In The Name of God The Most Compassionate, The Most Merciful

General Theory of Electric Machines

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Chapter 2
Transformers

Introduction

Transformers have the following characteristics

1. Transformers are **electromagnetic energy conversion** systems; as they receive electrical energy from the network; convert it to the magnetic energy; and then the magnetic energy is converted to the electrical energy with different voltage and current level.

2. A transformer has at least two windings: a primary and a secondary winding. **Primary winding** is the winding connected to the **power source** and the **secondary winding** is that connected to the **load**.

3. There is **no electrical connection** between the primary and secondary windings (except in auto-transformers); the connection is through a magnetic field.

Introduction

4. If the secondary voltage is lower than that of primary, the transformer is **step-down**; otherwise it is **step-up**.

5. **Swapping the primary and secondary windings** will change a step-down transformer to a step-up transformer and vice-versa.

6. In a **step-up** transformer, the **number of turns of the secondary winding** is **higher** than that of the **primary** winding.

7. In a **step-down** transformer, the **number of turns of the secondary winding** is **lower** than that of the **primary** winding.

8. Since transformers have **no mechanical part**, their **efficiency** is normally **very high**.

Applications of Transformers

1. Electric Power **Transmission Systems**.

2. **Impedance Matching** (e.g. in speakers).

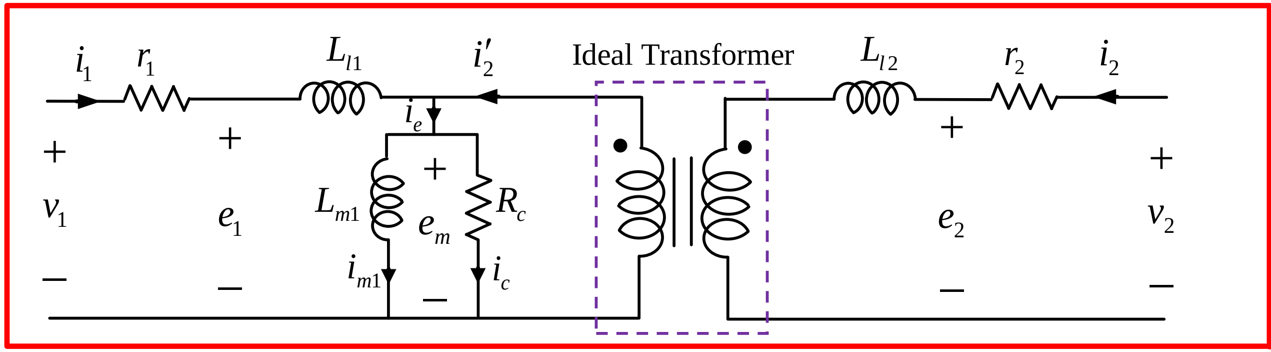
3. **Blocking** the dc component of an ac + dc signal or power.

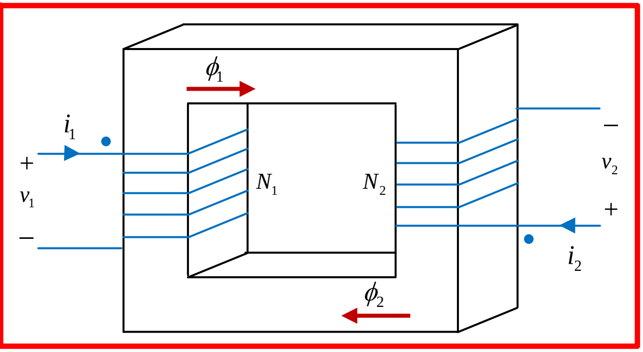
4. Voltage and current **measurement**: Voltage or potential transformers (VT) or (PT); Current transformers (CT).

Ideal Transformers

An ideal transformer has the following characteristics:

1. The **ohmic losses** due to the primary and secondary winding resistances are **neglected**. $r_1 = r_2 = 0$
2. The **core losses** are **neglected**. $R_c \to \infty$
3. The **magnetizing curve** of the transformer core is assumed to be **linear**.
4. The **leakage flux** of the windings is **neglected**. $L_{l1} = L_{l2} = 0$
5. The **core permeability** goes to **infinity**. $\mu_c \to \infty$ ⇒ $L_{m1} \to \infty$





Ideal Transformers

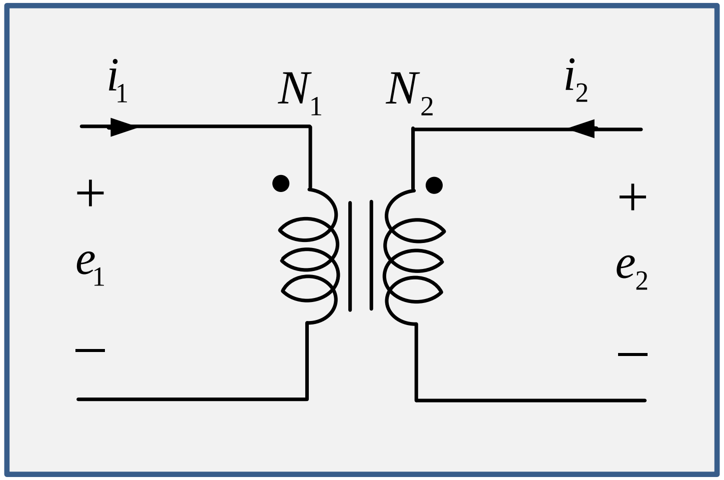
- The **induced voltage** in the primary and secondary windings will be

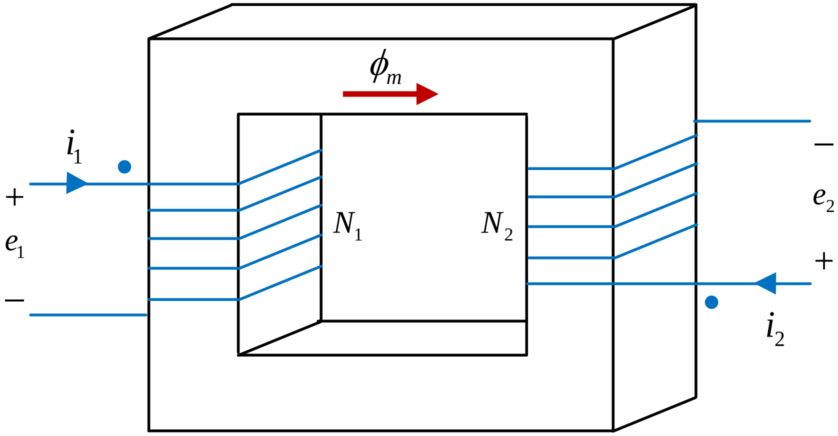
$$e_1 = N_1 \frac{d\phi_1}{dt}$$

$$e_2 = N_2 \frac{d\phi_2}{dt}$$

$$\phi_1 = \phi_2 = \phi_m$$

$$\Rightarrow \quad \frac{e_1}{e_2} = \frac{N_1}{N_2}$$

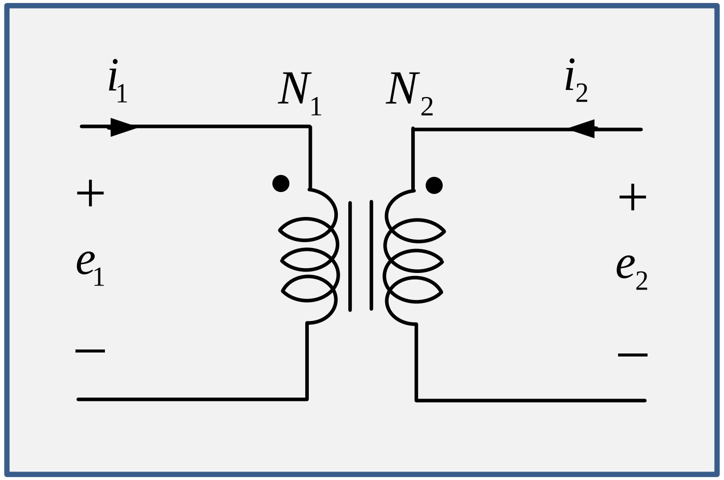


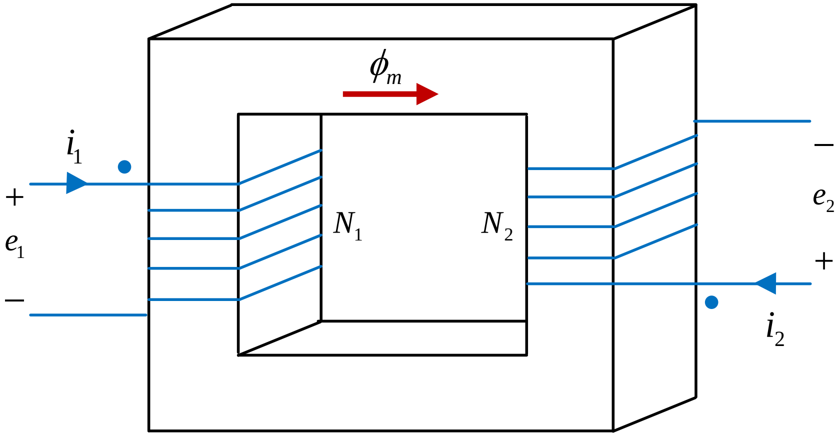


Ideal Transformers

- Since the algebraic sum of the magneto-motive forces (MMFs) is zero, in two-winding transformers it follows that:

$$\sum_k N_k i_k = 0 \Rightarrow N_1 i_1 + N_2 i_2 = 0 \Rightarrow \frac{i_1}{i_2} = -\frac{N_2}{N_1}$$

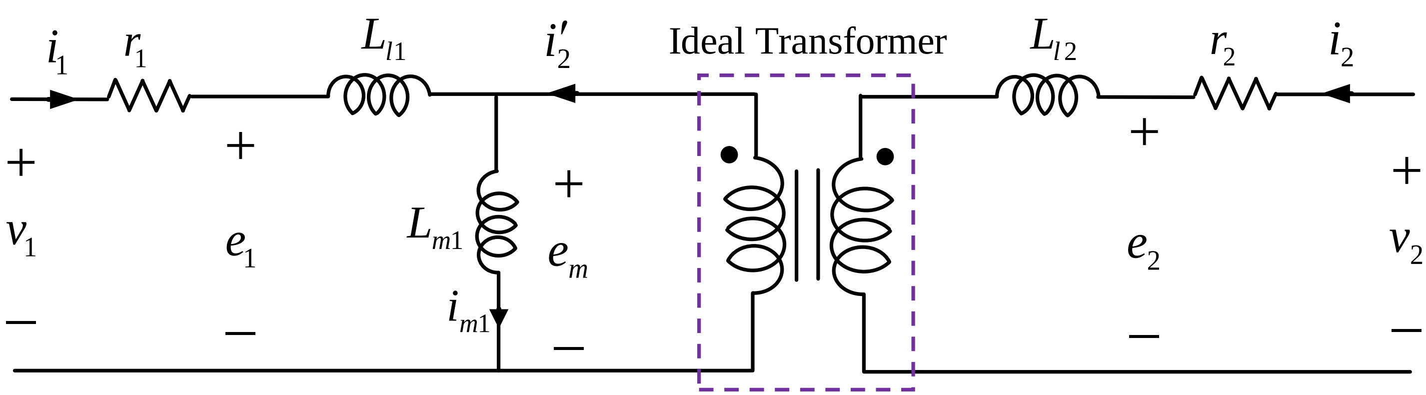




More Realistic Transformers

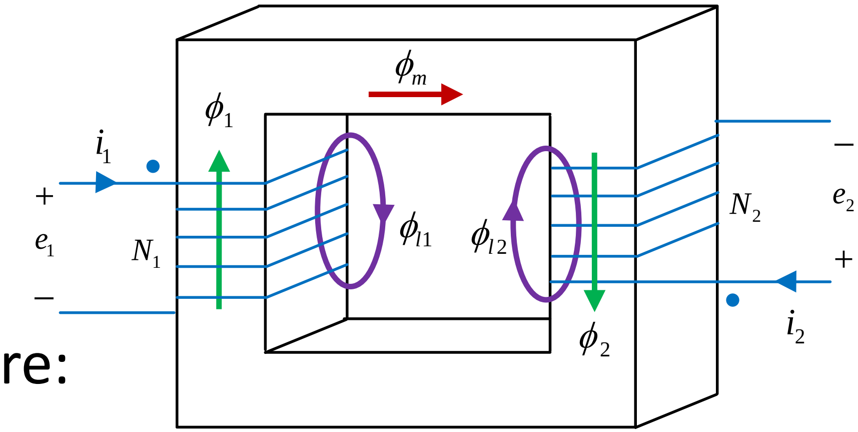
The transformer has the following characteristics:

1. The **ohmic losses** due to the primary and secondary winding resistances are **considered**.
2. The **core losses** are still **neglected**.
3. The **magnetizing curve** of the transformer core is still assumed to be **linear**.
4. The **leakage flux** of the windings is **considered**.
5. The **core permeability** is a **finite** value.



Model of Two-Winding Transformers

1. Flux linkage equations



- The fluxes of the windings are:

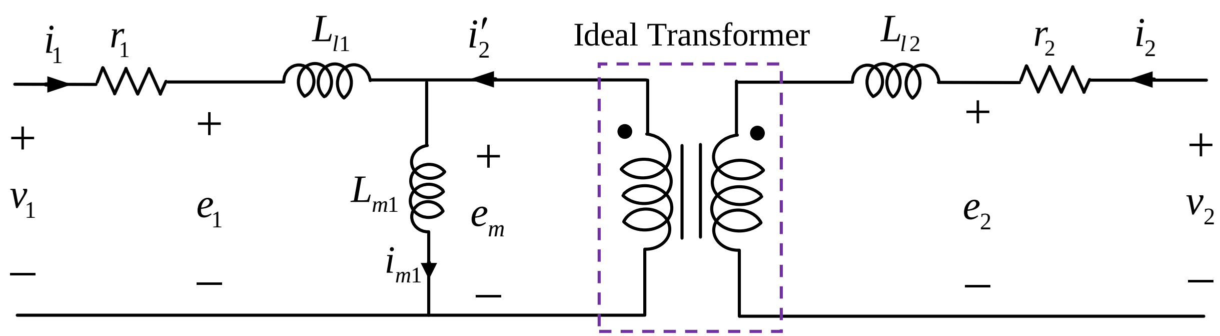
$$\phi_1 = \phi_{l1} + \phi_m$$

$$\phi_2 = \phi_{l2} + \phi_m$$

- The flux linkages of the windings are:

$$\lambda_1 = N_1\phi_1 = N_1\left(\phi_{l1} + \phi_m\right)$$

$$\lambda_2 = N_2\phi_2 = N_2\left(\phi_{l2} + \phi_m\right)$$



Model of Two-Winding Transformers

1. Flux linkage equations

- The relation between the **MMF**, magnetic **flux** and **permeance** is as follows:

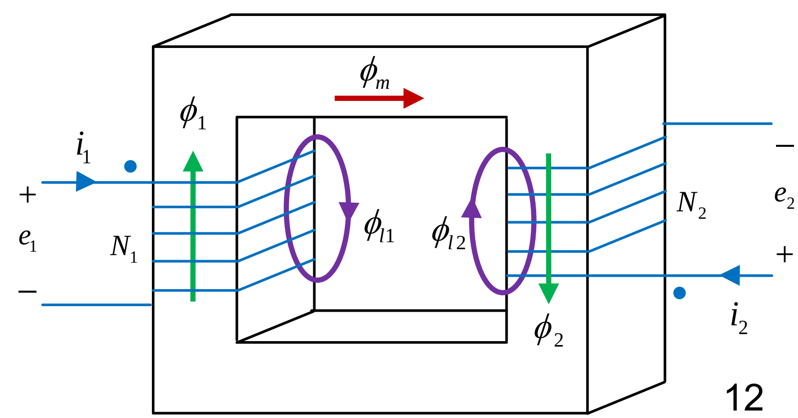
$$MMF = \sum Ni = \Re\phi = \frac{1}{P}\phi$$

$$\Re = \frac{1}{P}$$

where $\Re$ is the reluctance and P is the permeance.

