In The Name of God The Most Compassionate, The Most Merciful

Electric Machines II

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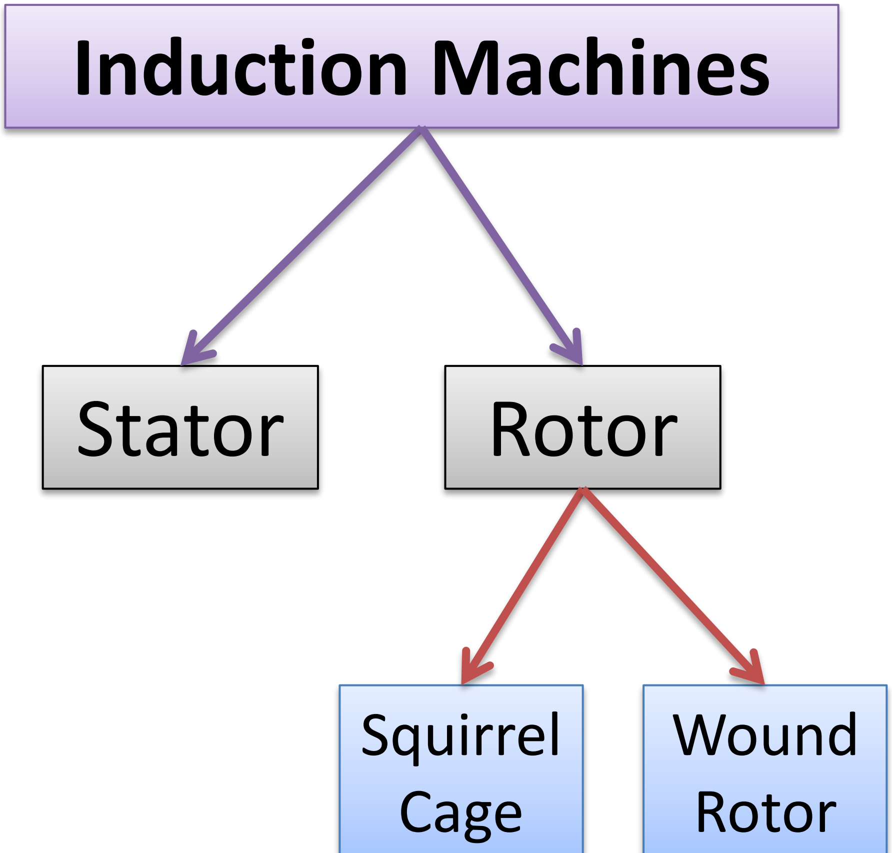
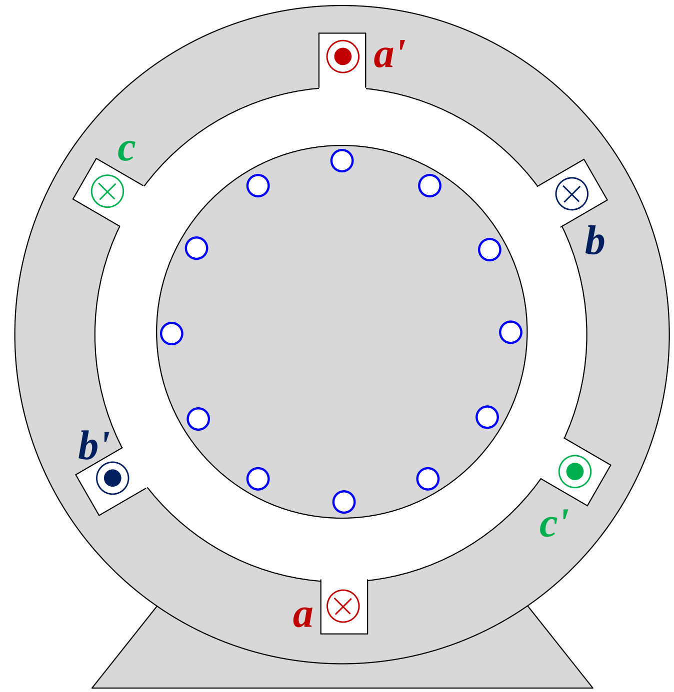
Chapter 3
Three-Phase Induction (Asynchronous) Machines

Structure of Induction Machines

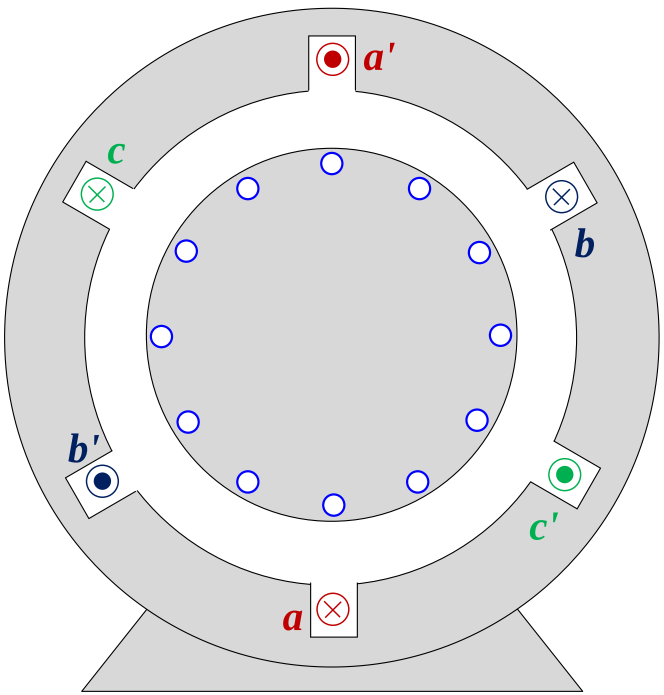


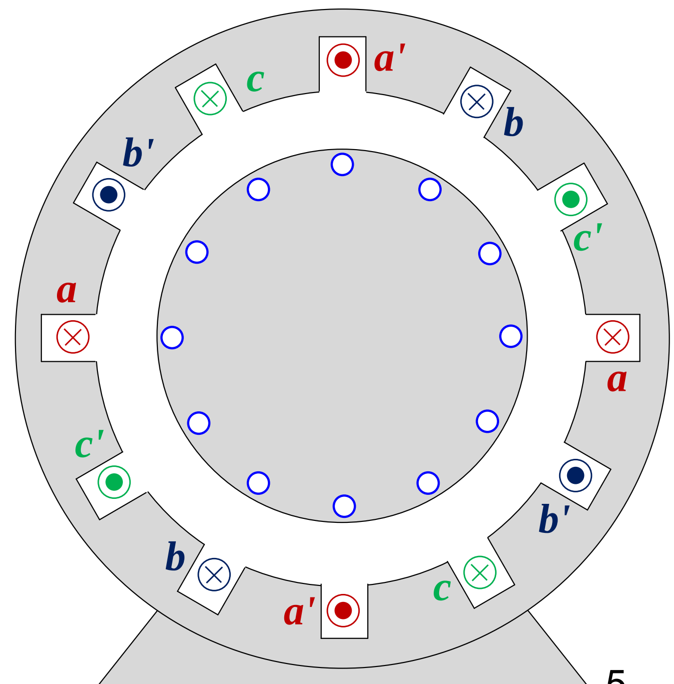


Structure of Induction Machines

1. Stator

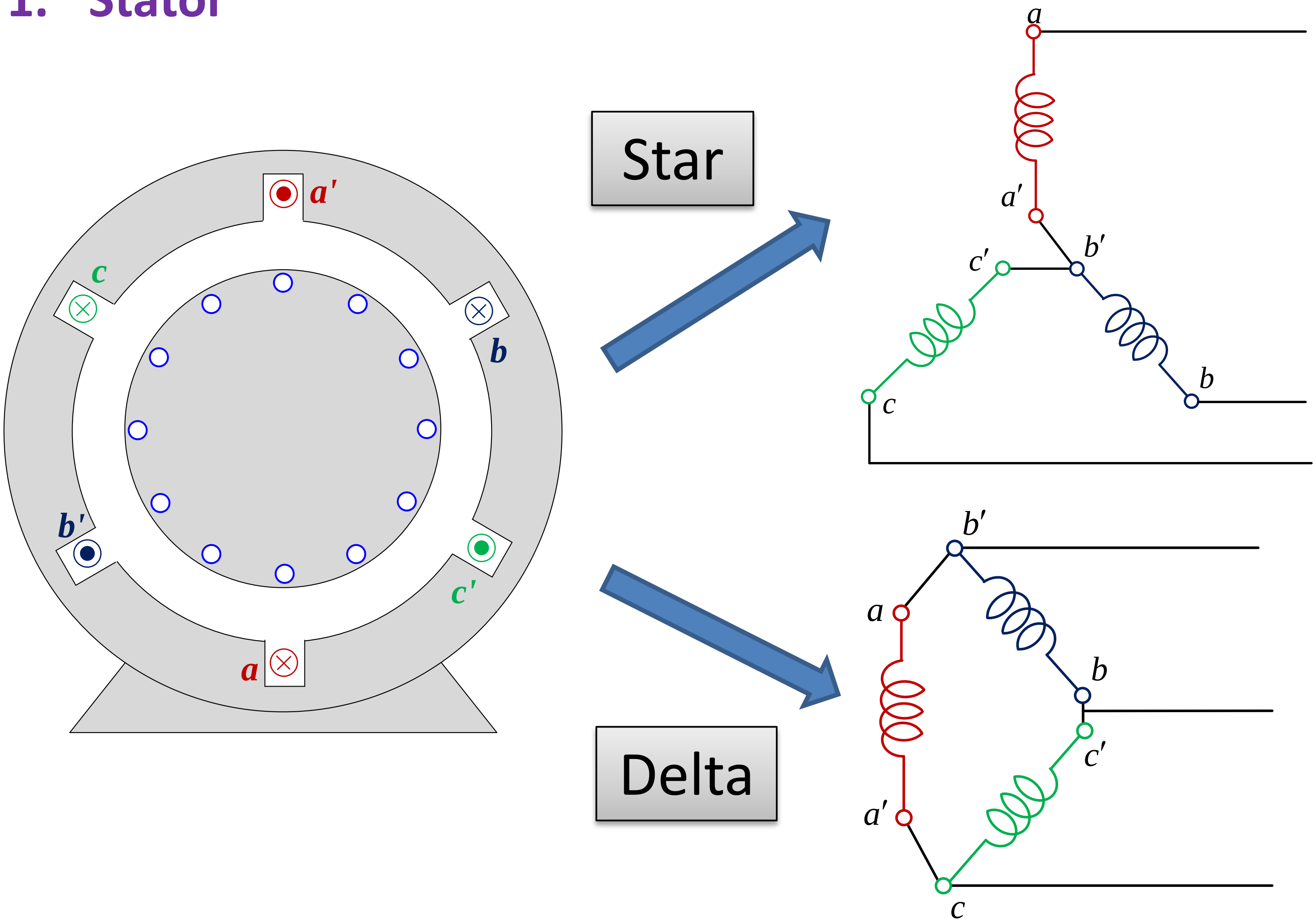
- The stator core is made from the laminated steel.
- The windings are located in slots.
- Three-phase windings are connected in star or delta configuration.





Structure of Induction Machines

1. Stator

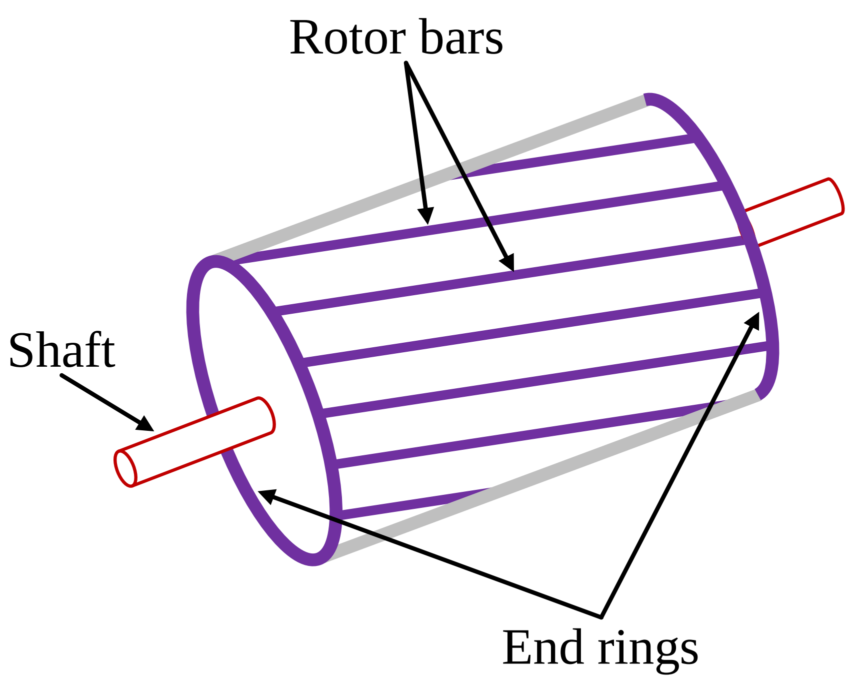


Structure of Induction Machines

2. Rotor

A. Squirrel Cage

- Aluminium or copper bars are located in rotor slots.
- The bars are short-circuited from both sides using end rings.

