F = -\left.\frac{\partial W_{field}}{\partial x}\right|_{\lambda = cte} = -A_g \frac{B^2}{2\mu_0}$$

$$F = -1456.4\ \ \text{N}$$



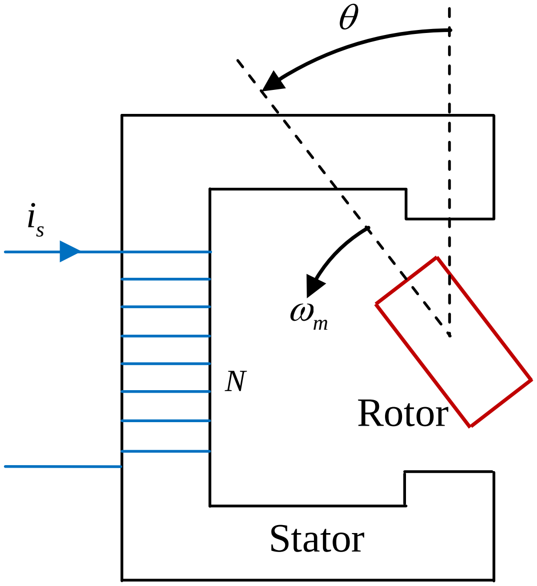
Developed Torque

Example 9: In the following figure the rotor has no winding and the self-inductance is assumed to be $L_s = 0.1 - 0.3\cos 2\theta - 0.2\cos 4\theta$ where θ is the rotor position. If stator current is 10 Aac with frequency of 60 Hz: .

a) In which rotational velocity the average torque is not zero if $\theta = \omega_m t + \delta$ where ω_m is the angular velocity of the rotor and δ is the initial position of the rotor.

b) In the velocity obtained above calculate the maximum torque and mechanical power.

c) Obtain maximum torque at zero velocity.

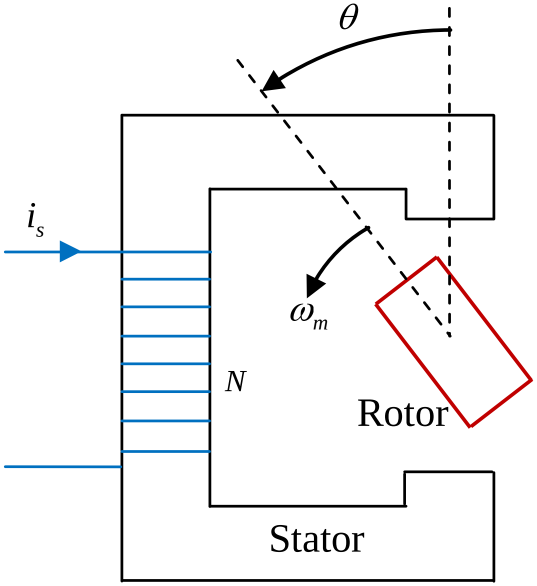


$$T = \tfrac{1}{2} i_s^2 \frac{dL_s}{d\theta} + i_s i_r \frac{dM_{sr}}{d\theta} + \tfrac{1}{2} i_r^2 \frac{dL_r}{d\theta}$$

Developed Torque

Solution 9: $L_s = 0.1 - 0.3\cos 2\theta - 0.2\cos 4\theta$ $\theta = \omega_m t + \delta$

Part a) $i_s = 10\cos\omega_s t$ $\omega_s = 2\pi \times 60 = 120\pi$



$$T = \tfrac{1}{2} i_s^2 \frac{dL_s}{d\theta}$$

$$T = \tfrac{1}{2}(10\cos\omega_s t)^2 \frac{d}{d\theta}(0.1 - 0.3\cos 2\theta - 0.2\cos 4\theta)$$

$$T = 50\cos^2 \omega_s t(0.6\sin 2\theta + 0.8\sin 4\theta)$$

$$T = 50\frac{1+\cos 2\omega_s t}{2}[0.6\sin 2(\omega_m t + \delta) + 0.8\sin 4(\omega_m t + \delta)]$$

Developed Torque

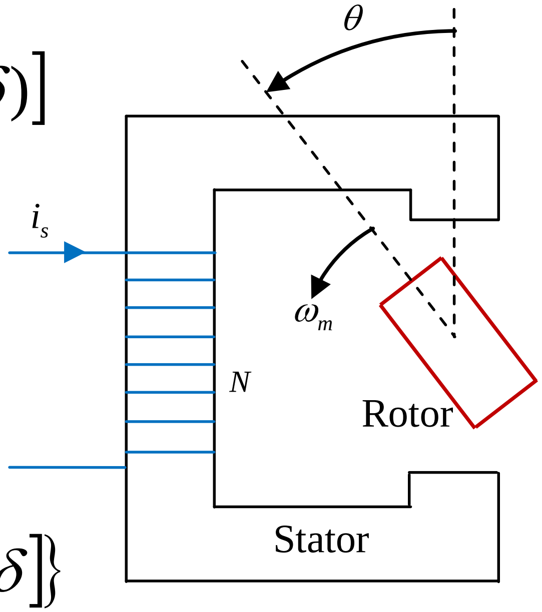
Solution 9: $$L_s = 0.1 - 0.3\cos 2\theta - 0.2\cos 4\theta \qquad \theta = \omega_m t + \delta$$