- It can be rewritten as

$$\phi = P\sum Ni$$



Model of Two-Winding Transformers

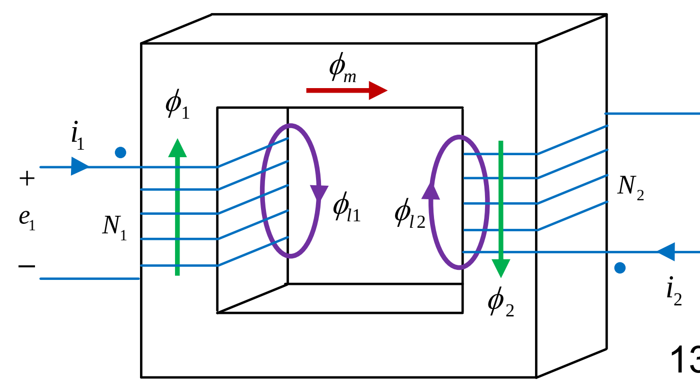
1. Flux linkage equations

- Substitution the permeance relation into the flux linkage expression for the primary winding yields

$$\left.\begin{array}{l}\lambda_1 = N_1\left(\phi_{l1} + \phi_m\right) \\ \phi = P\sum Ni\end{array}\right\} \Rightarrow \lambda_1 = N_1\left[\overbrace{\left(N_1\, i_1\, P_{l1}\right)}^{\phi_{l1}} + \overbrace{\left(N_1\, i_1 + N_2\, i_2\right)P_m}^{\phi_m}\right]$$

$$\Rightarrow \lambda_1 = \underbrace{\left(N_1^2\, P_{l1} + N_1^2\, P_m\right)}_{L_{11}}\, i_1 + \underbrace{N_1\, N_2\, P_m}_{L_{12}}\, i_2$$

$$\Rightarrow \lambda_1 = L_{11} i_1 + L_{12}\, i_2$$



Model of Two-Winding Transformers

1. Flux linkage equations

- Similarly for the secondary winding:

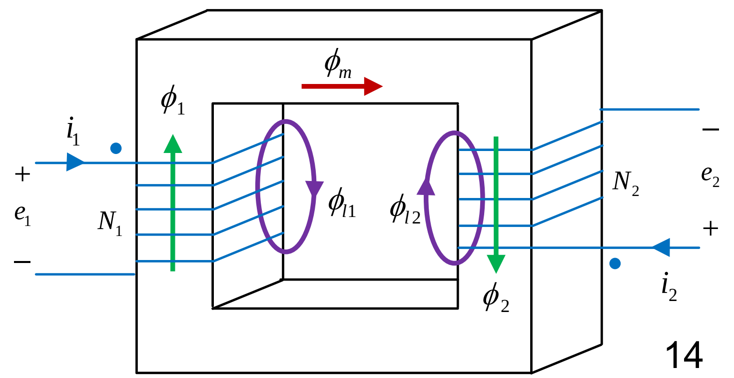
$$\lambda_2 = N_2\left(\phi_{l2} + \phi_m\right)$$

$$\phi = P\sum Ni$$

$$\lambda_2 = N_2\left[\underbrace{\left(N_2\, i_2\, P_{l2}\right)}_{\phi_{l2}} + \underbrace{\left(N_1\, i_1 + N_2\, i_2\right)P_m}_{\phi_m}\right]$$

$$\lambda_2 = \underbrace{\left(N_2^2\, P_{l2} + N_2^2\, P_m\right)}_{L_{22}} i_2 + \underbrace{N_1\, N_2\, P_m}_{L_{21}} i_1$$

$$\lambda_2 = L_{21}\, i_1 + L_{22}\, i_2$$



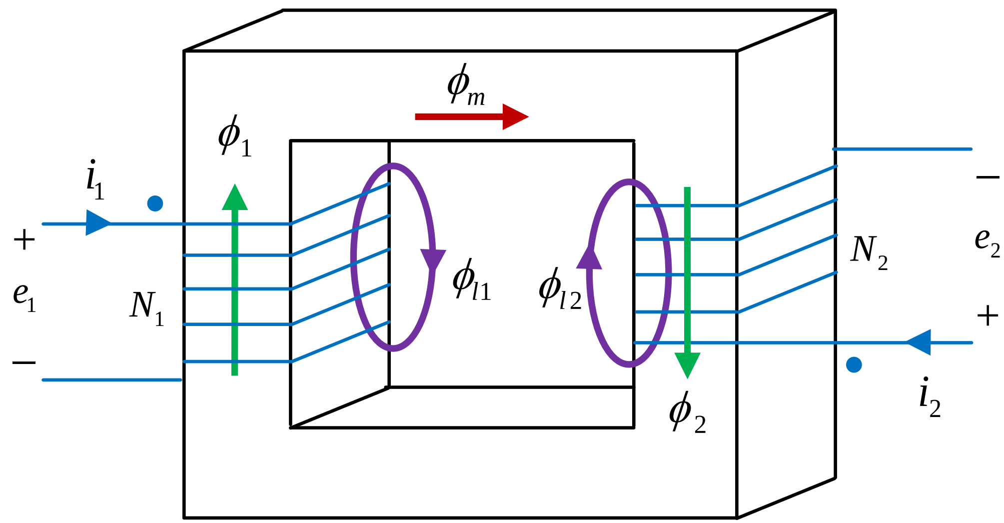
Model of Two-Winding Transformers

1. Flux linkage equations

- The resulting flux linkage equations for the two-winding transformers in terms of the winding inductances are:

$$\lambda_1 = L_{11} i_1 + L_{12}\, i_2$$

$$\lambda_2 = L_{21}\, i_1 + L_{22}\, i_2$$



- Or in the matrix form

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix}$$

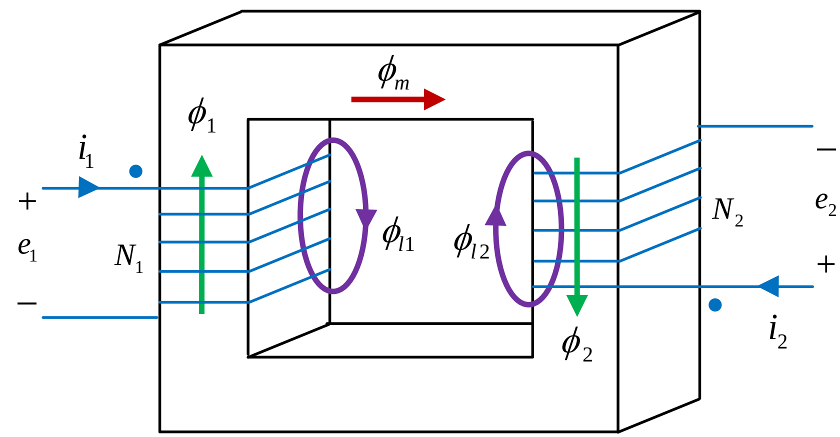
where L_{11} and L_{22} are the self-inductances of the windings, and L_{12} and L_{21} are the mutual inductances between them.

Model of Two-Winding Transformers

1. Flux linkage equations

- The self-inductance of winding 1 is obtained as

$$L_{11} = \frac{\lambda_1 |_{i_2=0}}{i_1} = \underbrace{N_1^2 P_{l1}}_{L_{l1}} + \underbrace{N_1^2 P_m}_{L_{m1}}$$



- Likewise for winding 2

$$L_{22} = \frac{\lambda_2 |_{i_1=0}}{i_2} = \underbrace{N_2^2 P_{l2}}_{L_{l2}} + \underbrace{N_2^2 P_m}_{L_{m2}}$$

$$\frac{L_{m1}}{L_{m2}} = \left(\frac{N_1}{N_2}\right)^2$$

where L_{l1} and L_{l2} are the leakage inductances of the windings and L_{m1} and L_{m2} are the magnetizing inductances

Model of Two-Winding Transformers

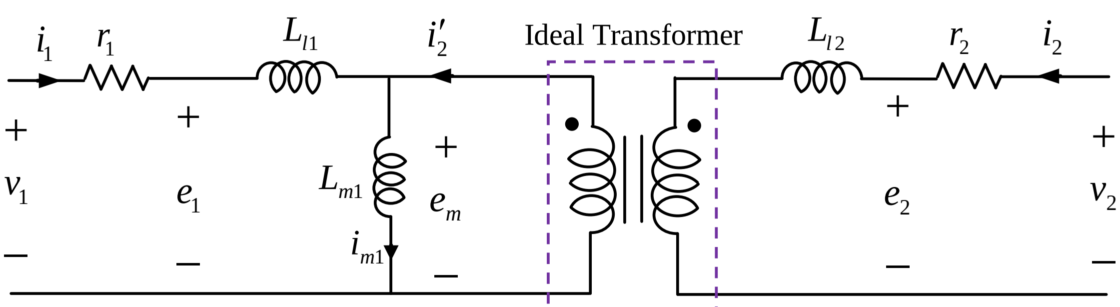
1. Flux linkage equations

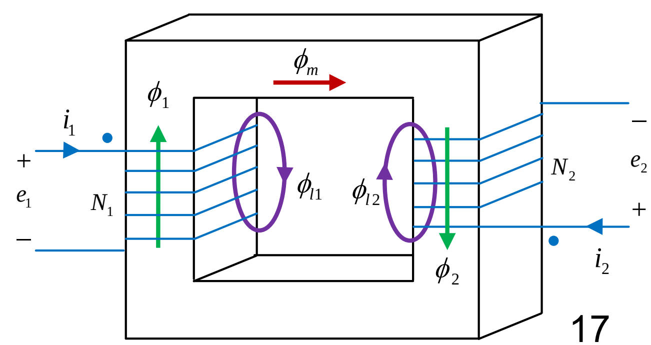
- The total mutual flux linked by winding 1 is expressed as

$$N_1\phi_m = N_1(\phi_{m1} + \phi_{m2}) = L_{m1}\left(i_1 + \frac{N_2}{N_1} i_2\right) = L_{m1}(i_1 + i_2') = L_{m1} i_{m1}$$

where i'_2 is the 2nd winding current referred to the 1st side,

$\phi_{m1} = N_1 i_1 P_m$ and $\phi_{m2} = N_2 i_2 P_m$ are the portions of the mutual flux magnetized by i_1 and i_2 respectively, and i_{m1} is the magnetizing current in the 1st side.



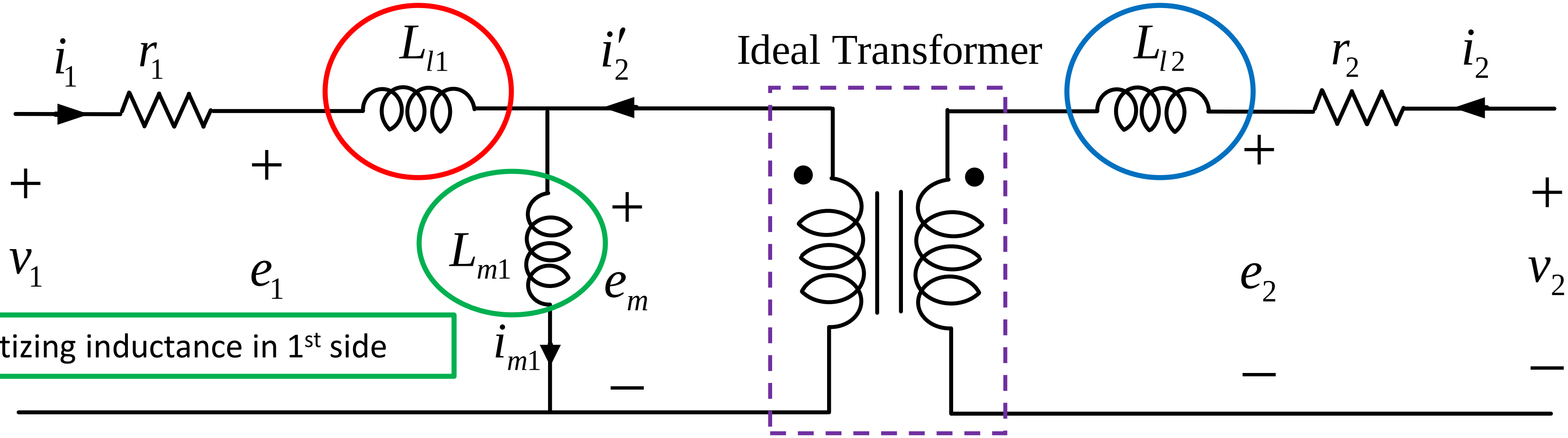


Model of Two-Winding Transformers

Review on the Inductances

Leakage inductance of the 1st winding

Leakage inductance of the 2nd winding



Magnetizing inductance in 1st side

Self-inductance of the 1st winding

Magnetizing inductance in 2nd side

$$L_{11} = L_{l1} + L_{m1}$$

$$L_{22} = L_{l2} + L_{m2}$$

Self-inductance of the 2nd winding

$$L_{12} = \frac{N_2}{N_1} L_{m1}$$

$$L_{21} = \frac{N_1}{N_2} L_{m2}$$

Mutual-inductances

$$L_{m2} = L_{m1}\left(\frac{N_2}{N_1}\right)^2$$

Model of Two-Winding Transformers

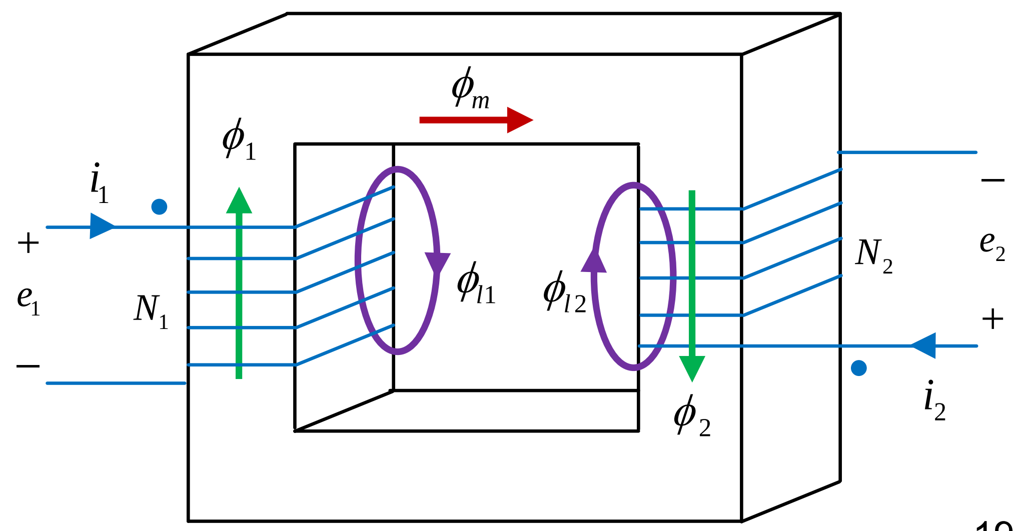
A Question

What would be the relation between L_{11} , L_{22} , L_{12} and L_{21} if the leakage inductances could be neglected?