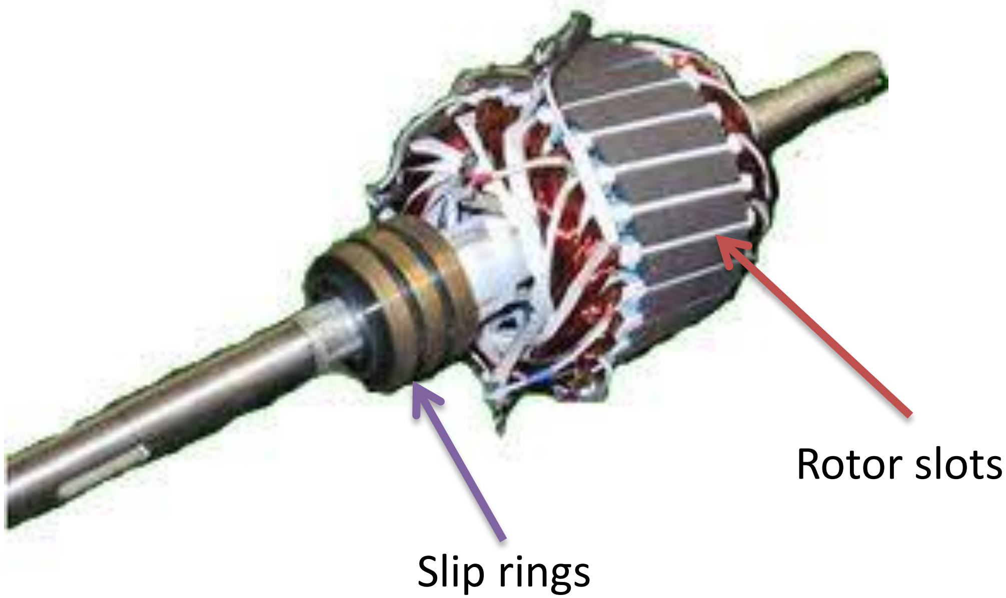


Structure of Induction Machines

2. Rotor

B. Wound Rotor

- Aluminium or copper windings are located in rotor slots.
- There are three slip-rings and brushes used for energy transfer.



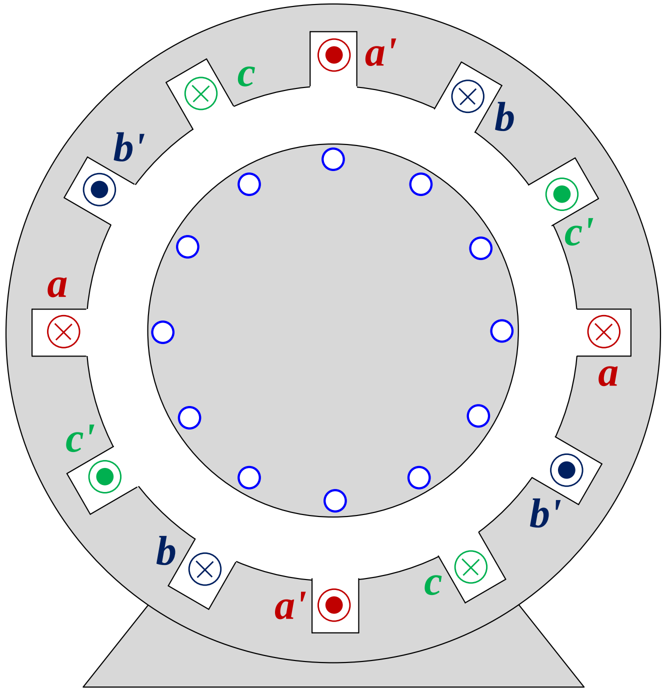
Basic Notions of Induction Machines

1. **Synchronous speed.** Assume the frequency of the applied voltage to the stator winding is f and the machine has p poles; the synchronous speed is defined as:

$$N_s = \frac{120 f}{p} \quad \text{rpm}$$

$$n_s = \frac{2 f}{p} \quad \text{rps}$$

$$\omega_s = \frac{4\pi f}{p} \quad \text{rad/s}$$



Synchronous speed is a mechanical quantity.

2. **Rotor speed** is the speed of the rotor.

In motoring mode

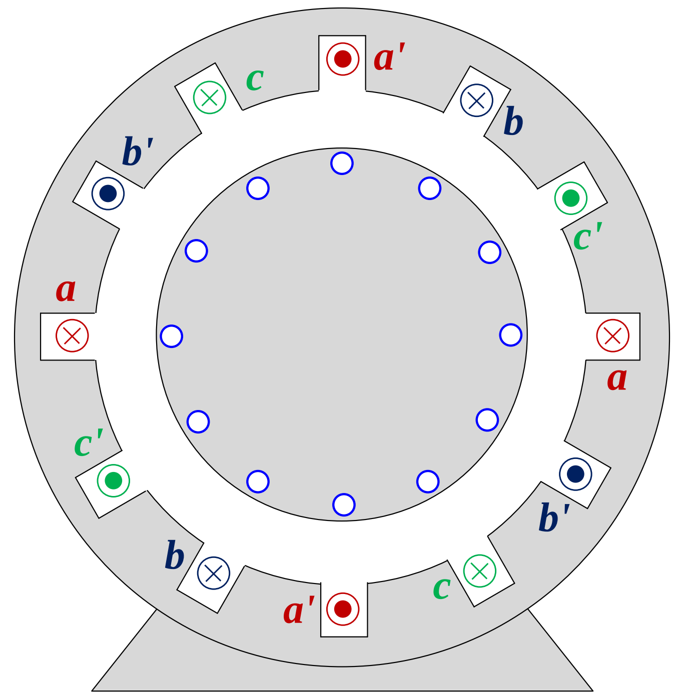
$N_r < N_s$ rpm $n_r < n_s$ rps

$\omega_r < \omega_s$ rad/s

In generating mode

$N_r > N_s$ rpm $n_r > n_s$ rps

$\omega_r > \omega_s$ rad/s



Rotor speed is a mechanical quantity.

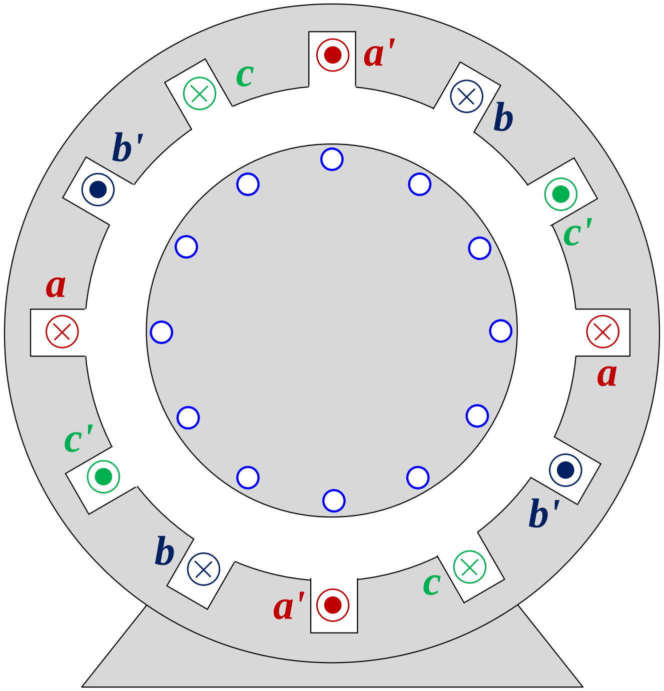
Basic Notions of Induction Machines

3. **Slip speed** is the difference between the synchronous speed and the rotor speed:

$$N_{slip} = N_s - N_r \quad \text{rpm}$$

$$n_{slip} = n_s - n_r \quad \text{rps}$$

$$\omega_{slip} = \omega_s - \omega_r \quad \text{rad/s}$$



Slip speed is a mechanical quantity.

Basic Notions of Induction Machines

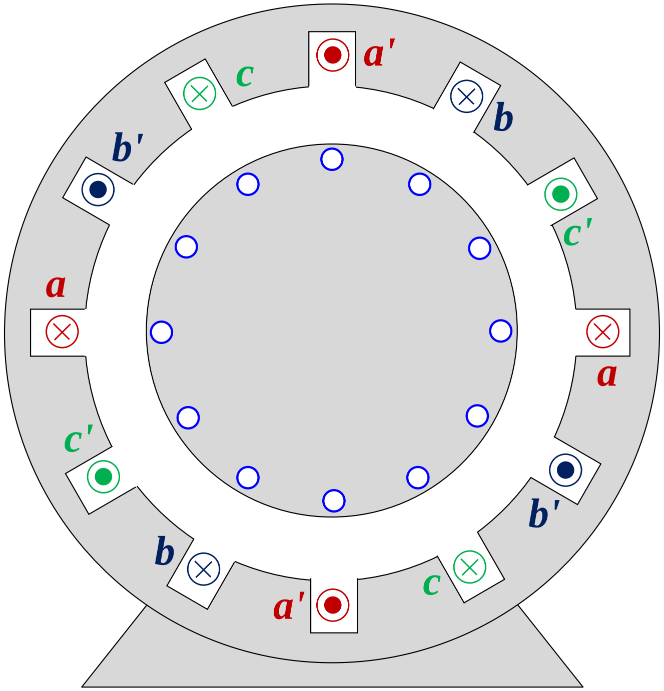
4. **Slip** is the slip speed divided by the synchronous speed:

$$S = \frac{N_s - N_r}{N_s}$$

$$= \frac{n_s - n_r}{n_s}$$

$$= \frac{\omega_s - \omega_r}{\omega_s}$$

$$N_r = (1 - S) N_s \quad \text{rpm}$$



Slip is a dimensionless quantity.

How Does an Induction Motor Work?

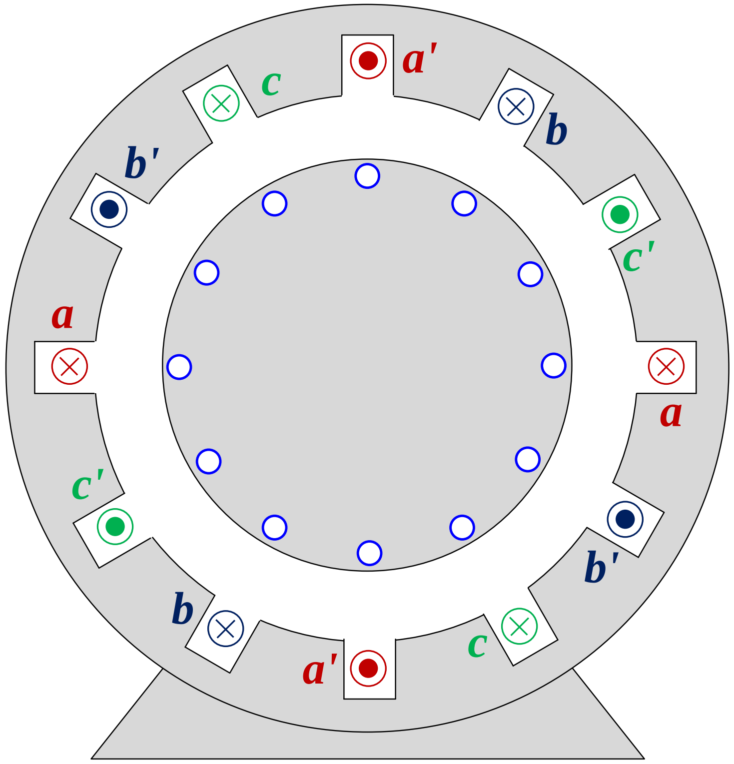
1. Connecting the 3-phase stator windings to a 3-phase AC source flows the **current in the stator windings**.
2. The stator current causes a **rotating magnetic field** with synchronous speed.
3. The rotating magnetic field, **induces a voltage** in the rotor bars.
4. Since the rotor bars are short-circuited by end-rings, a **current flows in the rotor bars**.
5. The rotor bar current produces **another rotating magnetic field** which rotates with synchronous speed in the same direction as the stator magnetic field.
6. Electromagnetic **torque** is developed due to the interaction between two magnetic fields.

$$T_{em} = kB_r \times B_s$$

7. The developed torque can **rotate** the rotor.

What is Meant by Asynchronous?