Part a) $$i_s = 10\cos\omega_s t \qquad \omega_s = 2\pi \times 60 = 120\pi$$

$$T = 50\frac{1+\cos 2\omega_s t}{2}\left[0.6\sin 2(\omega_m t+\delta) + 0.8\sin 4(\omega_m t+\delta)\right]$$

$$\sin a \cos b = \tfrac{1}{2}\left[\sin(a+b)+\sin(a-b)\right]$$

$$\begin{aligned} T = {} & 25\{0.6\sin 2(\omega_m t+\delta) + 0.8\sin 4(\omega_m t+\delta) \\ & +0.3\sin[2(\omega_m+\omega_s)t+2\delta] + 0.3\sin[2(\omega_m-\omega_s)t+2\delta] \\ & +0.4\sin[2(2\omega_m+\omega_s)t+4\delta] + 0.4\sin[2(2\omega_m-\omega_s)t+4\delta]\} \end{aligned}$$



$\omega_m = 0$ or $\omega_m = \pm\omega_s = \pm 120\pi$ or $\omega_m = \pm\frac{1}{2}\omega_s = \pm 60\pi$

Developed Torque

Solution 9: $L_s = 0.1 - 0.3\cos 2\theta - 0.2\cos 4\theta$ $\theta = \omega_m t + \delta$

Part b)

$\omega_s = 2\pi \times 60 = 120\pi$

$\omega_m = \pm\omega_s = \pm 120\pi$ $T_{ave} = 7.5\sin 2\delta$

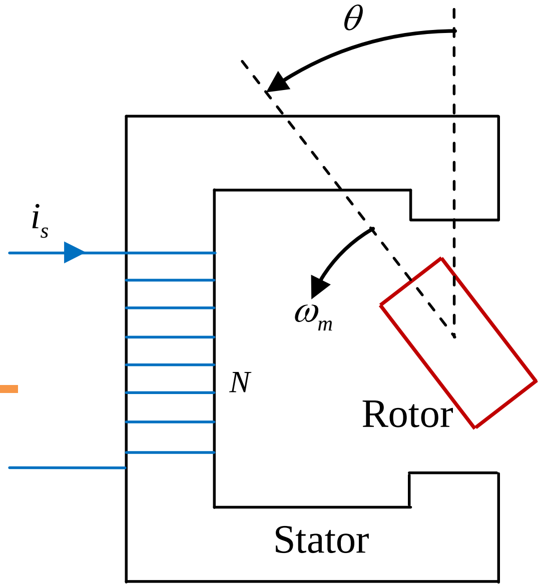
$T_{ave-\max} = 7.5 \text{ Nm}$ at $\delta = k\pi \pm \frac{\pi}{4}$

$P = T_{ave-\max}\,\omega_m = 7.5 \times 120\pi = 2827 \text{ W}$

$\omega_m = \pm\frac{1}{2}\omega_s = \pm 60\pi$ $T_{ave} = 10\sin 4\delta$

$T_{ave-\max} = 10 \text{ Nm}$ at $\delta = \frac{k\pi}{2} \pm \frac{\pi}{8}$

$P = T_{ave-\max}\,\omega_m = 10 \times 60\pi = 1885 \text{ W}$

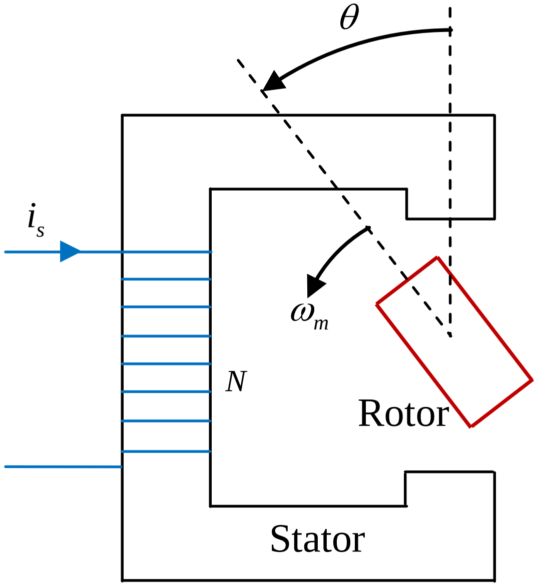


Developed Torque

Solution 9: $L_s = 0.1 - 0.3\cos 2\theta - 0.2\cos 4\theta$ $\qquad \theta = \omega_m t + \delta$

Part c)

$$\omega_s = 2\pi \times 60 = 120\pi$$



$$\omega_m = 0 \qquad\qquad T_{ave} = 15\sin 2\delta + 20\sin 4\delta$$

$$\frac{\partial T_{ave}}{\partial \delta} = 0 \Rightarrow 30\cos 2\delta + 80\cos 4\delta = 0$$

$$3\cos 2\delta + 8\left(2\cos^2 2\delta - 1\right) = 0$$

$$16\cos^2 2\delta + 3\cos 2\delta - 8 = 0 \left\{ \begin{array}{l} \cos 2\delta = 0.62 \\ \cos 2\delta = -0.81 \end{array} \right.$$

$$\Rightarrow \left\{ \begin{array}{l} \delta = 25.8^{\circ} \\ \delta = 71.9^{\circ} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} T_{ave-\max} = 31 \text{ Nm} \\ T_{ave-\max} = -10 \text{ Nm} \end{array} \right.$$