With this assumption, write the following expression in terms of the magnetizing inductance in the first side.

$$\lambda_1 = L_{11} i_1 + L_{12}\, i_2$$

$$\lambda_2 = L_{21}\, i_1 + L_{22}\, i_2$$



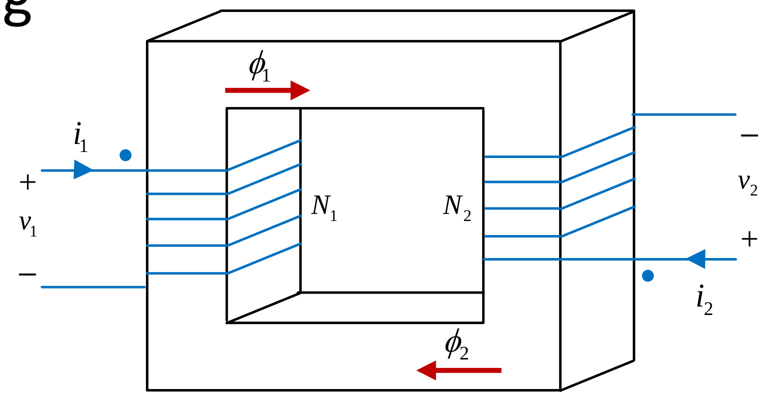
Model of Two-Winding Transformers

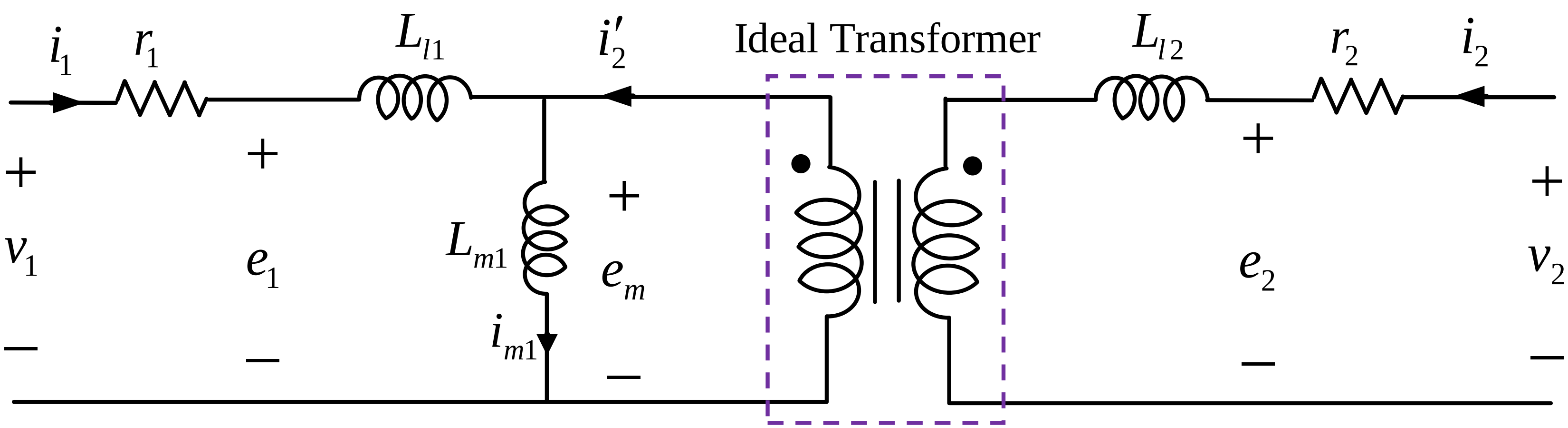
2. Voltage equations

- The terminal voltage can be expressed as the ohmic drop and the induced voltage for each winding

$$v_1 = r_1 i_1 + e_1$$

$$v_2 = r_2 i_2 + e_2$$





Model of Two-Winding Transformers

2. Voltage equations

- The induced voltage in winding 1 is equal to the time rate of change of the winding flux linkage:

$$e_1 = \frac{d\lambda_1}{dt} = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt}$$

- Since $L_{11} = L_{l1} + L_{m1}$ and $L_{12} = \frac{N_2}{N_1} L_{m1}$ it can be written as

$$e_1 = L_{l1}\frac{di_1}{dt} + L_{m1}\frac{d}{dt}\left(i_1 + \tfrac{N_2}{N_1} i_2\right) \Rightarrow e_1 = L_{l1}\frac{di_1}{dt} + L_{m1}\frac{d}{dt}\left(i_1 + i_2'\right)$$

$$\Rightarrow e_1 = L_{l1}\frac{di_1}{dt} + L_{m1}\frac{di_{m1}}{dt}$$

Model of Two-Winding Transformers

2. Voltage equations

- Similarly the induced voltage in winding 2 is obtained as:

$$e_2 = \frac{d\lambda_2}{dt} = L_{21}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt}$$

- Since $L_{22} = L_{l2} + L_{m2}$ and $L_{21} = \frac{N_1}{N_2}L_{m2}$ it can be written as

$$e_2 = L_{l2}\frac{di_2}{dt} + L_{m2}\frac{d}{dt}\left(\frac{N_1}{N_2}i_1 + i_2\right) \Rightarrow e_2 = L_{l2}\frac{di_2}{dt} + L_{m2}\frac{d}{dt}\left(i_1' + i_2\right)$$

$$\Rightarrow e_2 = L_{l2}\frac{di_2}{dt} + L_{m2}\frac{di_{m2}}{dt}$$

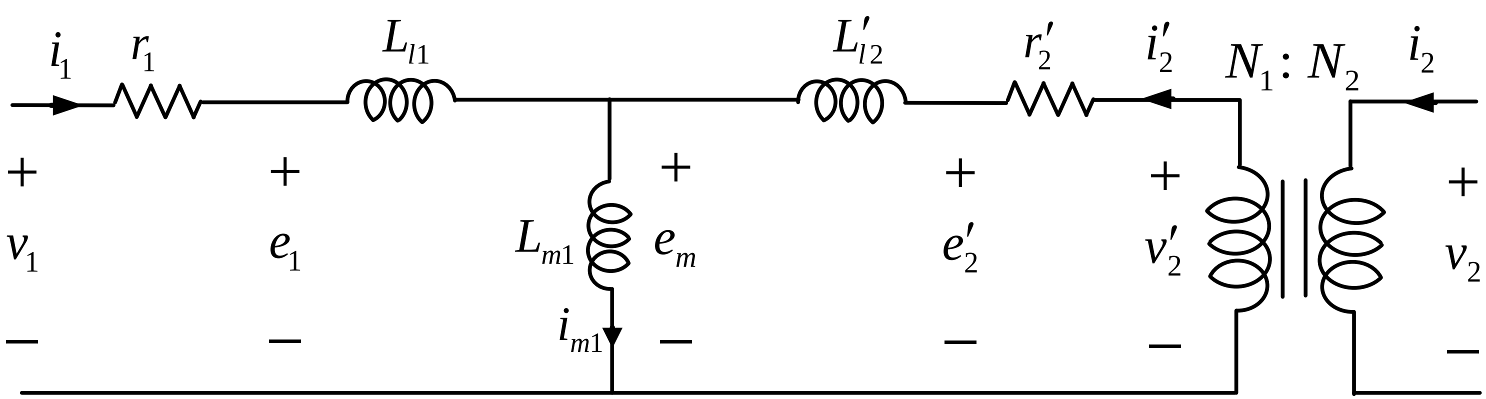
Model of Two-Winding Transformers

2. Voltage equations

$$e_2 = L_{l2}\frac{di_2}{dt} + L_{m2}\frac{d}{dt}\left(i_1' + i_2\right) \quad \Rightarrow \quad e_2 = L_{l2}\frac{di_2}{dt} + L_{m2}\frac{di_{m2}}{dt}$$

- The voltage e_2 can be referred to winding 1

$$e_2' = L_{l2}'\frac{di_2'}{dt} + L_{m1}\frac{d}{dt}\left(i_1 + i_2'\right) \quad \Rightarrow \quad e_2' = L_{l2}'\frac{di_2'}{dt} + L_{m1}\frac{di_{m1}}{dt}$$



Model of Two-Winding Transformers

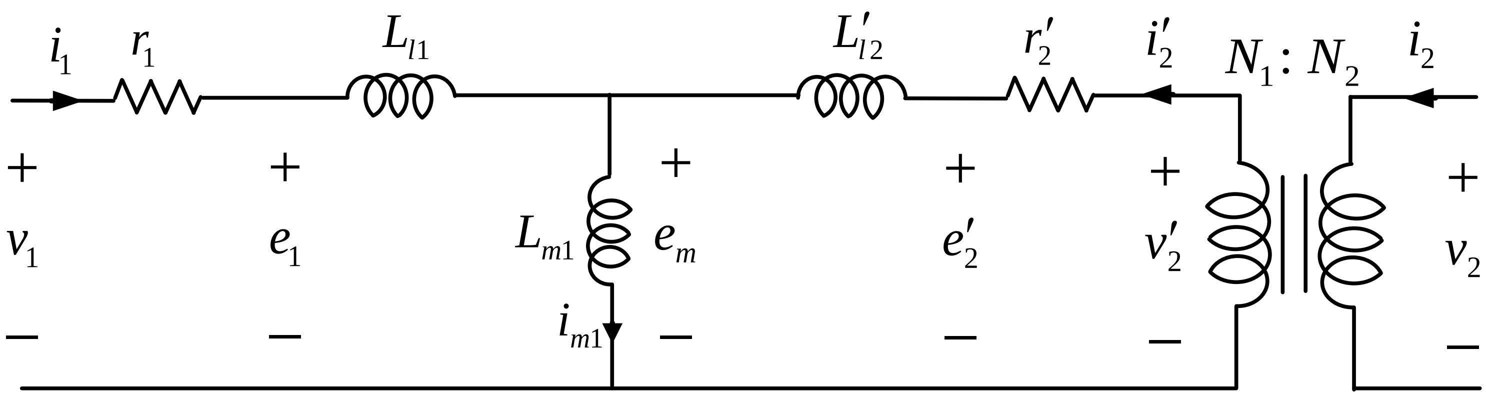
2. Voltage equations

- Therefore the voltage equations are

$$v_1 = r_1 i_1 + e_1 \Rightarrow v_1 = r_1 i_1 + L_{l1}\frac{di_1}{dt} + L_{m1}\frac{di_{m1}}{dt}$$

$$i_{m1} = i_1 + i_2'$$

$$v_2' = r_2' i_2' + e_2' \Rightarrow v_2' = r_2' i_2' + L_{l2}'\frac{di_2'}{dt} + L_{m1}\frac{di_{m1}}{dt}$$



Model of Two-Winding Transformers

3. Equivalent circuit

$$v_1 = r_1 i_1 + L_{l1}\frac{di_1}{dt} + L_{m1}\frac{di_{m1}}{dt}$$

$$v_2' = r_2' i_2' + L_{l2}'\frac{di_2'}{dt} + L_{m1}\frac{di_{m1}}{dt}$$

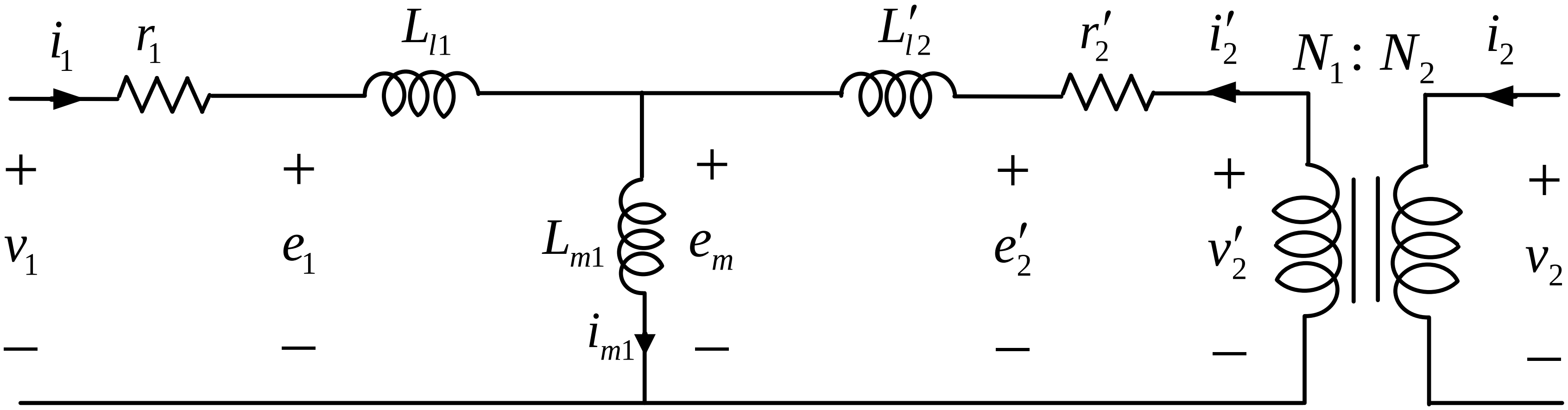
$$r_2' = r_2\left(\frac{N_1}{N_2}\right)^2$$

$$L_{l2}' = L_{l2}\left(\frac{N_1}{N_2}\right)^2$$

$$v_2' = v_2\frac{N_1}{N_2}$$

$$i_2' = i_2\frac{N_2}{N_1}$$

$$i_{m1} = i_1 + i_2'$$



Simulation of Two-Winding Transformers

- There are several ways to simulate a two-winding transformer, e.g.:
- The inputs are the primary and secondary voltages, both in the primary side.