- The induced voltage in rotor bars is due to the stator rotating magnetic field.
- Therefore if the rotor rotates with synchronous speed, no voltage is induced in the rotor bars and no torque can be developed.
- Hence, to develop torque, the rotor speed should be different from the synchronous speed.
- It is because induction motors are often called asynchronous motors.



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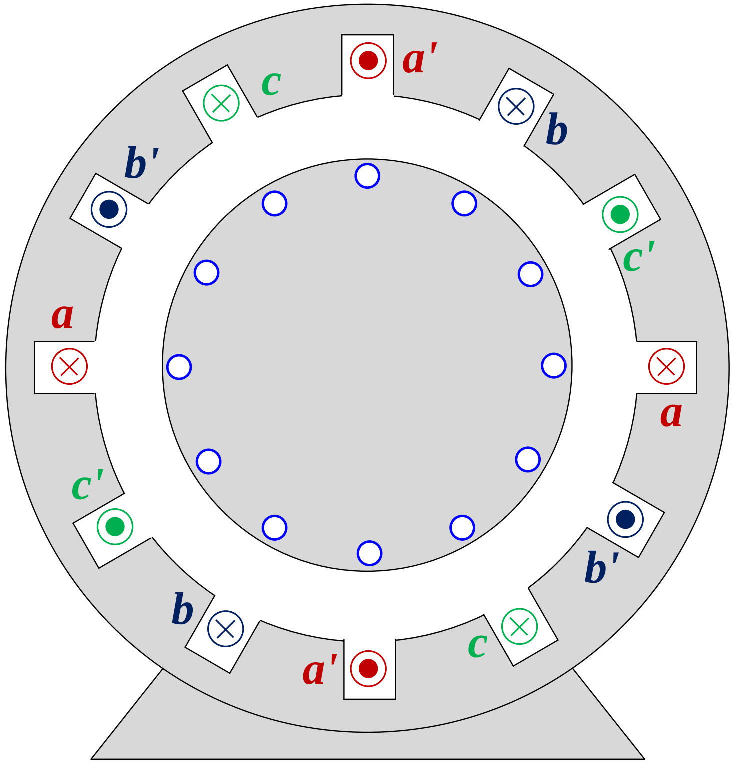
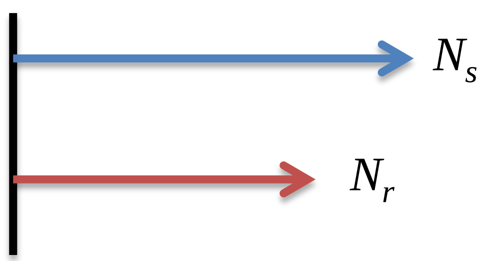
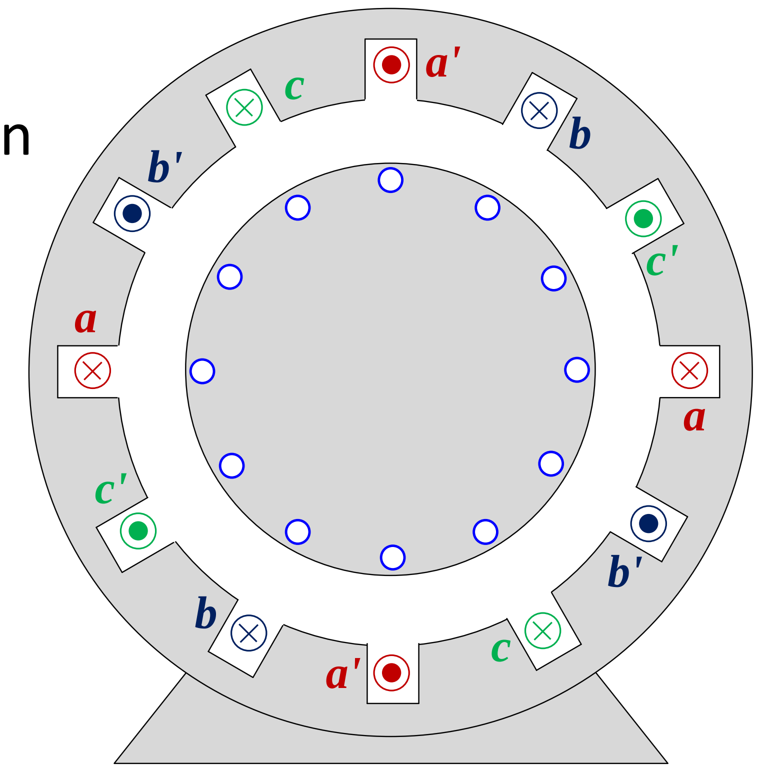


Different Operating Mode of Induction Machines

1. Motoring Mode

- The stator windings are connected to a 3-phase ac source.
- The mechanical energy is delivered on the motor shaft.





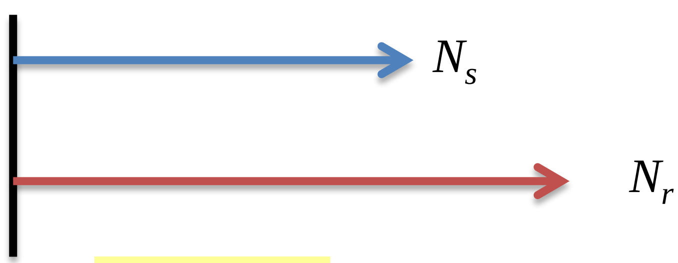
$$0 \le N_r \le N_s$$

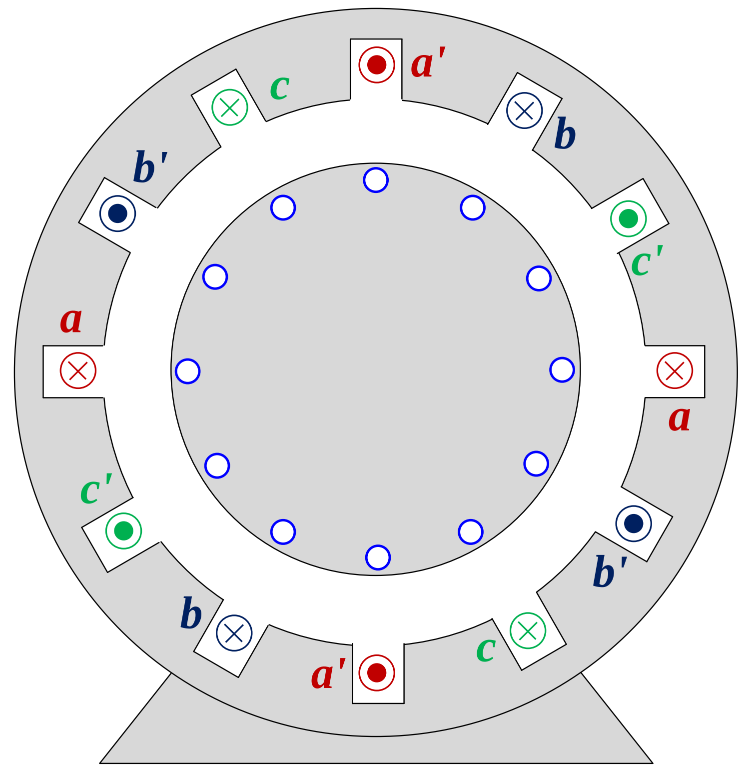
$$1 \ge S \ge 0$$

Different Operating Mode of Induction Machines

2. Generating Mode

- The stator windings are connected to a 3-phase ac source.
- The machine is in motoring mode.
- If by using a mechanical mover the rotor speed is increased to above synchronous speed the machine will be a generator.





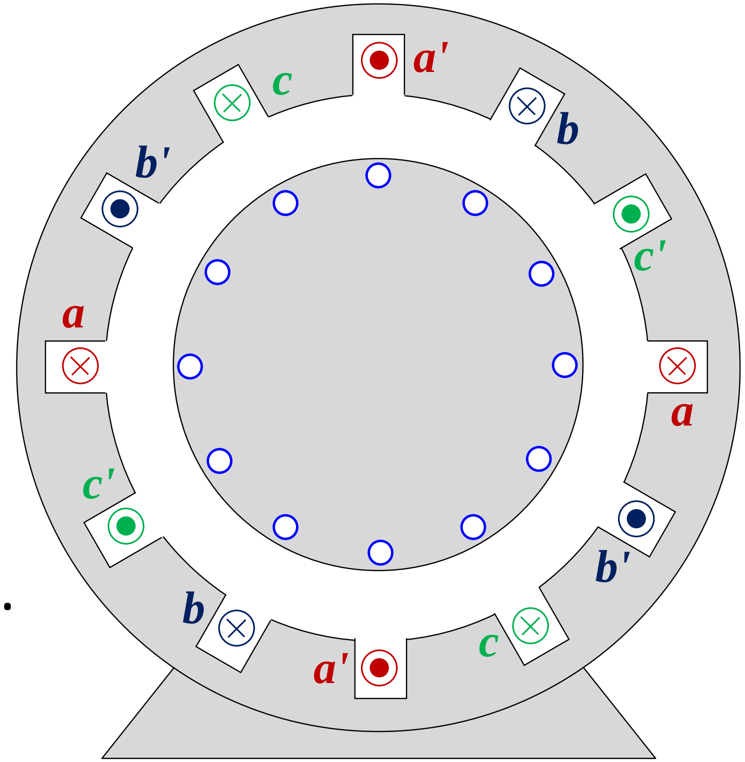
$$N_r > N_s$$

$$S < 0$$

Different Operating Mode of Induction Machines

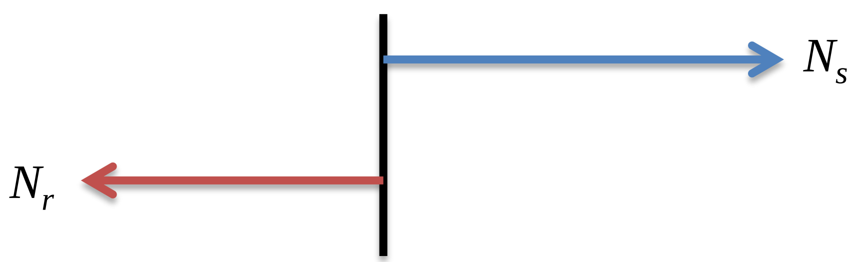
3. Plugging or Braking Mode

- If during the motoring mode the sequence of the applied voltage is changed,
- then the rotating magnetic field will change the direction,
- due to rotor inertia the magnetic field speed is in opposite of the rotor speed.
- The rotor will change the direction of rotation if it is not disconnected from the source.



$$N_r < 0$$

$$S > 1$$



Motoring Mode

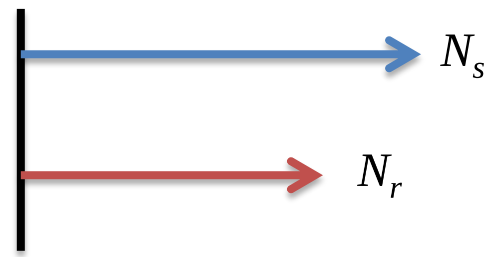
Assume the stator of a p-pole induction motor is connected to an ac source with frequency of f_1,

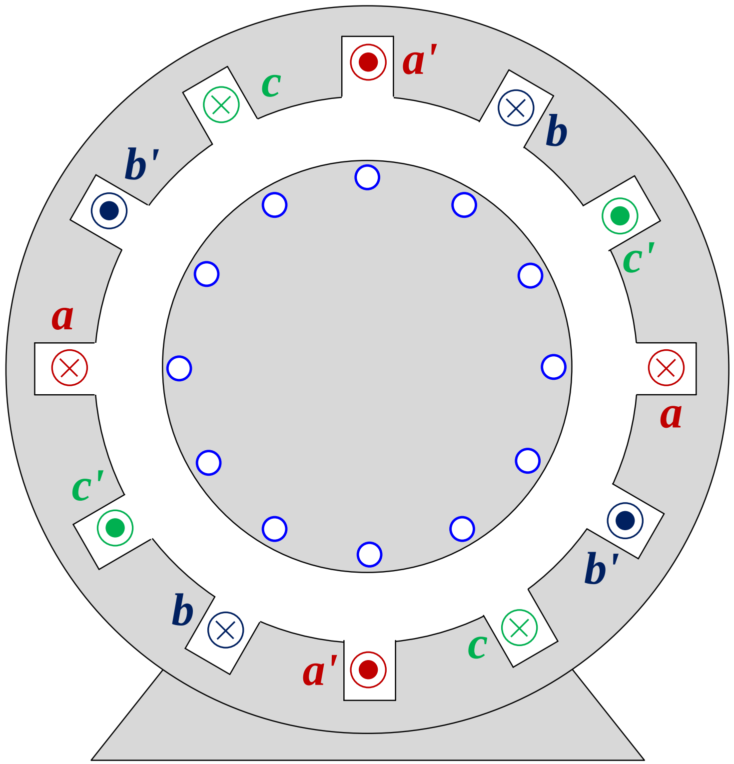
$$S = \frac{N_s - N_r}{N_s}$$

$$N_{slip} = N_s - N_r = SN_s$$

The rotor voltage/current frequency is

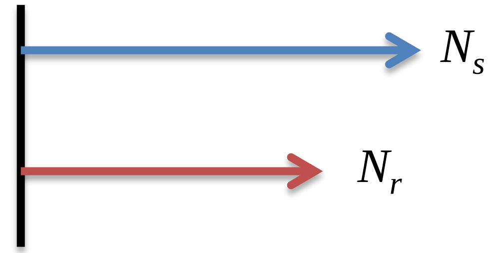
$$f_2 = \frac{p(N_s - N_r)}{120} = \frac{pSN_s}{120} = Sf_1$$