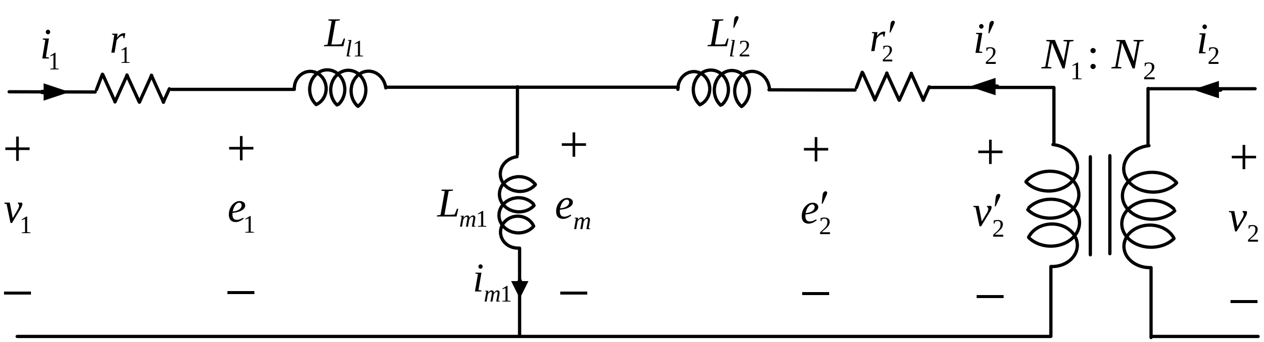
Inputs → v_1 v_2'

- The outputs are the primary and secondary current, both in the primary side.

Outputs → i_1 i_2'

- The states are the flux linkages of the primary and secondary windings.

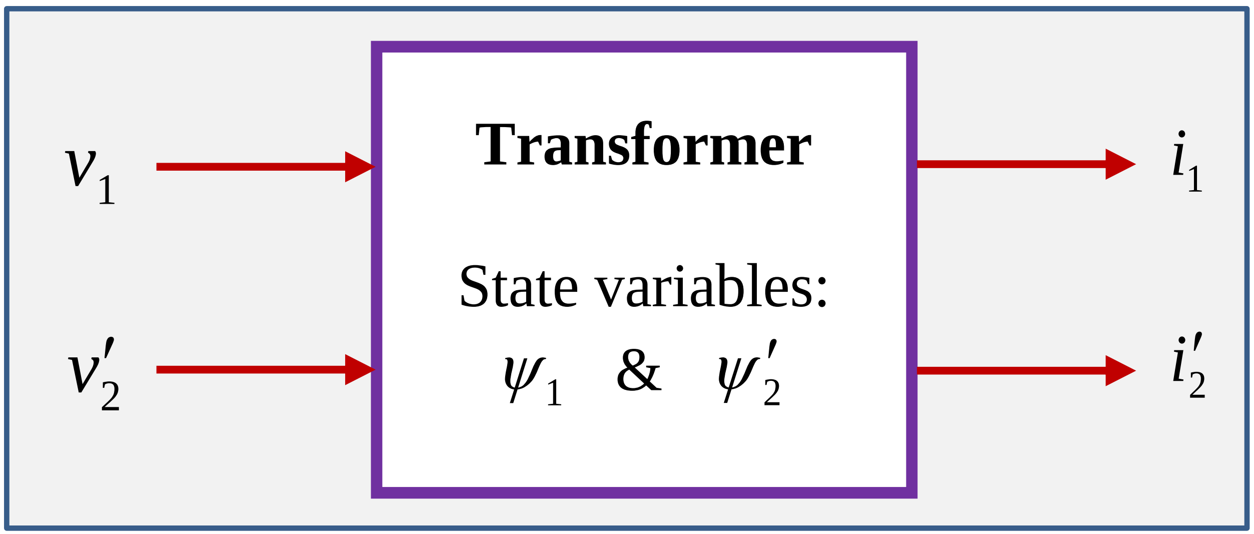
States → λ_1 λ_2'



Simulation of Two-Winding Transformers

- It's possible to consider the **flux linkage per second** of the windings as state variables

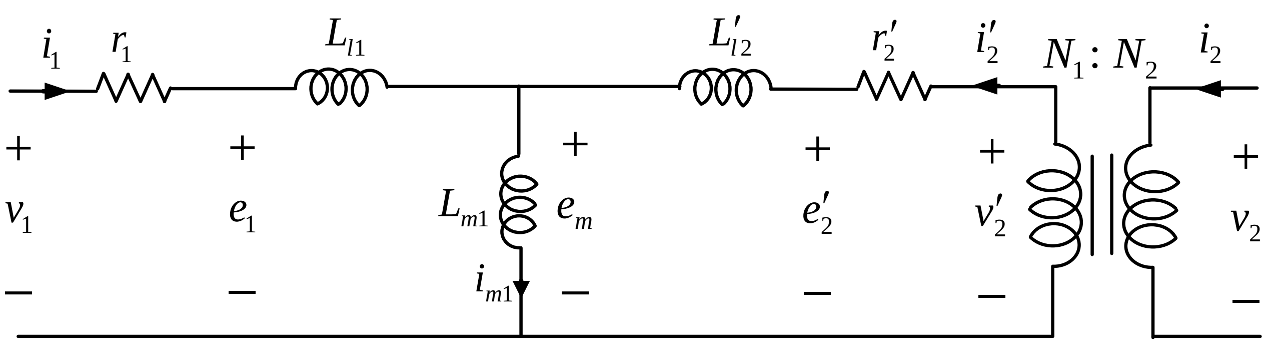
States → ψ_1 ψ_2'



$$\psi_1 = \omega_b \, \lambda_1$$

$$\psi_2' = \omega_b \, \lambda_2'$$

where ω_b is the base frequency at which the reactances are computed.



Simulation of Two-Winding Transformers

- The **voltage equations** will be

$$v_1 = r_1\, i_1 + \frac{1}{\omega_b}\frac{d\psi_1}{dt}$$

$$v_2' = r_2'\, i_2' + \frac{1}{\omega_b}\frac{d\psi_2'}{dt}$$

- The **flux linkage per second** of the windings are

$$\psi_1 = \omega_b\, \lambda_1 = x_{l1}\, i_1 + \psi_m$$

$$\psi_2' = \omega_b\, \lambda_2' = x_{l2}'\, i_2' + \psi_m$$

where ψ_m is the **magnetizing flux** referred to winding 1

$$\psi_m = \omega_b\, L_{m1}\left(i_1 + i_2'\right) = x_{m1}\left(i_1 + i_2'\right) = x_{m1}\, i_{m1}$$

and the **reactances** are defined as

$$x_{l1} = \omega_b\, L_{l1}$$

$$x_{l2}' = \omega_b\, L_{l2}'$$

$$x_{m1} = \omega_b\, L_{m1}$$

Simulation of Two-Winding Transformers

- The currents can be written in terms of the flux linkages

$$\psi_1 = x_{l1} i_1 + \psi_m \quad \Rightarrow \quad i_1 = \frac{\psi_1 - \psi_m}{x_{l1}} \qquad \mathbf{1}$$

$$\psi_2' = x_{l2}' i_2' + \psi_m \quad \Rightarrow \quad i_2' = \frac{\psi_2' - \psi_m}{x_{l2}'} \qquad \mathbf{2}$$

- Using the above two expressions and $\psi_m = x_{m1}\left(i_1 + i_2'\right)$ we have:

$$\frac{\psi_m}{x_{m1}} = \frac{\psi_1 - \psi_m}{x_{l1}} + \frac{\psi_2' - \psi_m}{x_{l2}'} \quad \Rightarrow \quad \psi_m\left(\frac{1}{x_{m1}} + \frac{1}{x_{l1}} + \frac{1}{x_{l2}'}\right) = \frac{\psi_1}{x_{l1}} + \frac{\psi_2'}{x_{l2}'}$$

Simulation of Two-Winding Transformers

- It can be rewritten as

$$\psi_m\left(\frac{1}{x_{m1}}+\frac{1}{x_{l1}}+\frac{1}{x'_{l2}}\right)=\frac{\psi_1}{x_{l1}}+\frac{\psi'_2}{x'_{l2}} \quad \Rightarrow \quad \psi_m = x_M\left(\frac{\psi_1}{x_{l1}}+\frac{\psi'_2}{x'_{l2}}\right) \qquad 3$$

where

$$\frac{1}{x_M}=\frac{1}{x_{m1}}+\frac{1}{x_{l1}}+\frac{1}{x'_{l2}}$$

- The above expression states the **magnetizing flux linkage per second** as a function of the **primary and secondary flux linkages per second**.

Simulation of Two-Winding Transformers

- The voltage equation of the primary side can be expressed as follows:

$$v_1 = r_1 i_1 + \frac{1}{\omega_b}\frac{d\psi_1}{dt}$$

$$i_1 = \frac{\psi_1 - \psi_m}{x_{l1}}$$

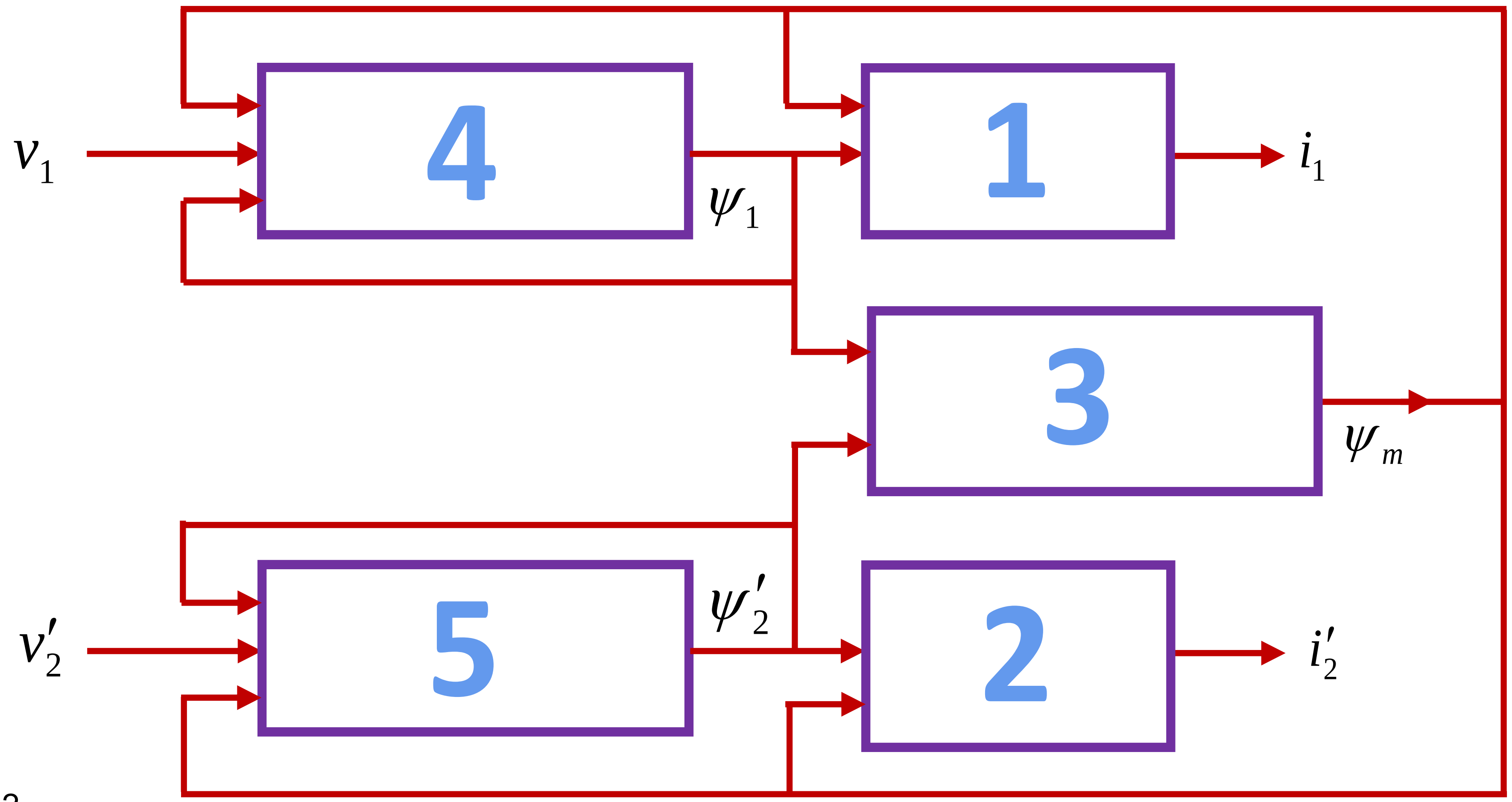
$$\psi_1 = \omega_b \int \left\{ v_1 - r_1 \left(\frac{\psi_1 - \psi_m}{x_{l1}} \right) \right\} dt \qquad (4)$$

- Similarly for the secondary side we have:

$$\psi_2' = \omega_b \int \left\{ v_2' - r_2' \left(\frac{\psi_2' - \psi_m}{x_{l2}'} \right) \right\} dt \qquad (5)$$