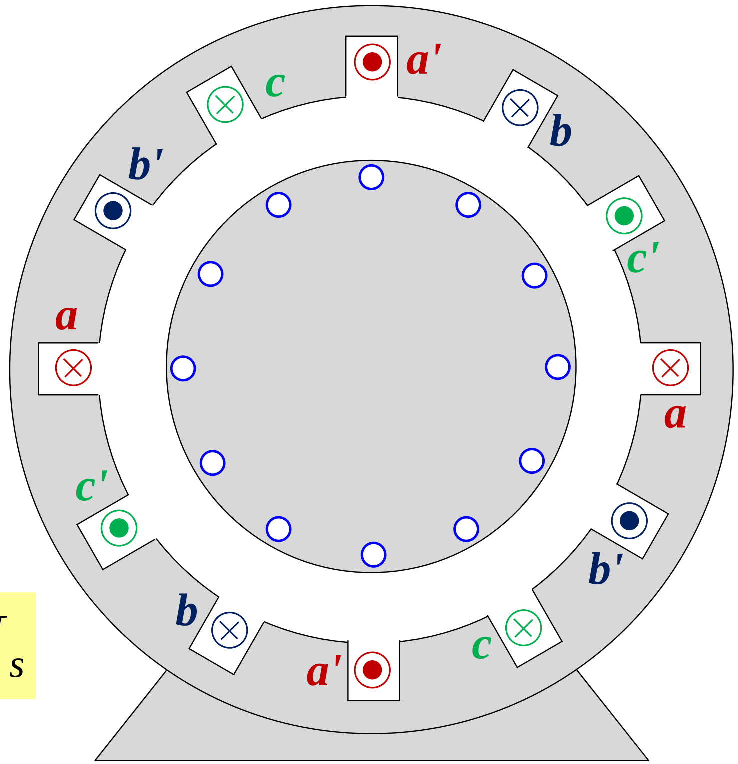


Motoring Mode

The speed of the rotor magnetic field with respect to rotor is



$$N_{frr} = \frac{120\, f_2}{p} = \frac{120 S f_1}{p} = SN_s$$



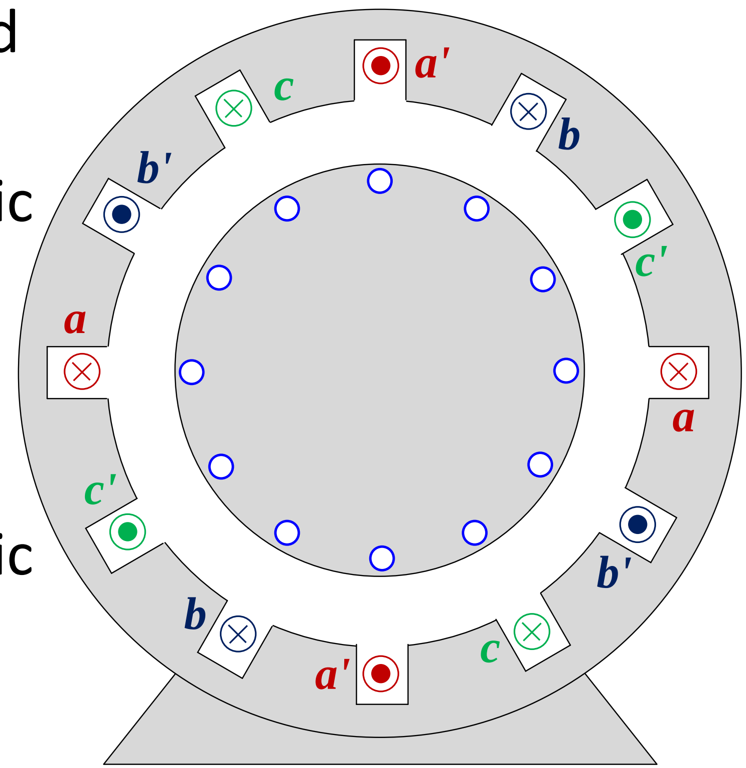
The speed of the rotor magnetic field with respect to stator is

$$N_{frs} = N_r + N_{frr} = (1-S)N_s + SN_s = N_s$$

Motoring Mode

Example: Consider a 3-phase induction motor with 460 V, 100 horsepower, 4-pole and 60 Hz which delivers the nominal power at the slip of 5%.

a) Calculate the synchronous speed and the rotor speed
b) Calculate the speed of rotor magnetic field
c) Calculate the rotor frequency
d) Calculate the slip speed
e) Calculate the speed of rotor magnetic field with respect to (1) rotor; (2) stator; and stator magnetic field.



Motoring Mode

Solution:

$$f_1 = 60 \ \text{Hz}$$

$$p_1 = 4 \qquad S = 0.05$$

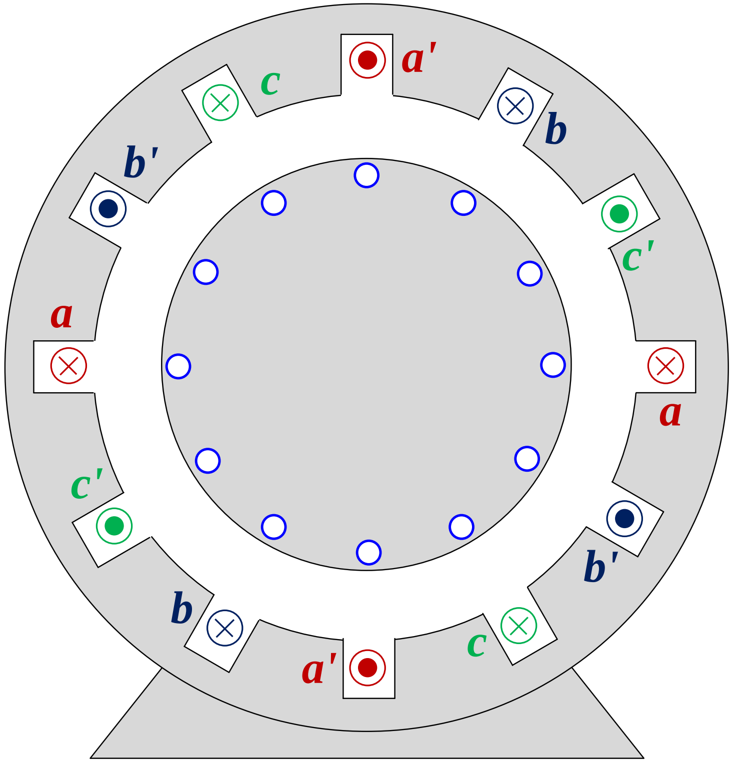
a) $$N_s = \frac{120 f_1}{p} = \frac{120 \times 60_1}{4} = 1800 \ \text{rpm}$$

$$N_r = (1 - S) N_s = 1710 \ \text{rpm}$$

b) $$N_{frs} = N_s = 1800 \ \text{rpm}$$

c) $$f_2 = s f_1 = 3 \ \text{Hz}$$

d) $$N_{slip} = S N_s = 90 \ \text{rpm}$$



Motoring Mode

Solution:

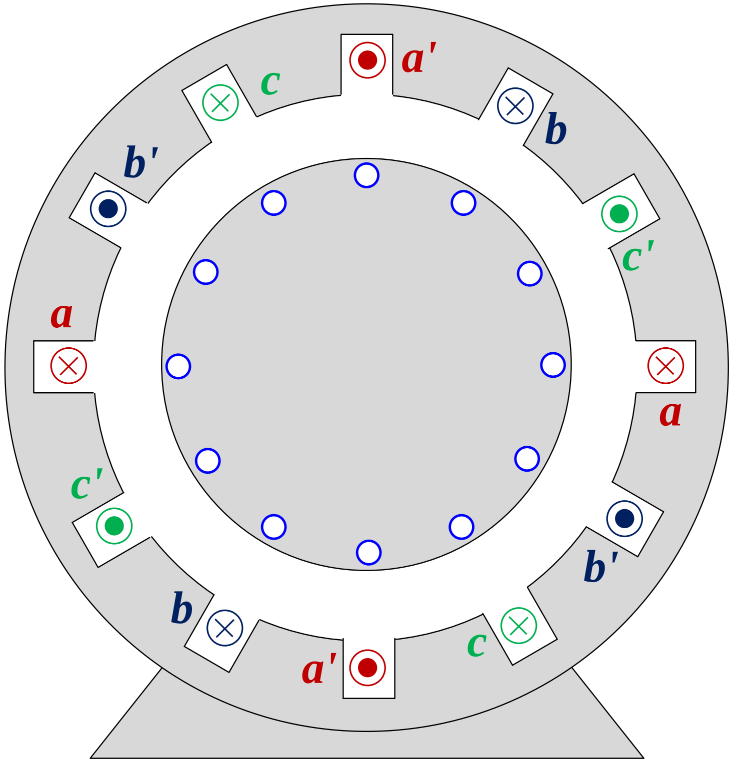
$$f_1 = 60 \ \text{Hz}$$

$$p_1 = 4 \qquad S = 0.05$$

e) $$N_{frr} = SN_s = 90 \ \text{rpm}$$

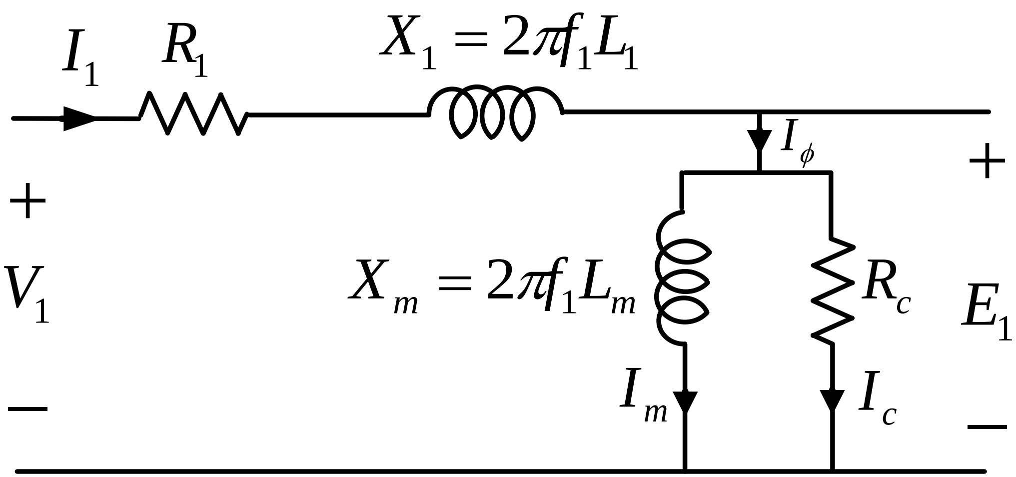
$$N_{frs} = N_s = 1800 \ \text{rpm}$$

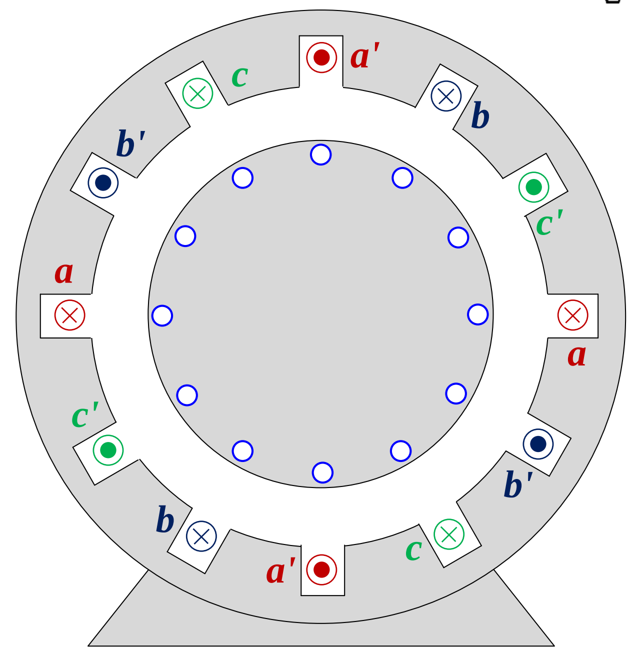
$$N_{frfs} = 0$$



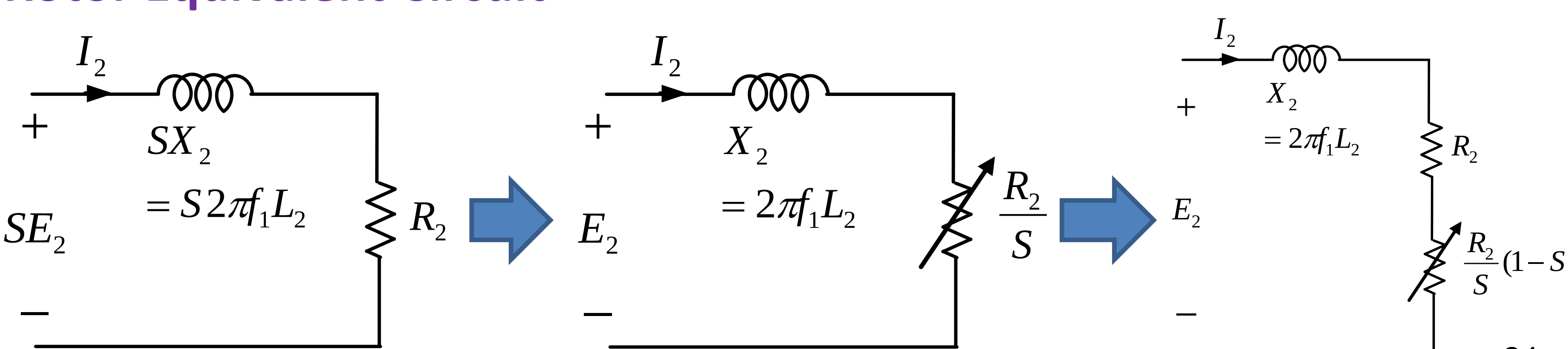
Equivalent Circuits of Induction Motors

Stator Equivalent Circuit





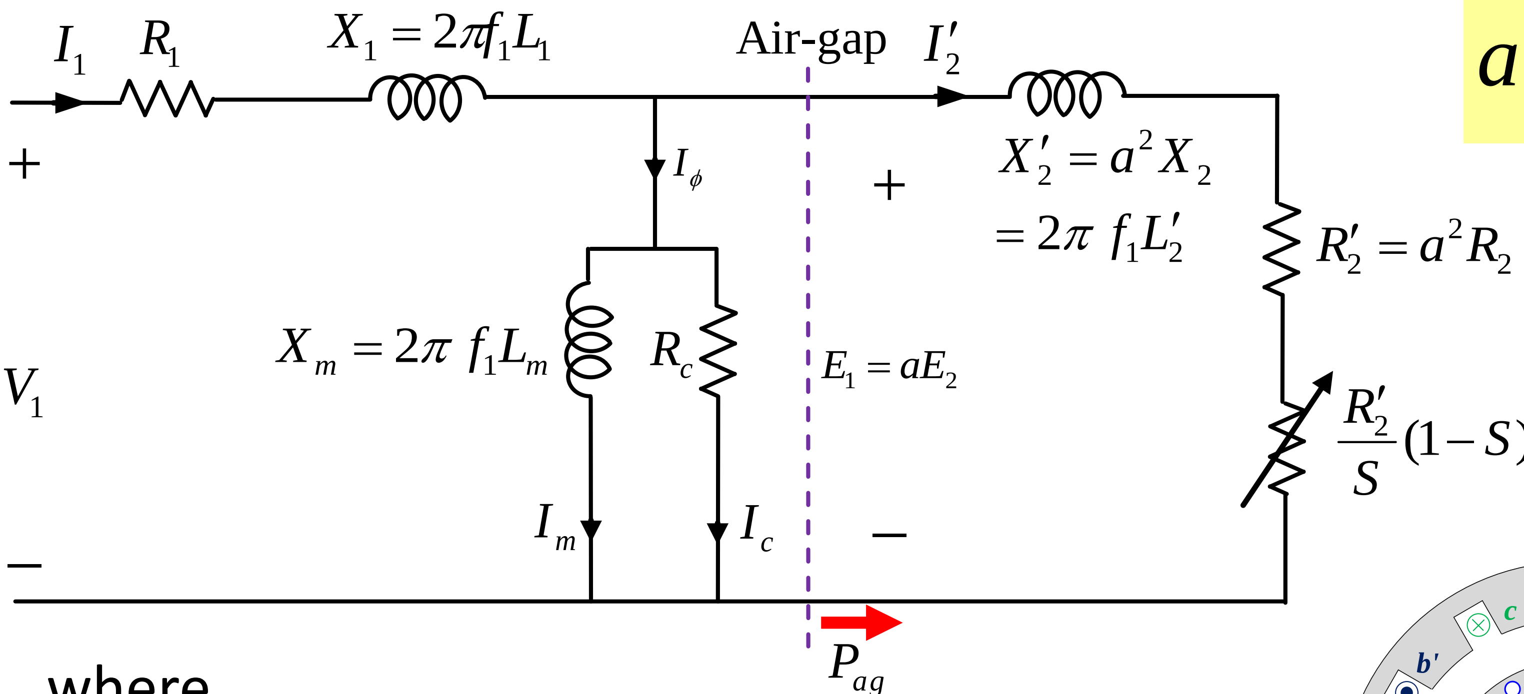
Rotor Equivalent Circuit

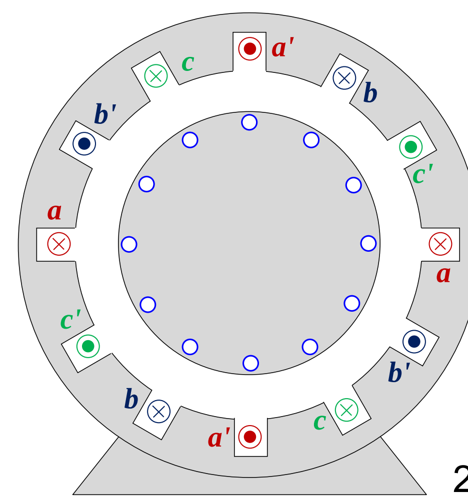


Equivalent Circuits of Induction Motors

Total Equivalent Circuit

$$a = \frac{N_1 k_{w1}}{N_2 k_{w2}}$$





where

f_1	is the stator applied source frequency.
V_1	is the stator phase voltage.
R_1	is the per-phase stator resistance.

Equivalent Circuits of Induction Motors

Total Equivalent Circuit

L_1 is the per-phase stator leakage inductance.

X_1 is the per-phase stator leakage reactance.

E_1 is the phase induced voltage in stator.

L_m is the per-phase magnetizing inductance.

X_m is the per-phase magnetizing reactance.

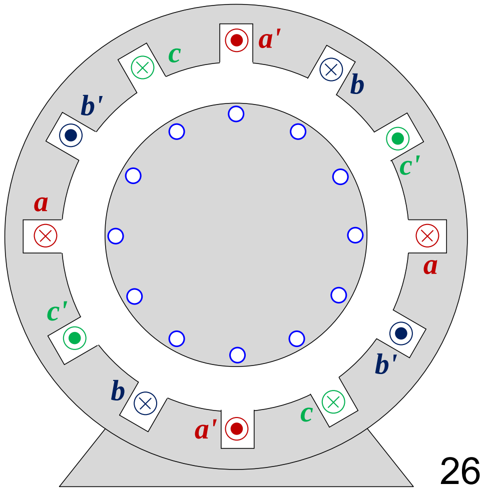
R_c is the per-phase equivalent resistance to model core losses.

I_1 is the phase stator current.

I_ϕ is the phase excitation (no-load) current.

I_m is the phase magnetizing current.

I_c is the phase equivalent current to model core losses.



Equivalent Circuits of Induction Motors

Total Equivalent Circuit

a is stator per rotor turn ratio.

E_2 is the phase induced voltage in rotor.

R_2 is the per-phase rotor resistance.

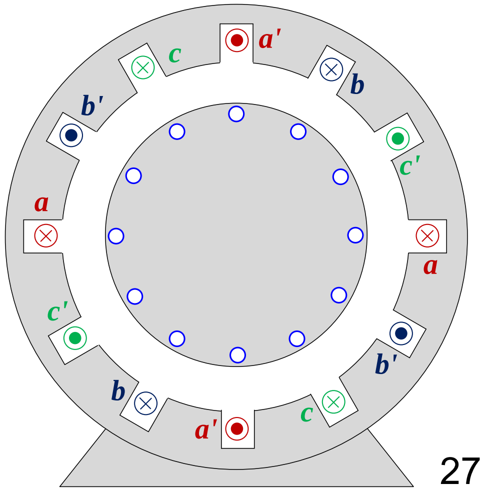
R'_2 is the per-phase rotor resistance referred to stator.

L_2 is the per-phase rotor leakage inductance.

L'_2 is the per-phase rotor leakage inductance referred to stator.

X_2 is the per-phase rotor leakage reactance under frequency of f_1.

X'_2 is the per-phase rotor leakage reactance referred to stator under frequency of f_1.



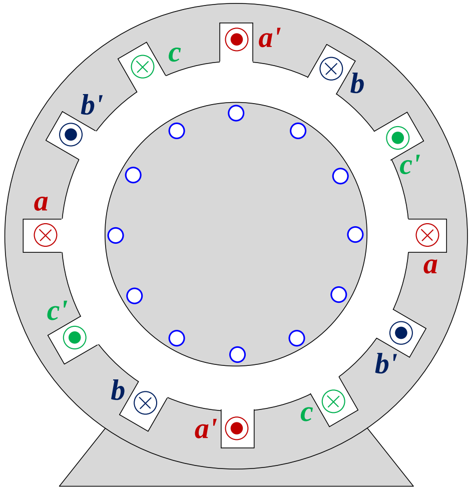
Equivalent Circuits of Induction Motors

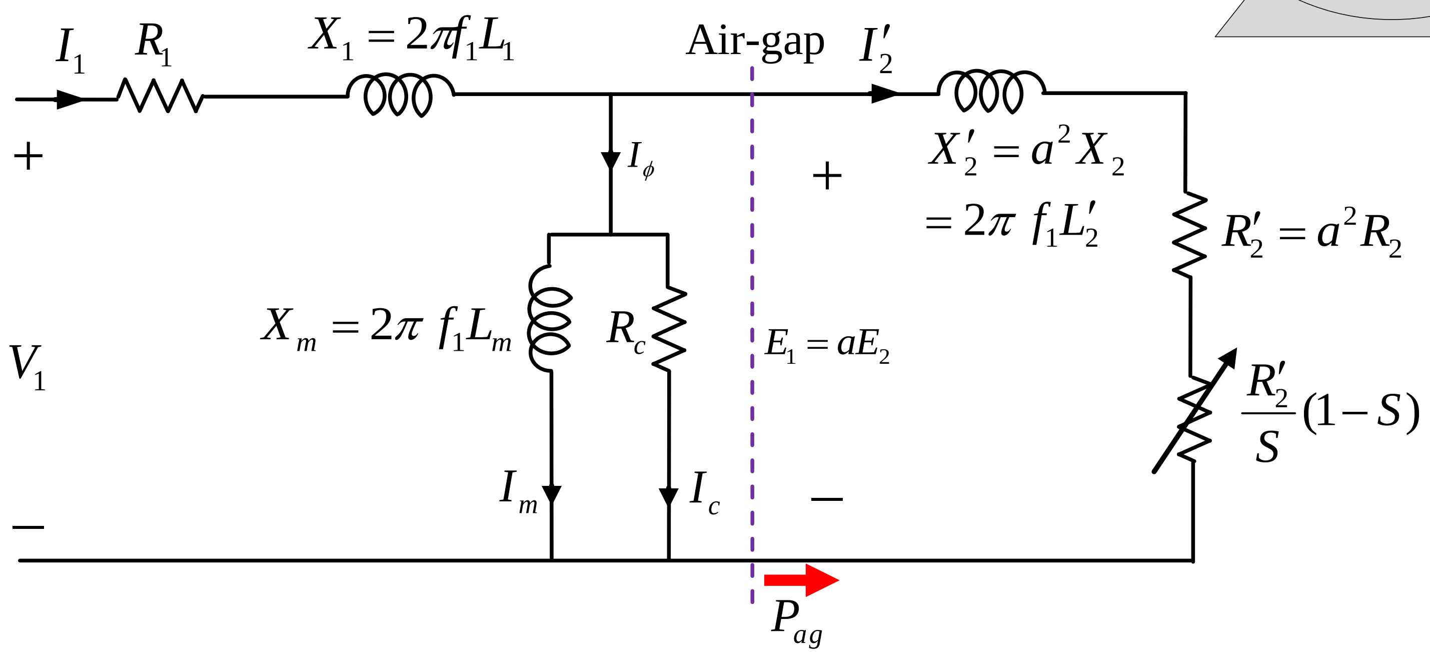
Total Equivalent Circuit

I_2 is the phase rotor current.