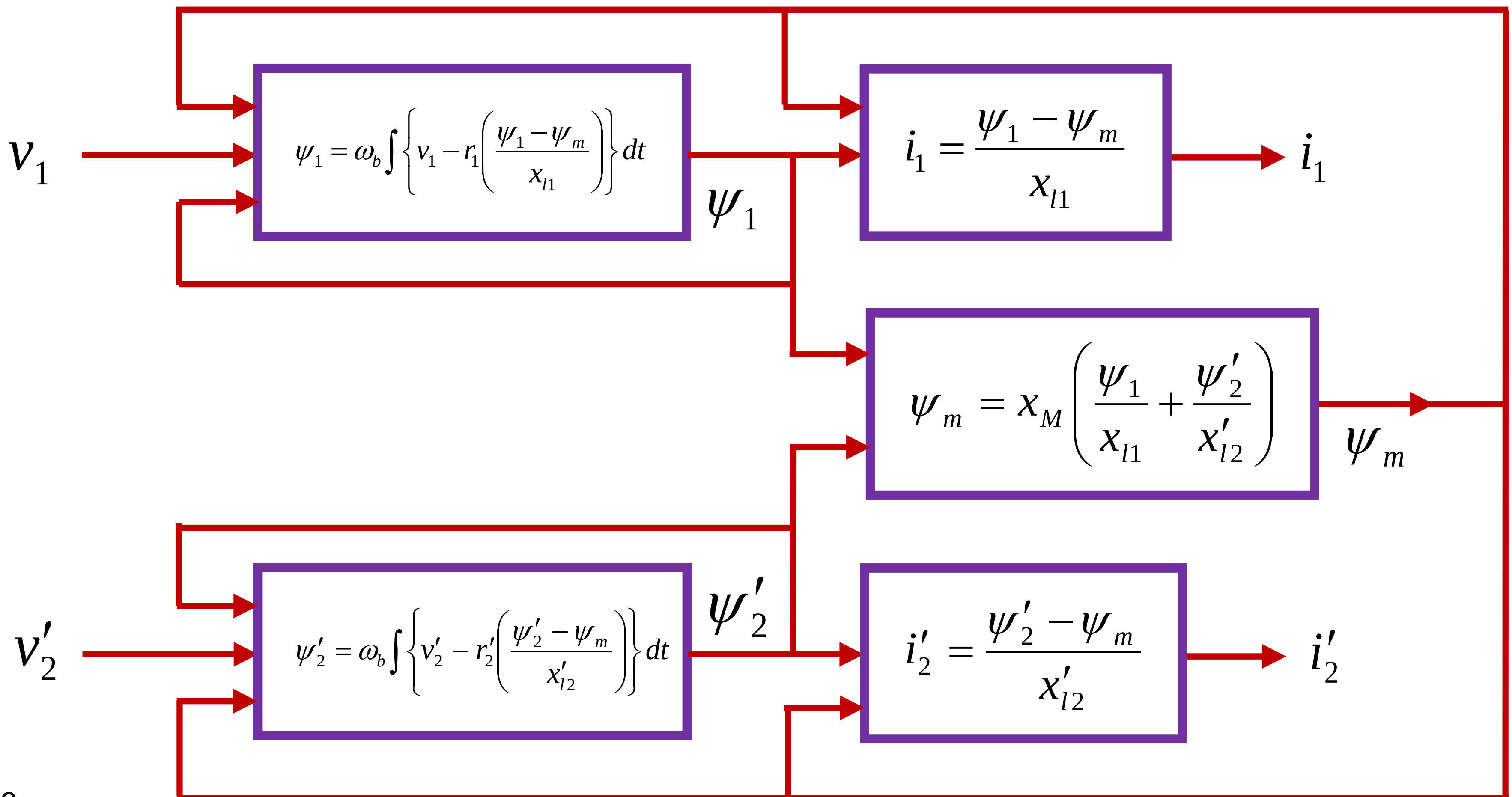
Simulation of Two-Winding Transformers

- The block diagram of the simulation can be shown as



Simulation of Two-Winding Transformers

- The block diagram with equations



Simulation of Two-Winding Transformers

- In order to simulated the transformer, the following **parameters** are required:

r_1 : the primary winding resistance in Ω

r_2' : the secondary winding resistance referred to the primary

ω_b : the base frequency in rad/s

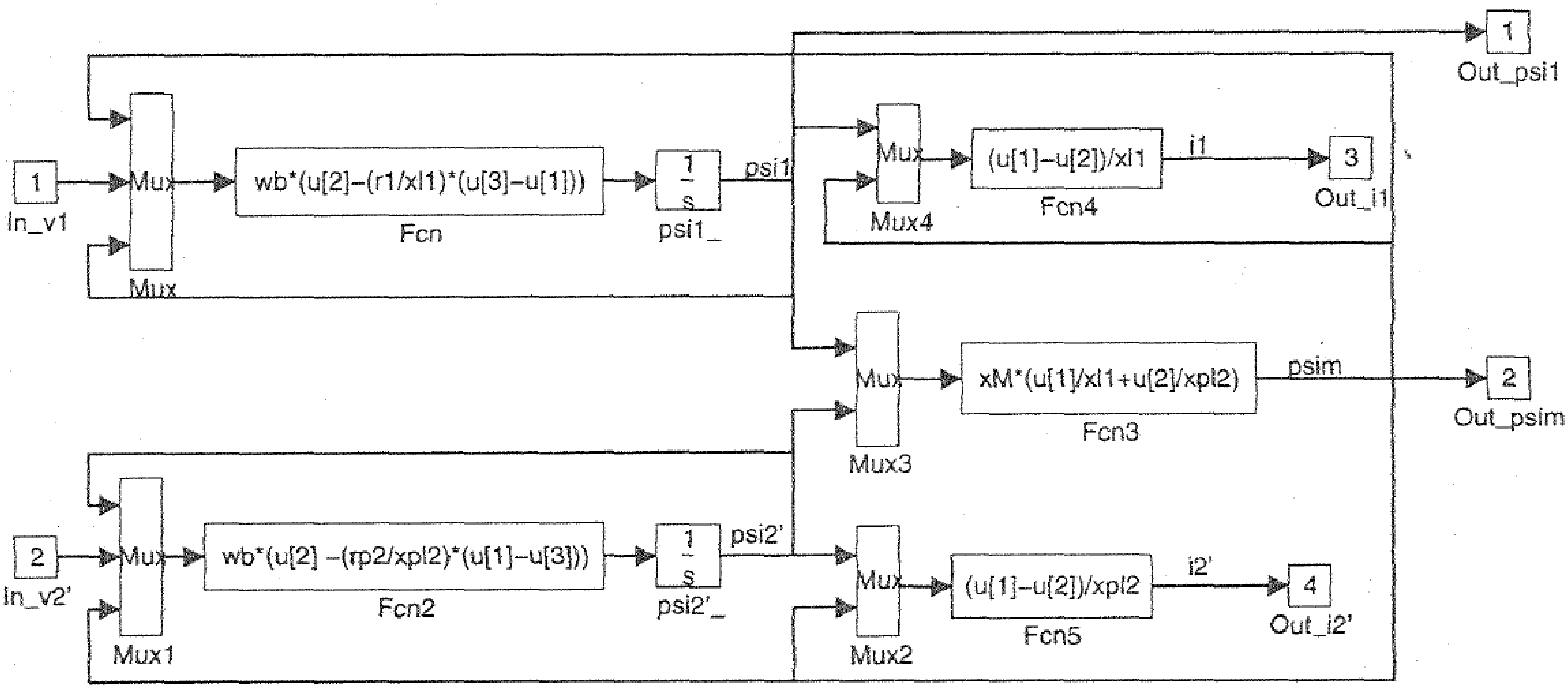
x_{l1} : the primary leakage reactance in Ω

x_{l2}' : the secondary leakage reactance referred to the primary

x_{m1} : the magnetizing reactance in the primary side in Ω

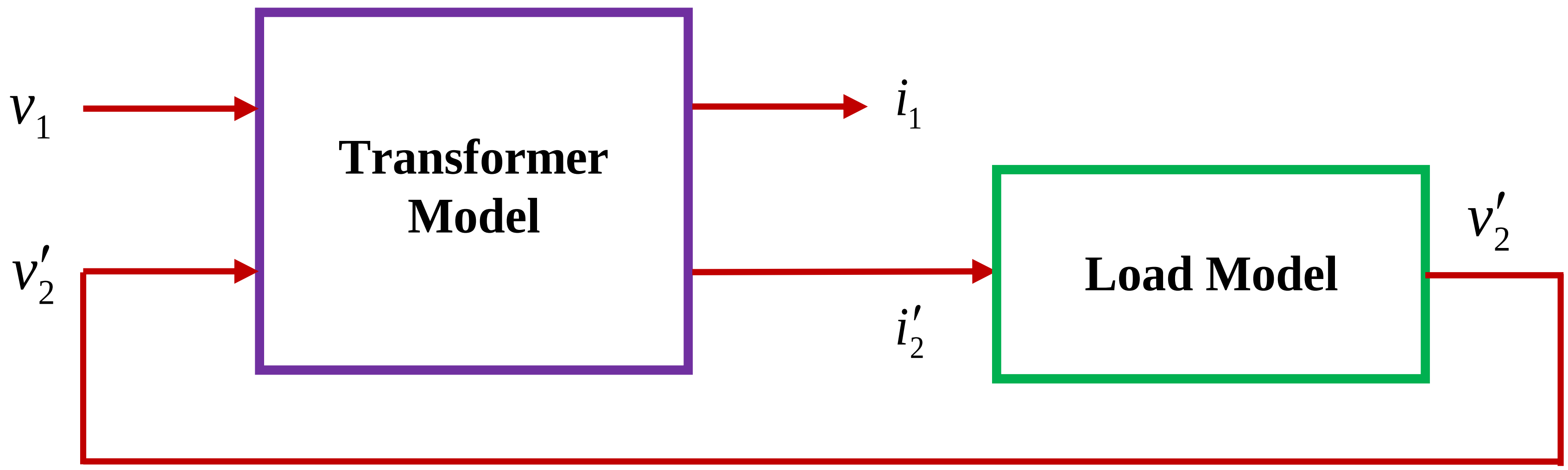
Simulation of Two-Winding Transformers

- The simulation of the two-winding transformer in MATLAB/SIMULINK



Load Modelling

- It is noted that the secondary terminal voltage is **not an explicit input** and depends on the load connected to this terminal.
- Therefore the **load** should be modelled in modular form.
- Then the load model is combined with the transformer model to obtain the overall response of the system:



Load Modelling

Two Special Cases

1. **Short Circuit** $v_2' = 0$

It's straightforward to implement the short-circuit condition by setting the secondary voltage to zero.

2. **Open Circuit** $i_2' = 0$

- It's not as easy as the short-circuit condition and the open-circuit secondary voltage is expressed as follows:

$$v_{2oc}' = \frac{1}{\omega_b}\frac{d\psi_m}{dt} = \frac{1}{\omega_b}\frac{x_{m1}}{x_{l1}+x_{m1}}\frac{d\psi_1}{dt} = \frac{x_{m1}}{x_{l1}+x_{m1}}\left(v_1 - r_1 i_1\right)$$

Why?

Load Modelling

Two Special Cases

2. Open Circuit

$$\psi_2' = x_{l2}'\, i_2' + \psi_m \quad \xrightarrow{i_2' = 0} \quad \psi_2' = \psi_m$$

$$v_2' = r_2'\, i_2' + \frac{1}{\omega_b}\frac{d\psi_2'}{dt} \quad \xrightarrow{i_2' = 0} \quad v_2' = \frac{1}{\omega_b}\frac{d\psi_2'}{dt}$$

$$\Rightarrow \quad v_2' = \frac{1}{\omega_b}\frac{d\psi_m}{dt}$$

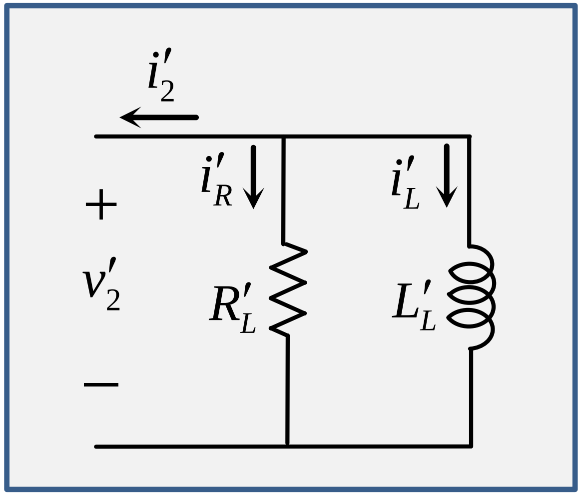
$$\psi_m = x_{m1}\left(i_1 + i_2'\right) \quad \xrightarrow{i_2' = 0} \quad \psi_m = x_{m1} i_1$$

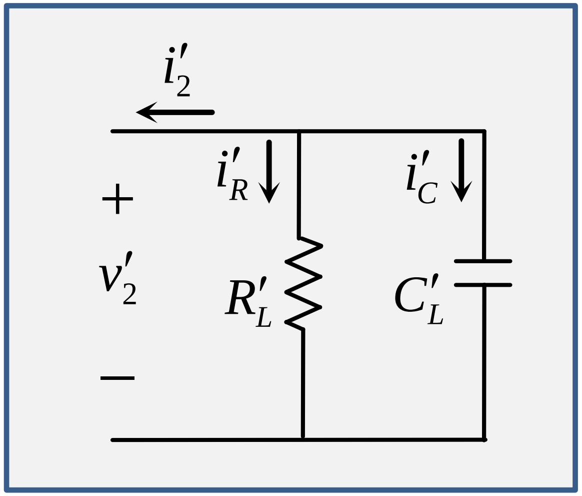
$$\psi_1 = x_{l1}\, i_1 + \psi_m$$

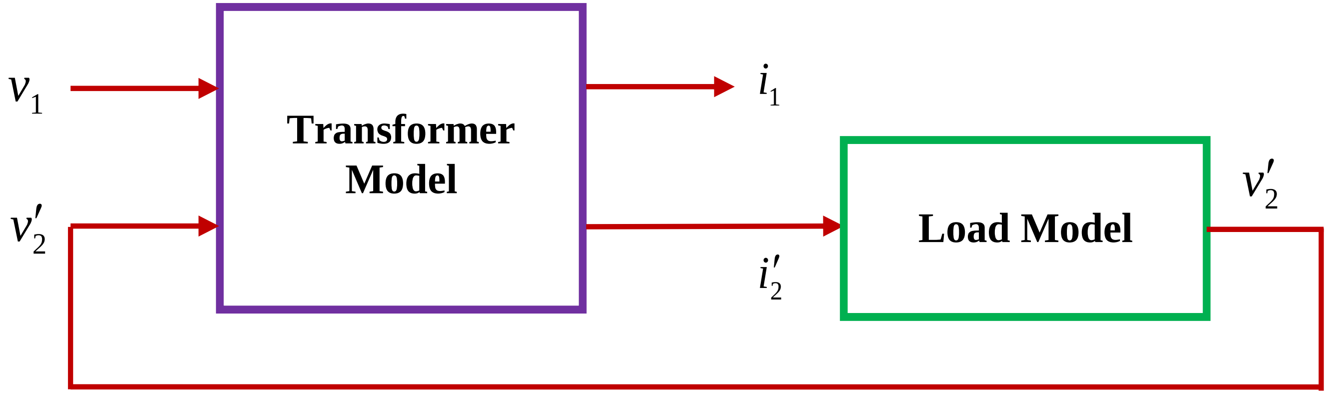
$$\Rightarrow \quad \psi_m = \frac{x_{m1}}{x_{l1} + x_{m1}}\psi_1$$

Load Modelling

- The load can be represented by an equivalent impedance or admittance in the form of either RL or RC circuit.