I'_2 is the phase rotor current referred to stator.

P_{ag} is the per-phase air-gap power.

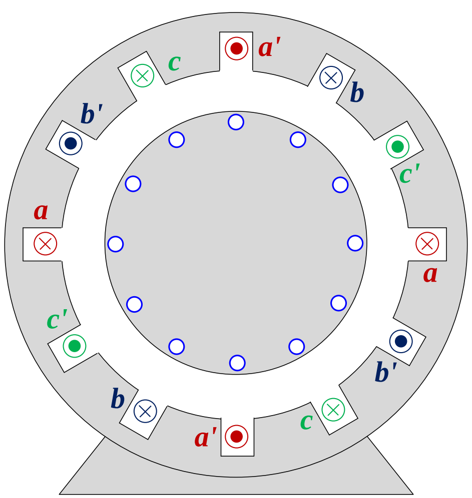


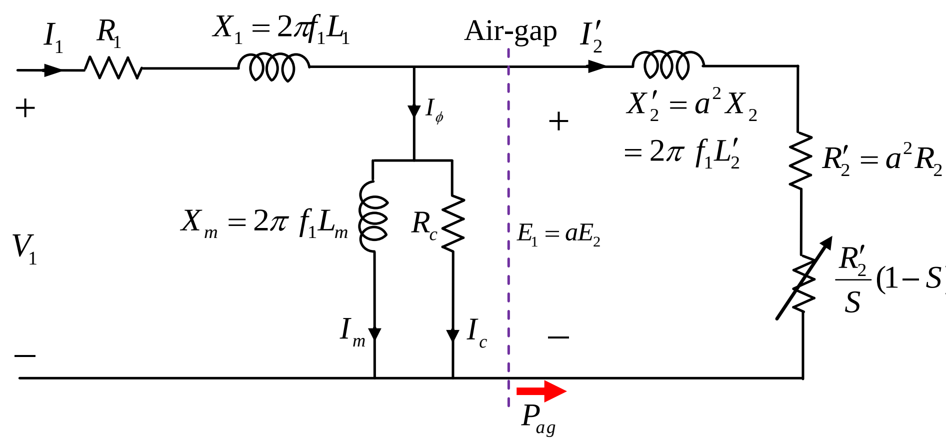


Equivalent Circuits of Induction Motors

A few points

- The mentioned equivalent circuit is valid for single-phase of a 3-phase induction machine.
- In this equivalent circuit all quantities are per-phase.
- All quantities referred to stator side.
- The frequency of the equivalent circuit is f_1.
- This equivalent circuit is similar to that of a transformer.

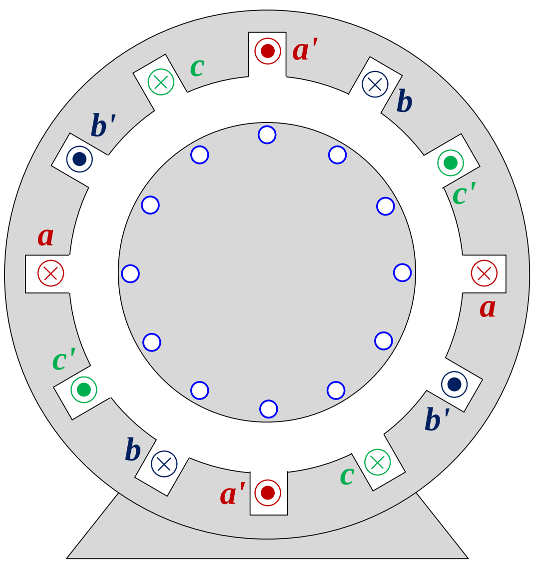


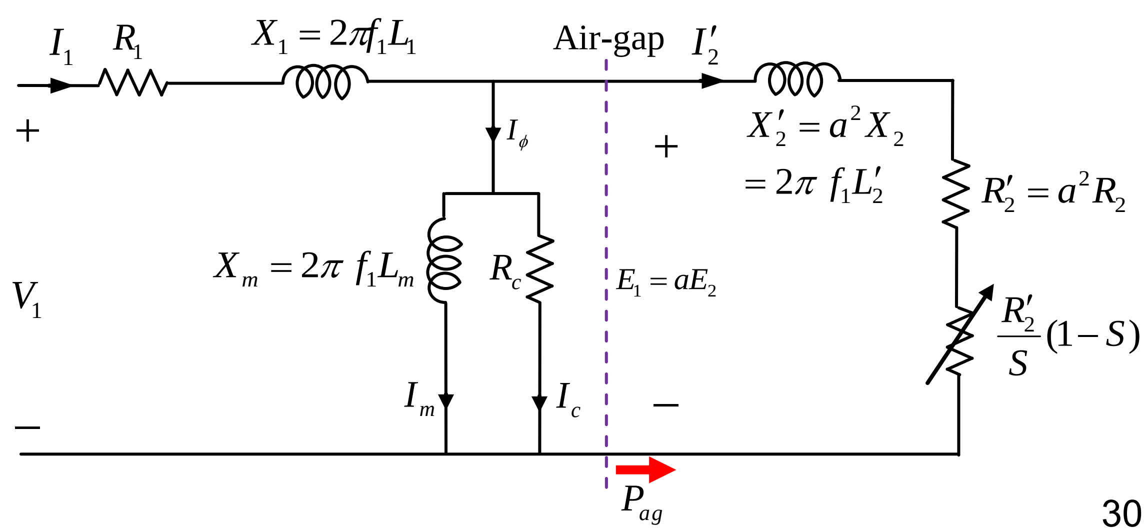


Air-Gap Power

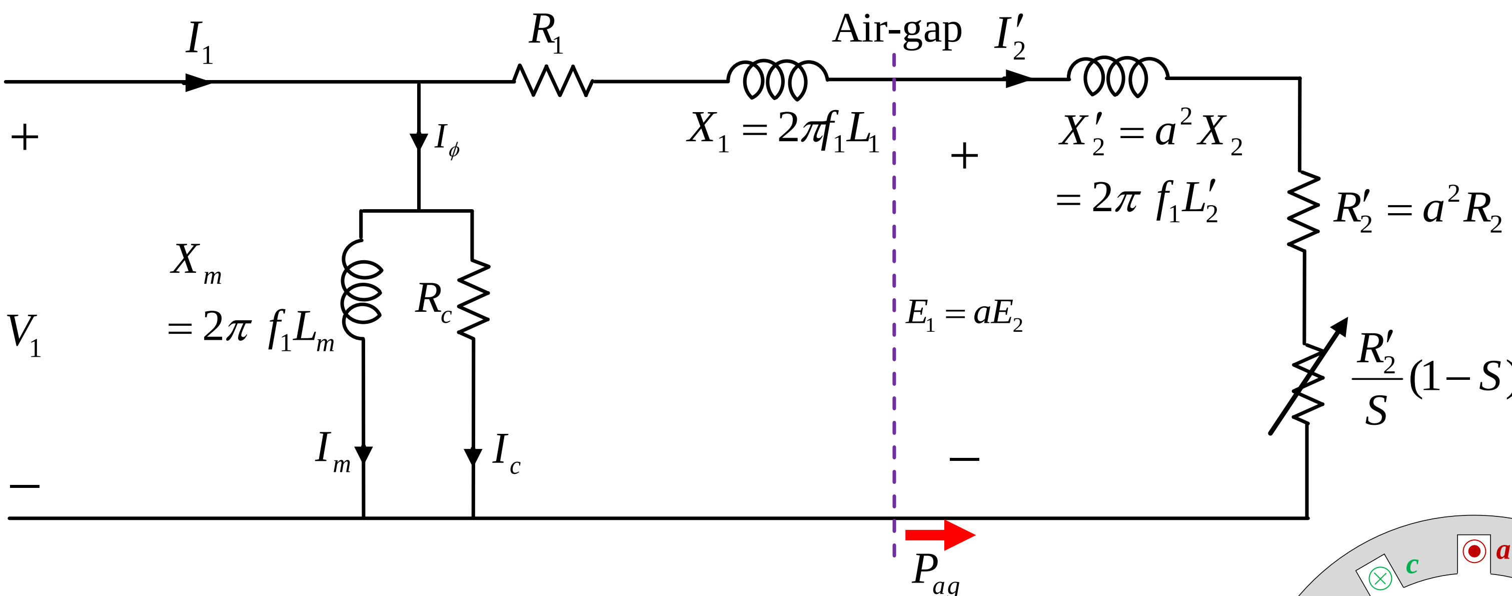
Air-gap power = rotor copper losses + developed mechanical power

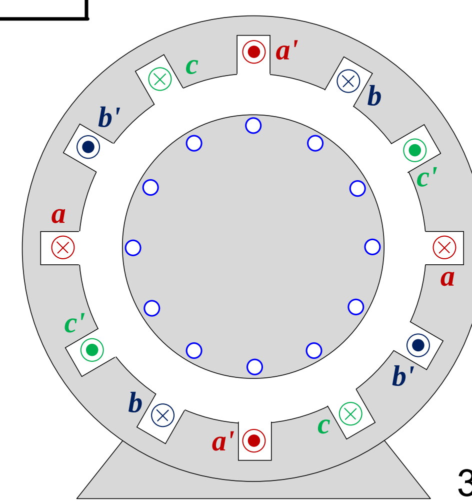
$$P_{ag} = R_2 I_2^2 + \frac{R_2}{S}(1-S) I_2^2$$



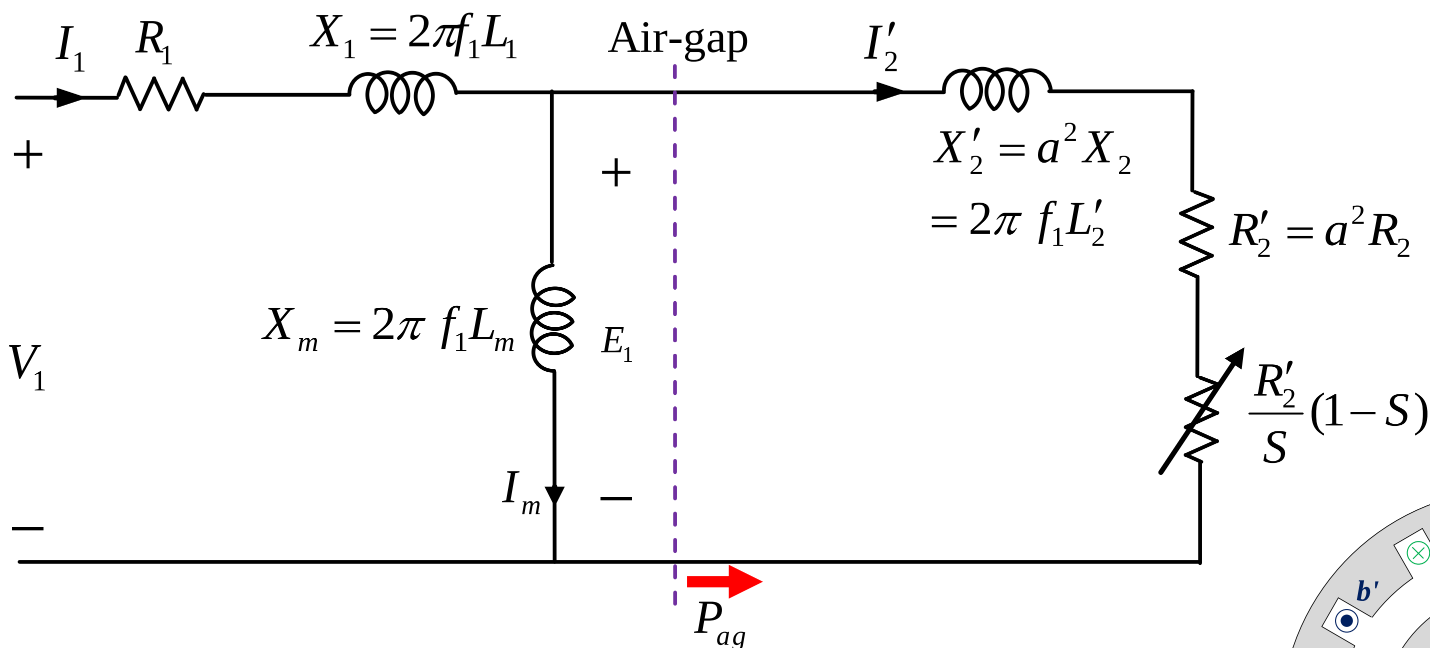


Approximated Equivalent Circuits of Induction Motors

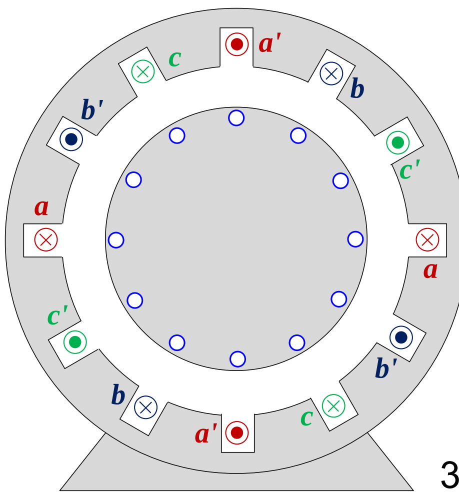




IEEE Equivalent Circuits of Induction Motors

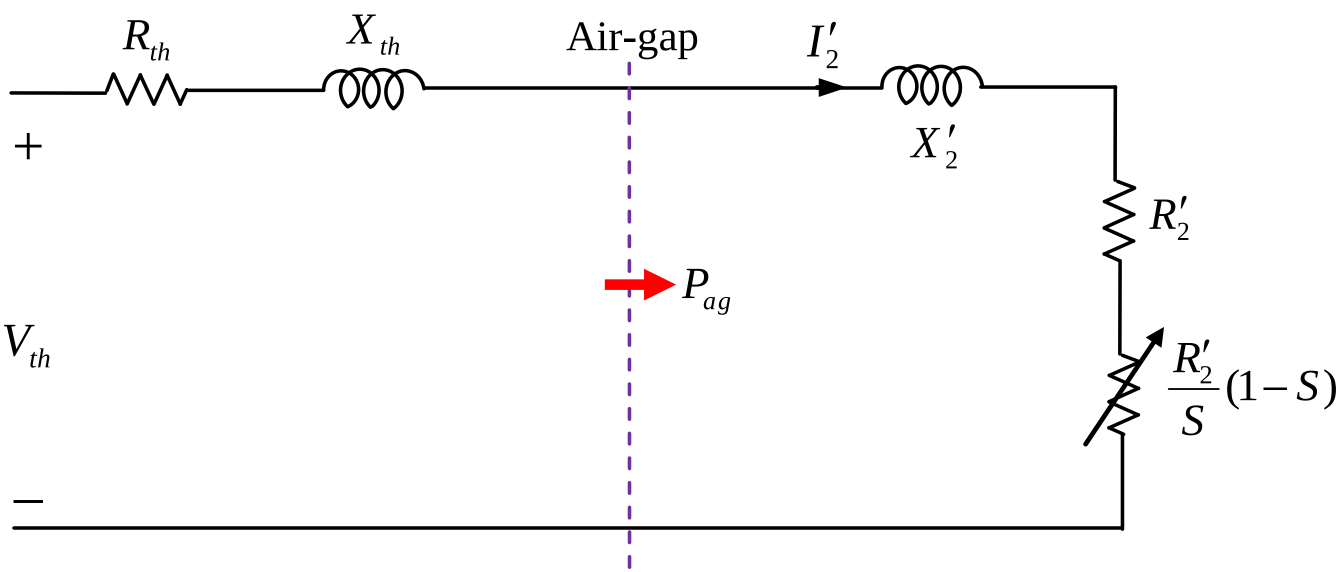


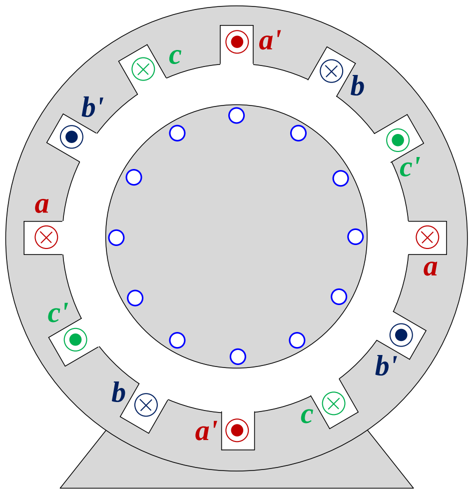
R_c has been removed from the circuit and core losses assumed to be a constant value.



Thevenin's Equivalent Circuits of Induction Motors

It is obtained from the IEEE equivalent circuit.



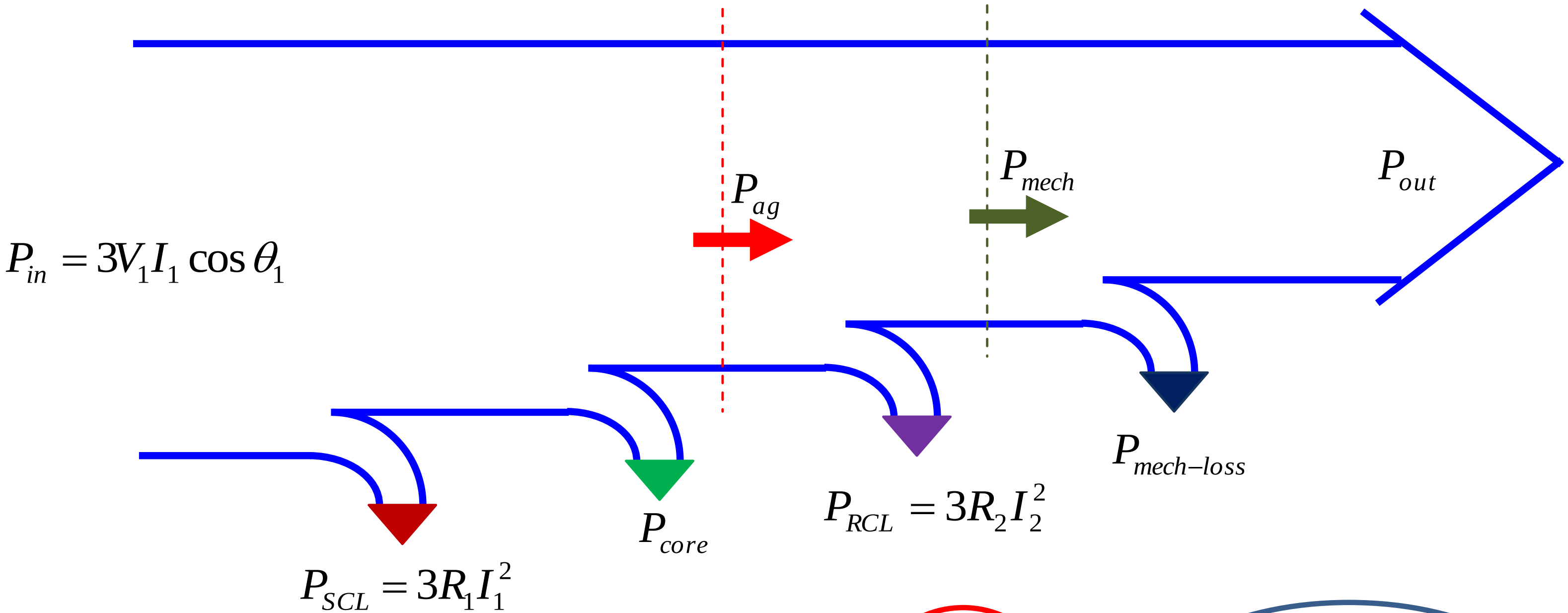


$$V_{th} = \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}} V_1 \quad \Rightarrow \quad V_{th} \approx \frac{X_m}{X_1 + X_m} V_1$$

$$Z_{th} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)} = R_{th} + jX_{th} \quad \Rightarrow \quad R_{th} \approx \left(\frac{X_m}{X_1 + X_m}\right)^2 R_1$$

$$X_{th} \approx X_1$$

Power and Torque in 3-phase Induction Motors



Air-gap power

$$P_{ag} = 3\frac{R_2}{S} I_2^2 = 3R_2 I_2^2 + 3\frac{R_2}{S}(1-S) I_2^2$$

Rotor copper losses

$$P_{RCL} = SP_{ag}$$

Mechanical power

$$P_{mech} = (1-S) P_{ag}$$

Power and Torque in 3-phase Induction Motors

Input power = Stator copper losses + Core losses + Air-gap power

Air-gap power = Rotor copper losses + Developed mechanical power

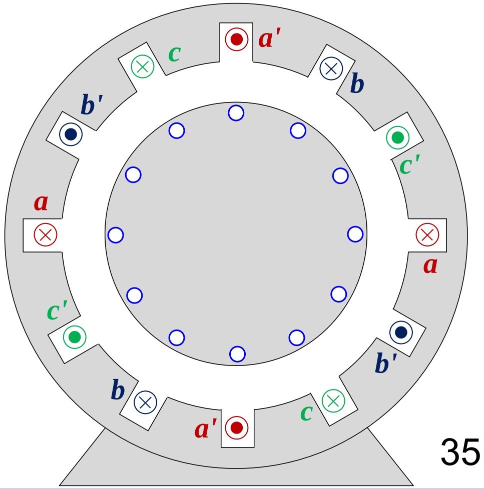
Developed mechanical power = Mechanical losses + Output power

$$P_{mech} = (1 - S) P_{ag}$$

$$P_{mech} = T_{mech}\, \omega_r$$

$$\omega_r = (1 - S)\omega_s$$

$$P_{ag} = T_{mech}\, \omega_s$$

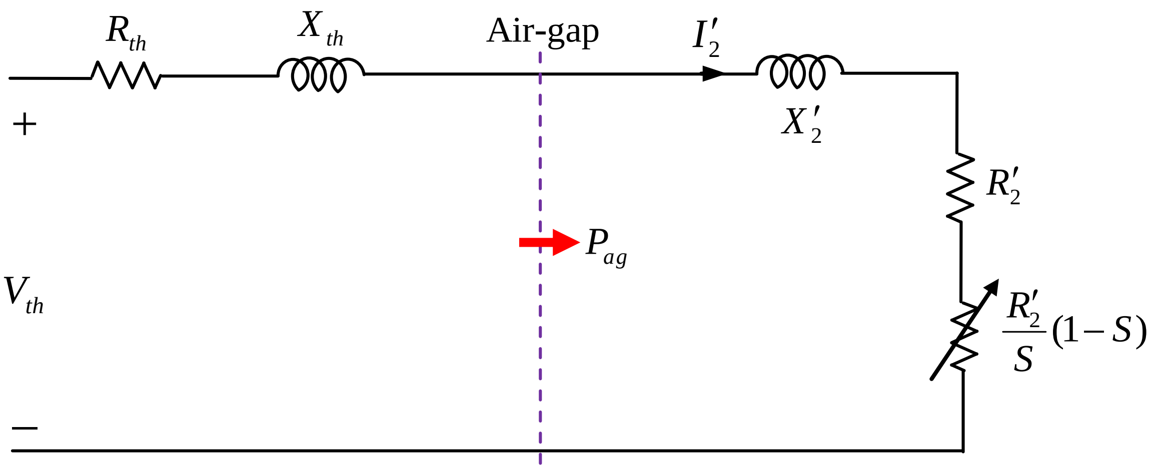


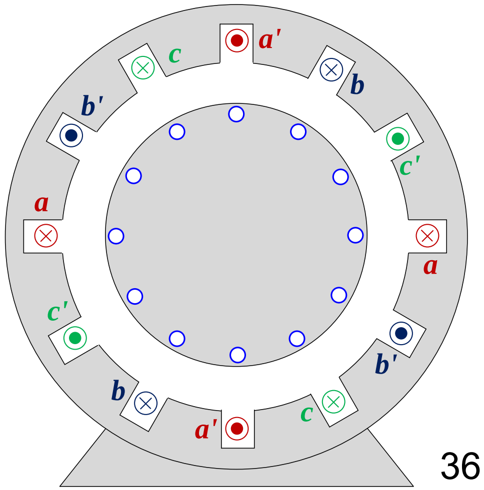
Power and Torque in 3-phase Induction Motors