Load Modelling

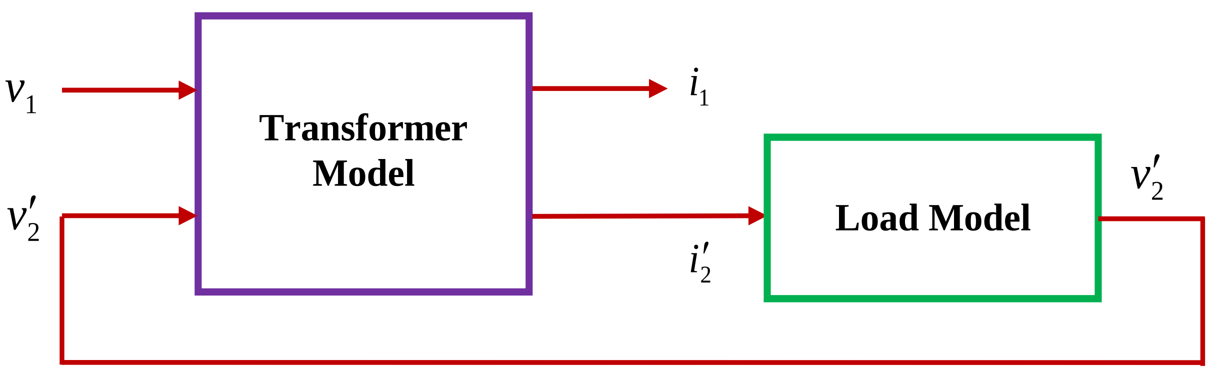
- Consider a specified loading of S_L at the rated voltage of v_{2rated}.
- It can be translated into an equivalent circuit load admittance referred to the primary side:

$$\left(G'_L \pm jB'_L\right)^{-1} = \left(\frac{N_1}{N_2}\right)^2 \frac{v_{2rated}^2}{S_L^*}$$

Conductance **Susceptance**

- For **lagging power factor** loads, the parallel **RL** circuit is used.

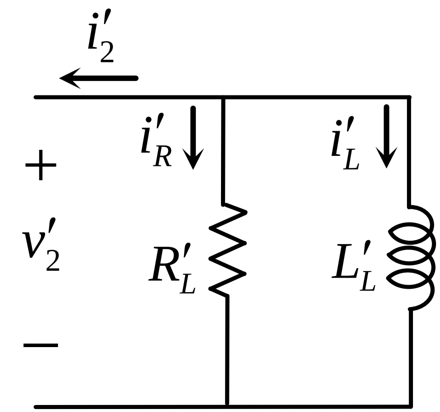
- For **leading power factor** loads, the parallel **RC** circuit is used.



Load Modelling

Lagging power factor loads (parallel RL circuit)

$$\left(G'_L - jB'_L\right)^{-1} = \left(\frac{N_1}{N_2}\right)^2 \frac{v_{2rated}^2}{S_L^*}$$



- Assume that the conductance and susceptance are calculated based on the complex power and rated voltage

$$v'_2 = R'_L\, i'_R = R'_L\left(-i'_2 - i'_L\right)$$

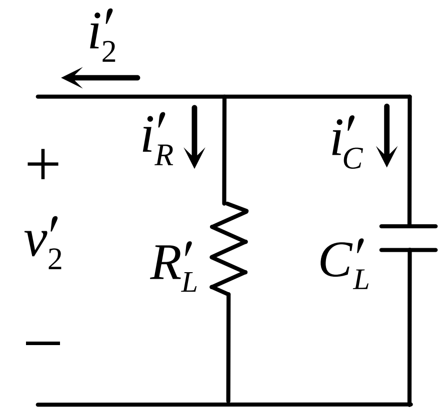
$$i'_L = \frac{1}{L'_L}\int v'_2\, dt = \omega_b\, B'_L \int v'_2\, dt$$

$$v'_2 = R'_L\left(-i'_2 - \omega_b B'_L \int v'_2\, dt\right)$$

Load Modelling

Leading power factor loads (parallel RC circuit)

$$\left(G_L' + jB_L'\right)^{-1} = \left(\frac{N_1}{N_2}\right)^2 \frac{v_{2rated}^2}{S_L^*}$$



- Assume that the conductance and susceptance are calculated based on the complex power and rated voltage

$$i_R' = v_2' / R_L'$$

$$v_2' = \frac{1}{C_L'} \int i_C' \, dt = \frac{\omega_b}{B_L'} \int \left(-i_2' - i_R'\right) dt$$

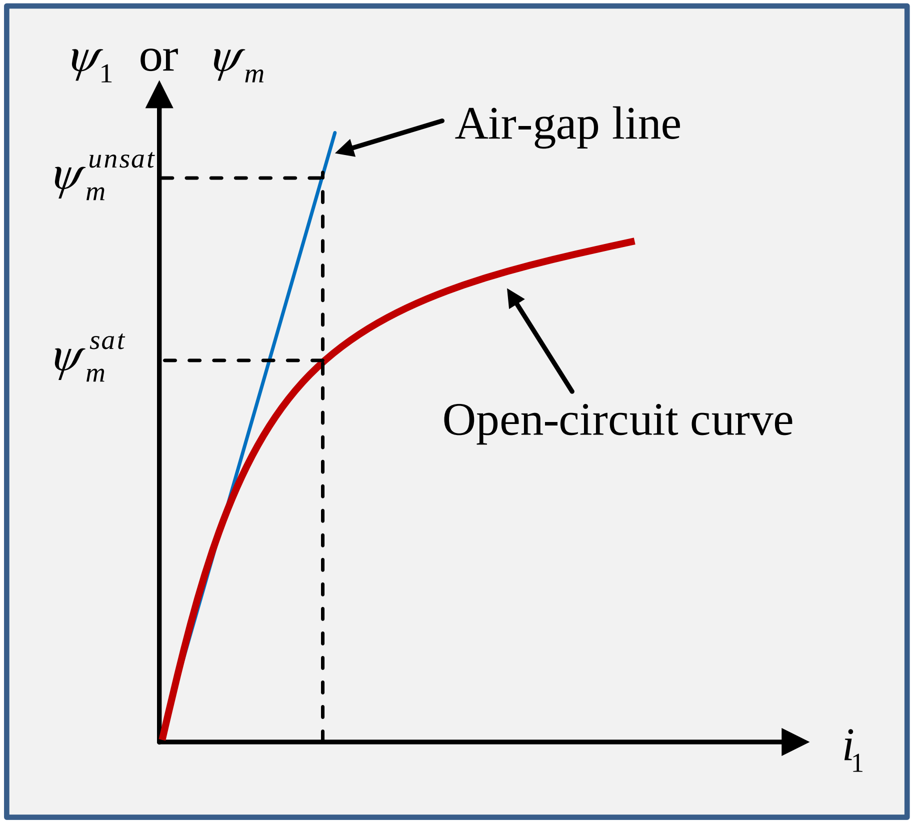
$$v_2' = \frac{\omega_b}{B_L'} \int \left(-i_2' - \frac{v_2'}{R_L'}\right) dt$$

Core Saturation Effects

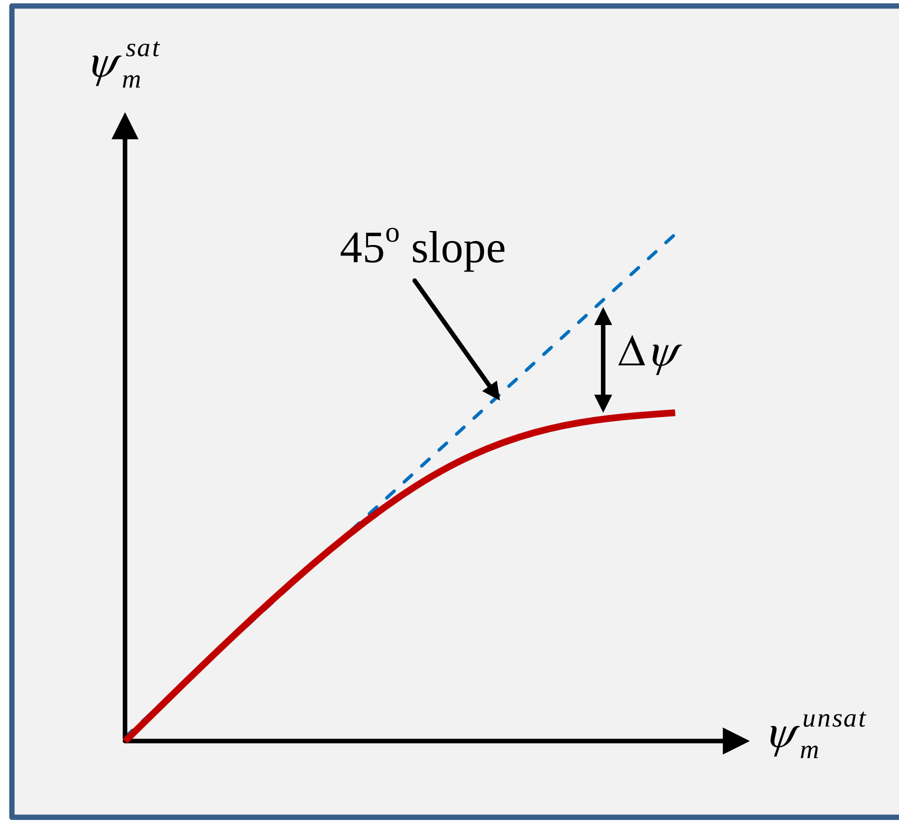
- Core saturation mainly affects the value of the **magnetizing inductance** and, to a much lesser extent, the **leakage inductances**.
- Considering the core saturation effects on the **leakage inductances** is complex and **neglected** here.
- Therefore the core saturation effects on the **magnetizing inductance** are only **considered**.
- Core saturation behaviour can be determined from just the **open-circuit magnetization curve** of the transformer.

Core Saturation Effects

Saturation characteristics



(a) Open-circuit curve



(b) Saturated vs. unsaturated flux linkage

Core Saturation Effects

Some of the methods to incorporate the core saturation effects in the dynamic simulation are:

1. Using the appropriate **saturated value of the magnetizing reactance** at each time step of the simulation.

2. Approximating the magnetizing current by some **analytic function** of the saturated flux linkage.

3. Using the **relation between saturated and unsaturated values** of the magnetizing flux linkage.

Core Saturation Effects

Method 1: Using the appropriate **saturated value of the magnetizing reactance** at each time step of the simulation.

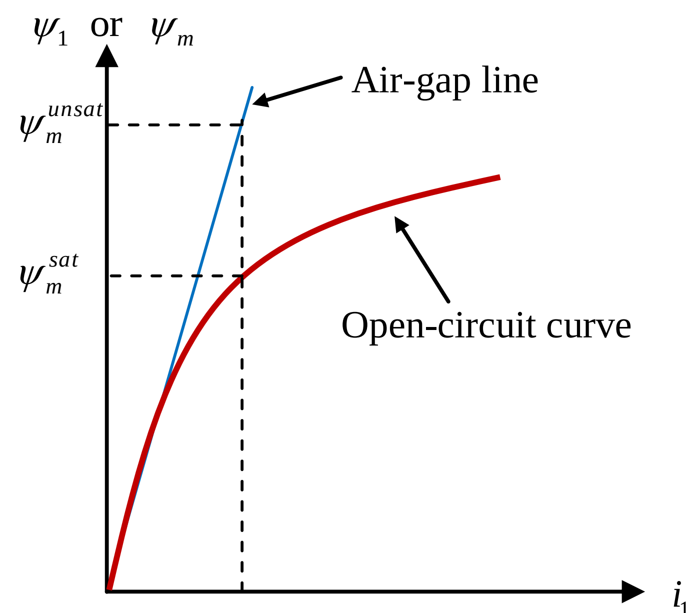
- The saturated value of the magnetizing reactance x_{m1}^{sat} can be updated using the product of the unsaturated value of the magnetizing reactance x_{m1}^{unsat} times a saturation factor k_s.

$$x_{m1}^{sat} = k_s \, x_{m1}^{unsat}$$

- k_s is determined from the open-circuit curve:

$$k_s = \frac{\psi_m^{sat}}{\psi_m^{unsat}} \leq 1$$

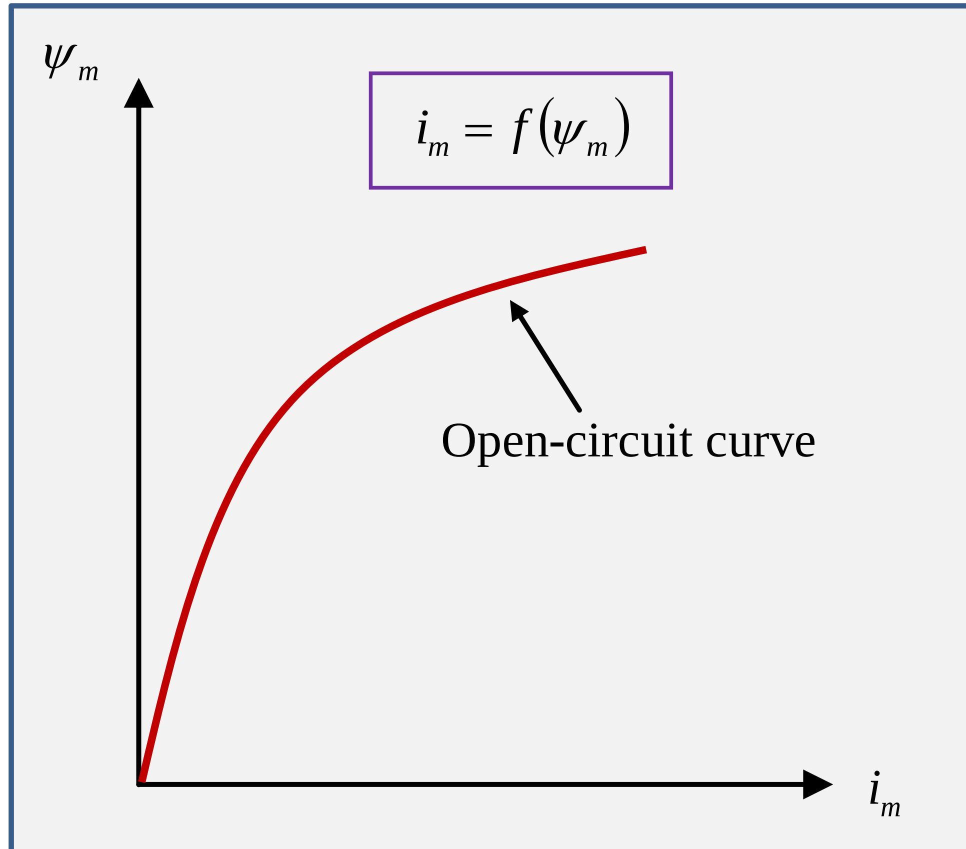
- x_{m1}^{unsat} is a **constant** value.