Developed mechanical power

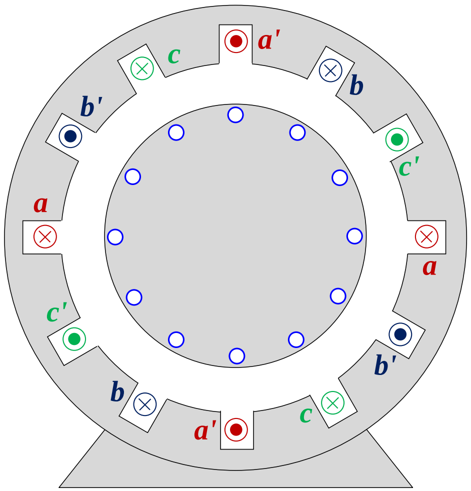
$$P_{ag} = T_{mech}\,\omega_s \quad \& \quad P_{ag} = 3\frac{R_2}{S} I_2^2 \quad \Rightarrow \quad T_{mech} = \frac{3}{\omega_s}\frac{R_2}{S} I_2^2 = \frac{3}{\omega_s}\frac{R_2'}{S} I_2'^2$$

$$T_{mech} = \frac{3}{\omega_s}\frac{R_2'}{S}\frac{V_{th}^2}{\left(R_{th} + \frac{R_2'}{S}\right)^2 + \left(X_{th} + X_2'\right)^2}$$



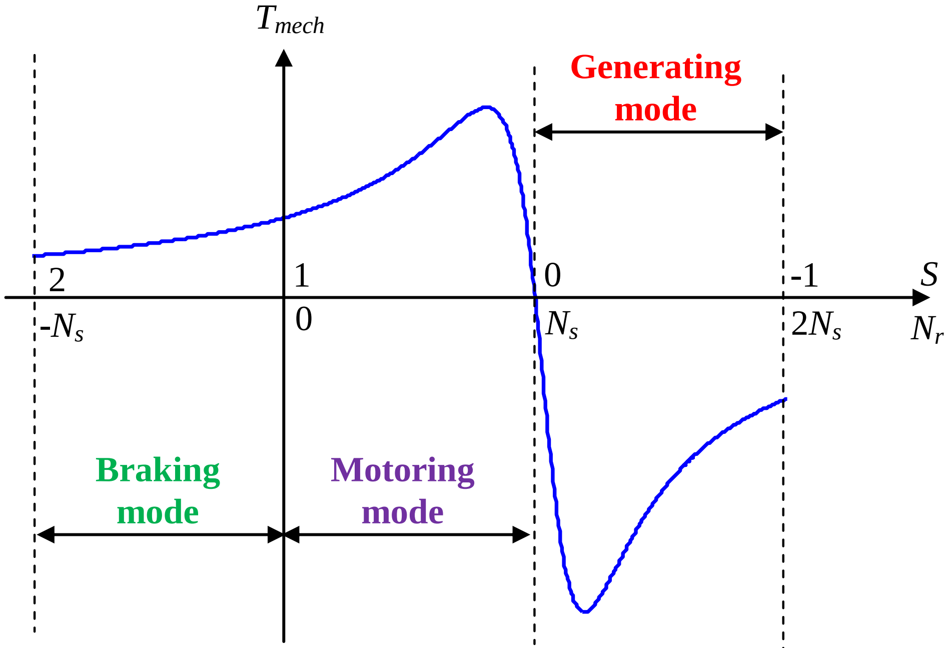


Power and Torque in 3-phase Induction Motors



Developed mechanical power

$$T_{mech} = \frac{3}{\omega_s} \frac{R_2'}{S} \frac{V_{th}^2}{\left(R_{th} + \frac{R_2'}{S}\right)^2 + \left(X_{th} + X_2'\right)^2}$$

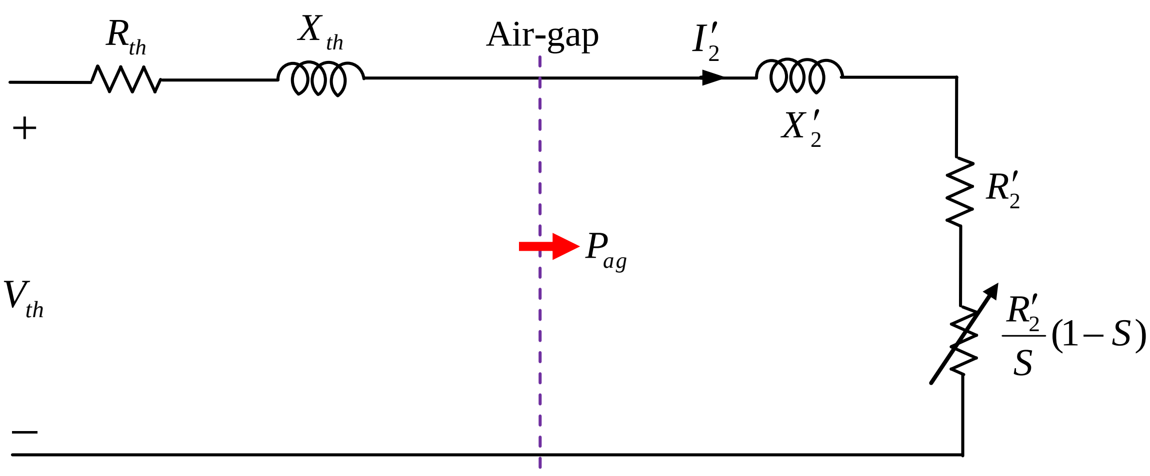


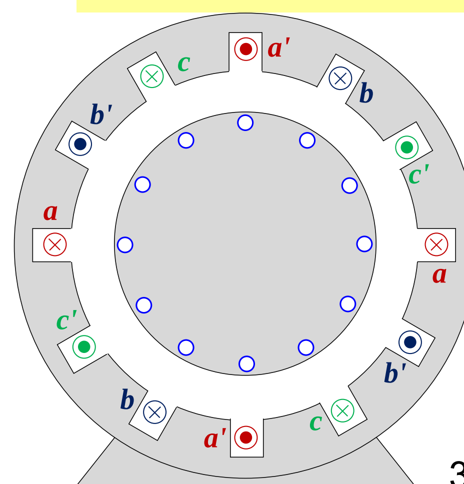
Power and Torque in 3-phase Induction Motors

Developed mechanical power

$$T_{mech} = \frac{3}{\omega_s}\frac{R_2'}{S}\frac{V_{th}^2}{\left(R_{th}+\frac{R_2'}{S}\right)^2+\left(X_{th}+X_2'\right)^2}$$

If $R_{th}+\frac{R_2'}{S} >> X_{th}+X_2'$ & $\frac{R_2'}{S} >> R_{th}$ ⇒ $T_{mech} = \frac{3}{\omega_s}\frac{V_{th}^2}{R_2'}S$

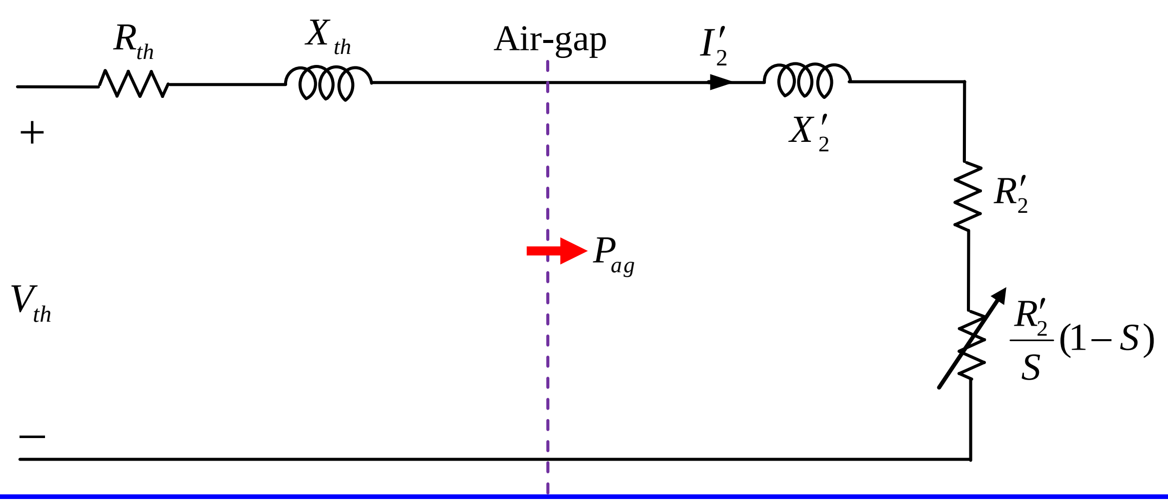


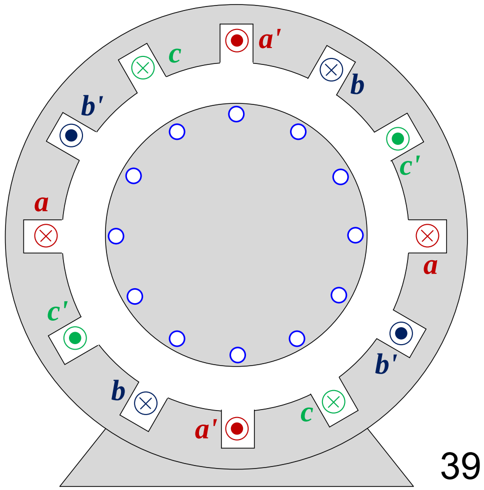


Power and Torque in 3-phase Induction Motors

Output or External Power

$$P_{out} = T_{out}\,\omega_r \;\Rightarrow\; T_{out} = \frac{P_{out}}{\omega_r} \;\Rightarrow\; T_{out} = \frac{P_{mech} - P_{mech-loss}}{\omega_r}$$





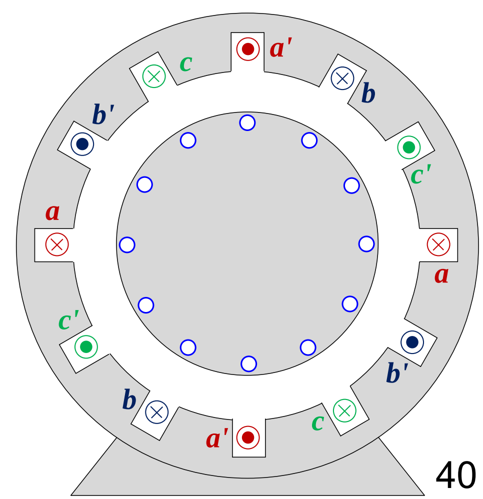
Power and Torque in 3-phase Induction Motors

Maximum Torque

$$\frac{dT_{mech}}{dS} = 0 \quad \Rightarrow \quad \frac{R'_2}{S_{T_{\max}}} = \sqrt{R_{th}^2 + \left(X_{th} + X'_2\right)^2}$$

$$\Rightarrow \quad S_{T_{\max}} = \frac{R'_2}{\sqrt{R_{th}^2 + \left(X_{th} + X'_2\right)^2}} \quad \Rightarrow \quad S_{T_{\max}} \approx \frac{R'_2}{X_{th} + X'}$$

The **slip of the maximum torque depends** on the **rotor resistance**.



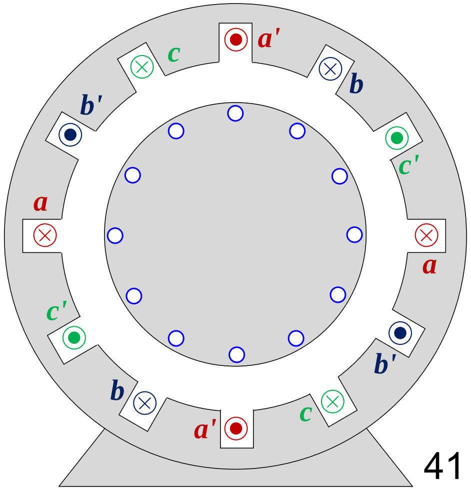
Power and Torque in 3-phase Induction Motors

Maximum Torque

$$S_{T_{\max}} = \frac{R_2'}{\sqrt{R_{th}^2 + \left(X_{th} + X_2'\right)^2}} \quad \Rightarrow \quad S_{T_{\max}} \approx \frac{R_2'}{X_{th} + X'}$$

$$T_{\max} = \frac{3}{2\omega_s} \frac{V_{th}^2}{R_{th} + \sqrt{R_{th}^2 + \left(X_{th} + X_2'\right)^2}} \quad \Rightarrow \quad T_{\max} = \frac{3}{2\omega_s} \frac{V_{th}^2}{X_{th} + X_2'}$$

The **maximum torque** is **independent** of the **rotor resistance**.



Power and Torque in 3-phase Induction Motors

Starting Torque

Starting torque is the torque when the rotor speed is zero or $S=1$.

$$T_{start} = \frac{3}{\omega_s} R_2' \frac{V_{th}^2}{\left(R_{th} + R_2'\right)^2 + \left(X_{th} + X_2'\right)^2}$$