Core Saturation Effects

Method 2: Approximating the magnetizing current by some **analytic function** of the saturated flux linkage.

$$i_m = f(\psi_m)$$



For more details see

pages 101-105 of Reference [1]

(Chee-Mun Ong)

Core Saturation Effects

Method 3: Using the **relation between saturated and unsaturated values** of the magnetizing flux linkage.

$$\psi_m^{unsat} = x_{m1}^{unsat}\left(i_1 + i_2'\right)$$

$$i_1 = \frac{\psi_1 - \psi_m^{sat}}{x_{l1}}$$

$$i_2' = \frac{\psi_2' - \psi_m^{sat}}{x_{l2}'}$$

$$\psi_m^{unsat} = \psi_m^{sat} + \Delta\psi$$

$$\frac{\psi_m^{unsat}}{x_{m1}^{unsat}} = \frac{\psi_1 - \psi_m^{sat}}{x_{l1}} + \frac{\psi_2' - \psi_m^{sat}}{x_{l2}'}$$

$$\psi_m^{sat} = x_M\left(\frac{\psi_1}{x_{l1}} + \frac{\psi_2'}{x_{l2}'} - \frac{\Delta\psi}{x_{m1}^{unsat}}\right)$$

where

$$\frac{1}{x_M} = \frac{1}{x_{m1}^{unsat}} + \frac{1}{x_{l1}} + \frac{1}{x_{l2}'}$$

Core Saturation Effects

Method 3: Using the **relation between saturated and unsaturated values** of the magnetizing flux linkage.

$$\psi_m^{sat} = x_M \left(\frac{\psi_1}{x_{l1}} + \frac{\psi_2'}{x_{l2}'} - \frac{\Delta\psi}{x_{m1}^{unsat}} \right)$$

where

$$\frac{1}{x_M} = \frac{1}{x_{m1}^{unsat}} + \frac{1}{x_{l1}} + \frac{1}{x_{l2}'}$$

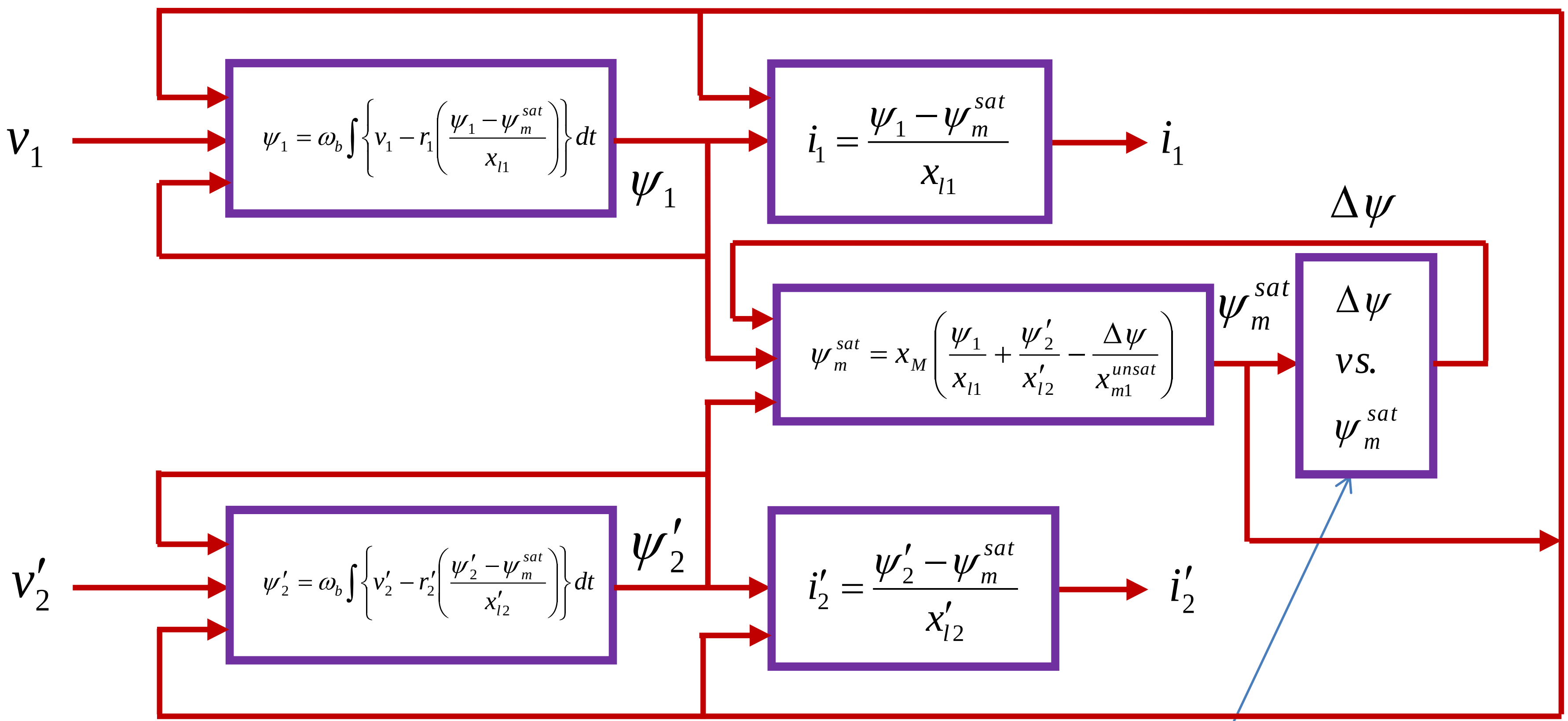
- Comparing to the case with no saturation effects, the only additional term appeared in the above expression is: $\frac{\Delta\psi}{x_{m1}^{unsat}}$

- Therefore similar model with minor modification is used.

- Only $\Delta\psi$ has to be obtained.

Core Saturation Effects

Block diagram of Method 3



Now we need to find the way to represent the **saturation block**

Core Saturation Effects

Method 3: It is required to compute $\Delta\psi$ from ψ_m^{sat} .

The approaches to represent the **saturation block** are as follows:

1. Using a **look-up table** and interpolation technique

input	ψ_m^{sat}	___	___	___	___
output	$\Delta\psi$	___	___	___	___

2. Approximate **analytic functions**

   2.1. Three-segment linear-exponential approximation

   2.2. Three-segment linear approximation

Core Saturation Effects

2. Approximate **analytic functions**

   2.1. Three-segment linear-exponential approximation

Linear region ($\psi_m^{sat} < B_1$ **):**

$$\Delta\psi = 0$$

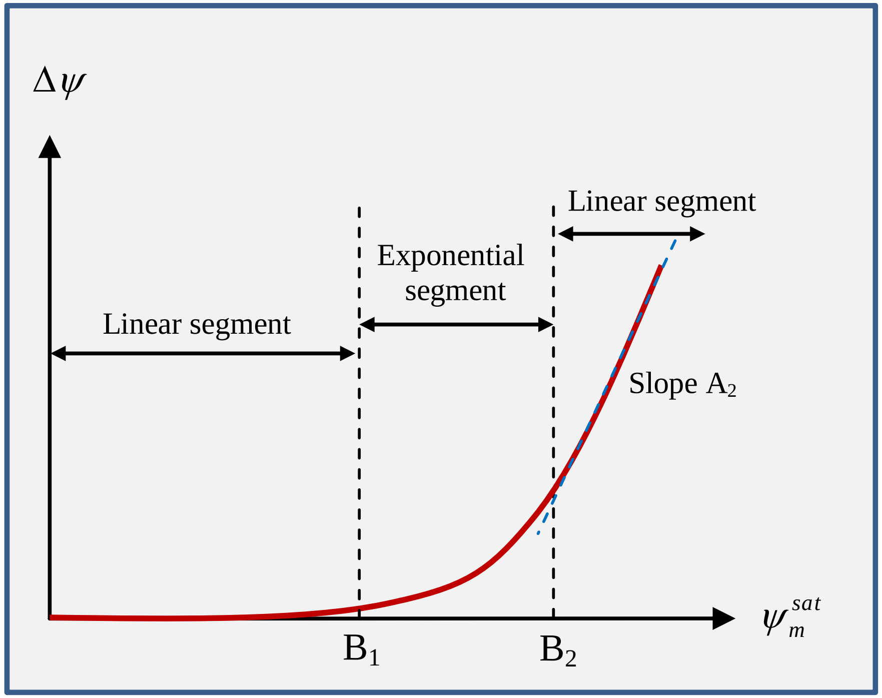
Knee region ($B_1 < \psi_m^{sat} < B_2$ **):**

$$\Delta\psi = ae^{b\left(\psi_m^{sat} - B_1\right)}$$

Fully saturated region ($\psi_m^{sat} > B_2$ **):**

$$\Delta\psi = A_2\left(\psi_m^{sat} - B_2\right) + \Delta\psi\left(B_2\right)$$

where $\Delta\psi\left(B_2\right) = ae^{b\left(B_2 - B_1\right)}$



Core Saturation Effects

2. Approximate **analytic functions**

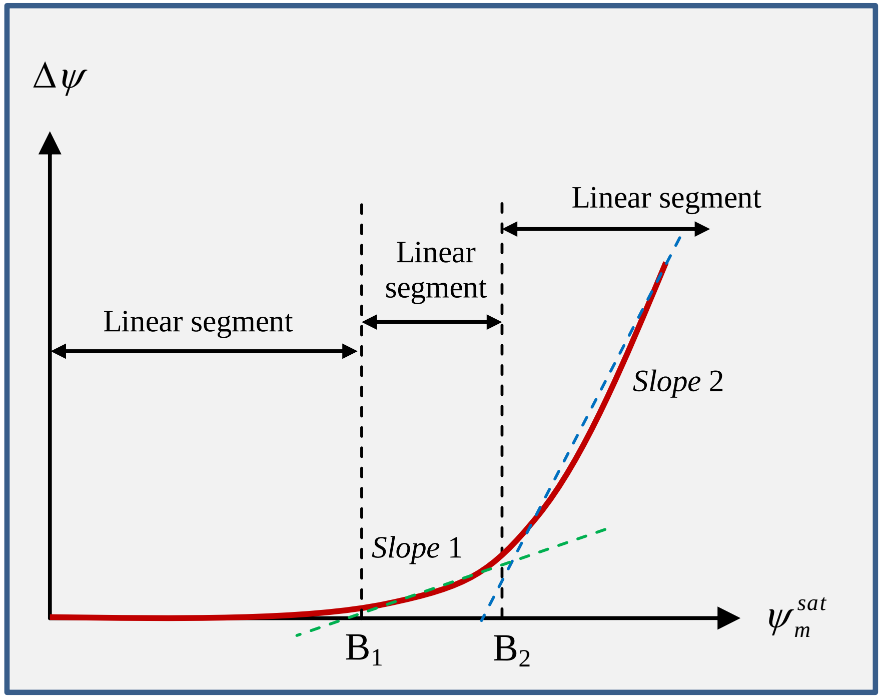
    2.2. Three-segment linear approximation

$$\Delta\psi = A_1\left(\psi_m^{sat} - B_1\right) + A_2\left(\psi_m^{sat} - B_2\right)$$

where

$$A_1 = \begin{cases} slope1 & \text{if} \quad \psi_m^{sat} > B_1 \\ 0 & \text{otherwise} \end{cases}$$

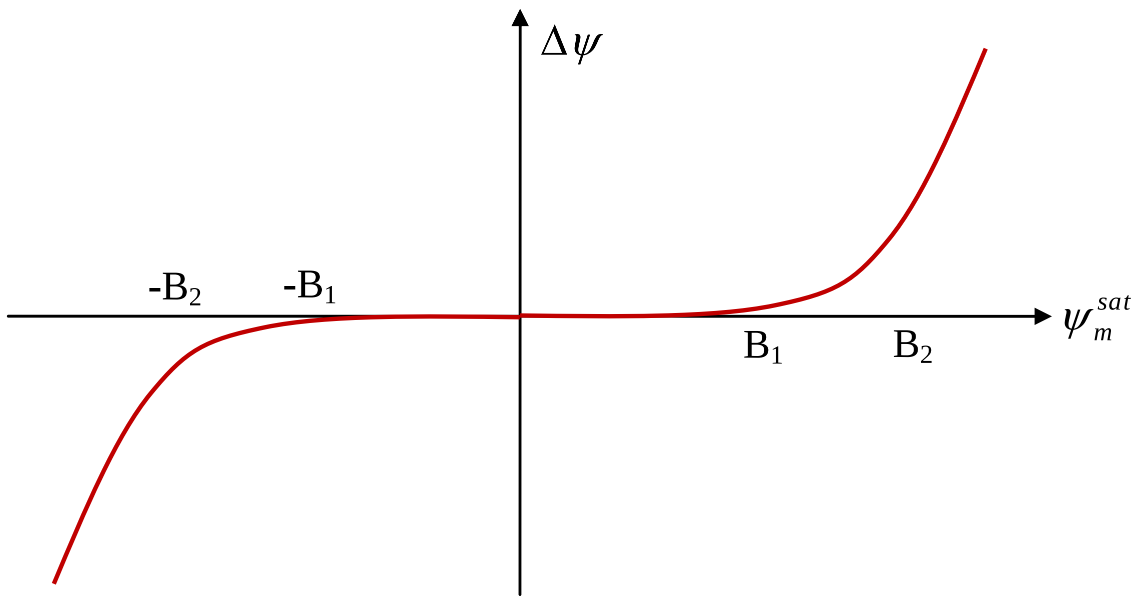
$$A_2 = \begin{cases} slope2 - slope1 & \text{if} \quad \psi_m^{sat} > B_2 \\ 0 & \text{otherwise} \end{cases}$$



Core Saturation Effects

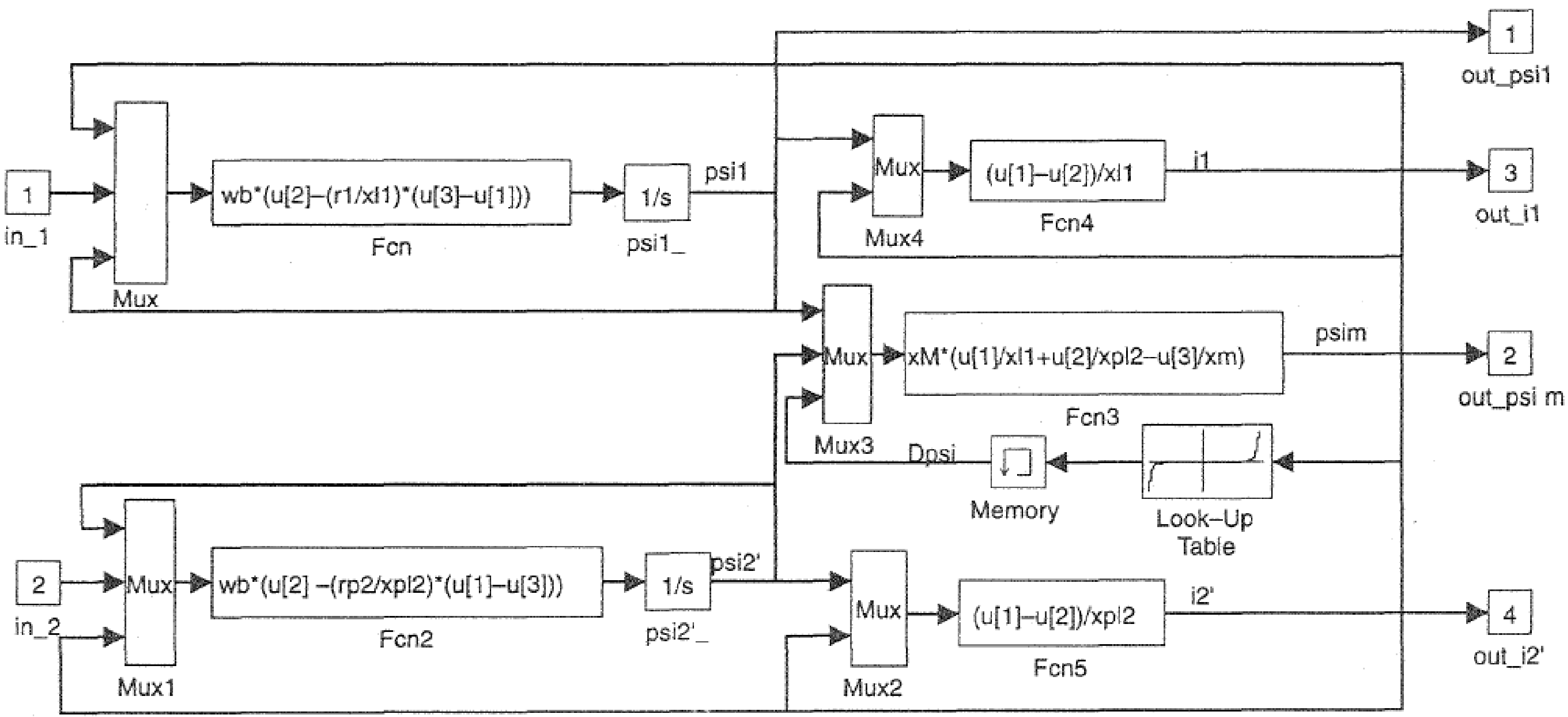
2. Approximate **analytic functions**

- Since ψ_m is alternating, saturation for negative ψ_m must be taken care of by a similar approximation in the third quadrant.
- In the third quadrant the slope of the linear approximation remains the same but the sign of B_1 and B_2 changes.



Core Saturation Effects

Implementation in MATLAB/SIMULINK using Look-up Table

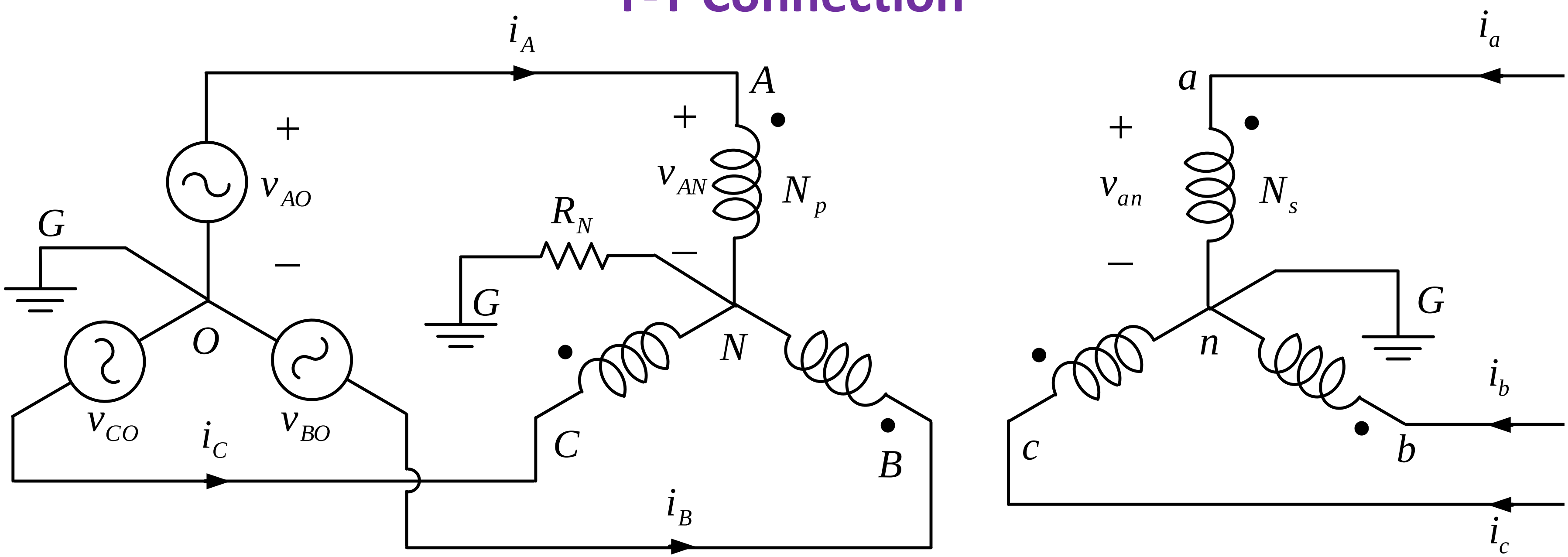


Three-Phase Connections

- The **generation**, **transmission** and **distribution** of AC electric power systems are mostly in **three-phase**.
- When the three-phase system is operating under **balanced conditions**, it can be represented by a **single phase equivalent circuit**.
- For **unbalanced** operating conditions, the three-phase system must be represented as it is.
- The operating characteristic of a 3-phase transformer depends on both **winding connections** and the **magnetic circuit of its core**.
- Here only the **winding connection** effects are considered and the effects of the magnetic circuit of the core (e.g. mutual effects of the primary windings when share a common core) are not considered.

Three-Phase Connections

Y-Y Connection



Terminal & ground point Voltage expressions

$$v_{AG} = v_{AO}$$

$$v_{BG} = v_{BO}$$

$$v_{CG} = v_{CO}$$

$$v_{AN} = v_{AG} - v_{NG}$$

$$v_{BN} = v_{BG} - v_{NG}$$

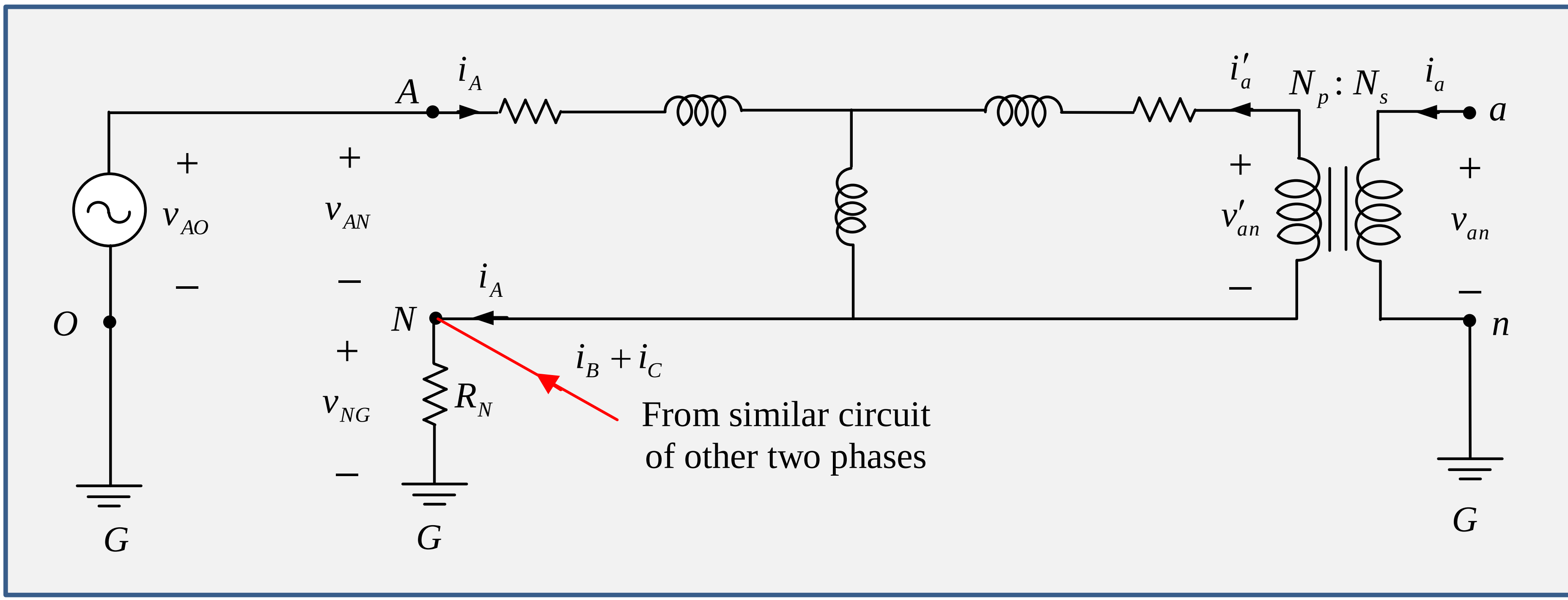
$$v_{CN} = v_{CG} - v_{NG}$$

$$v_{NG} = R_N\left(i_A + i_B + i_C\right)$$

Three-Phase Connections

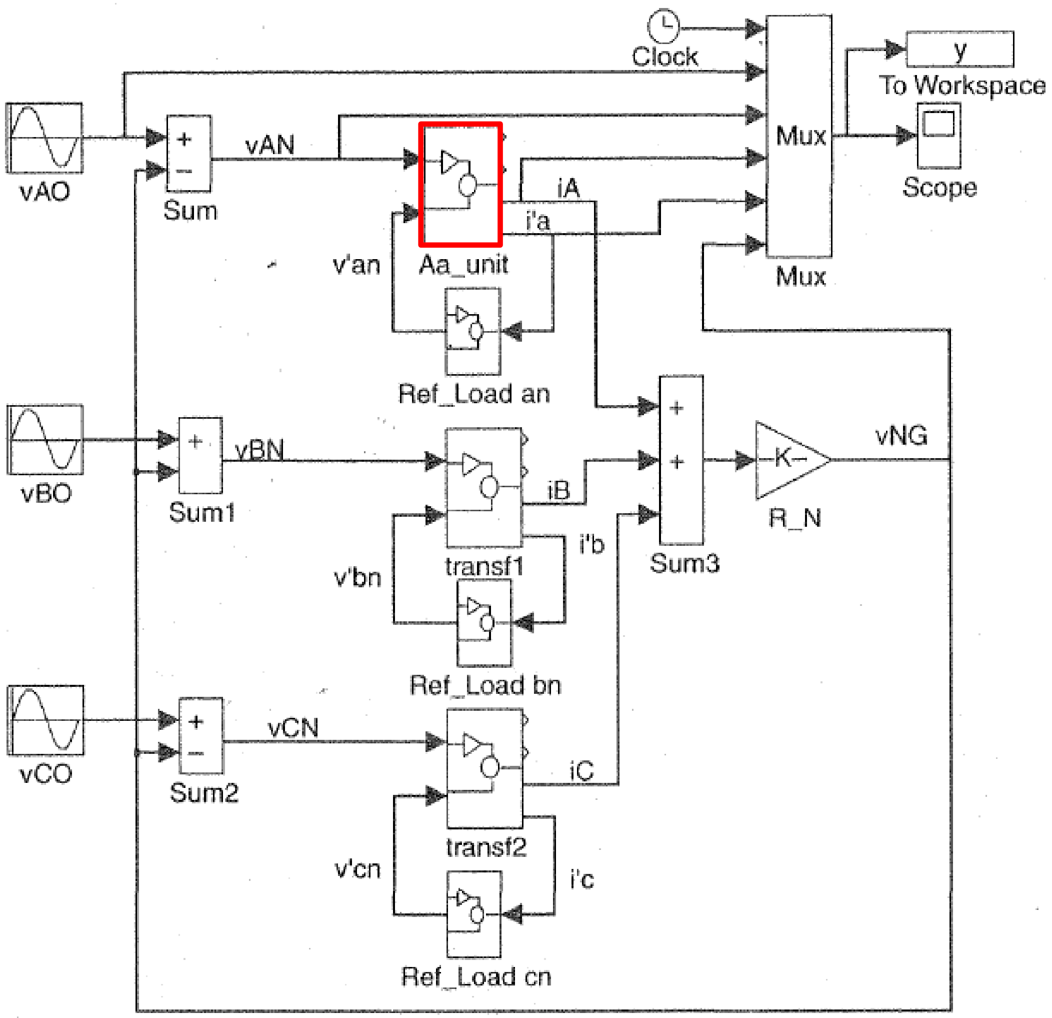
Y-Y Connection

Equivalent circuit representation of one transformer in detail



Three-Phase Connections

Y-Y Connection



Simulation in MATLAB/SIMULINK

$$v_{AN} = v_{AG} - v_{NG}$$

$$v_{BN} = v_{BG} - v_{NG}$$

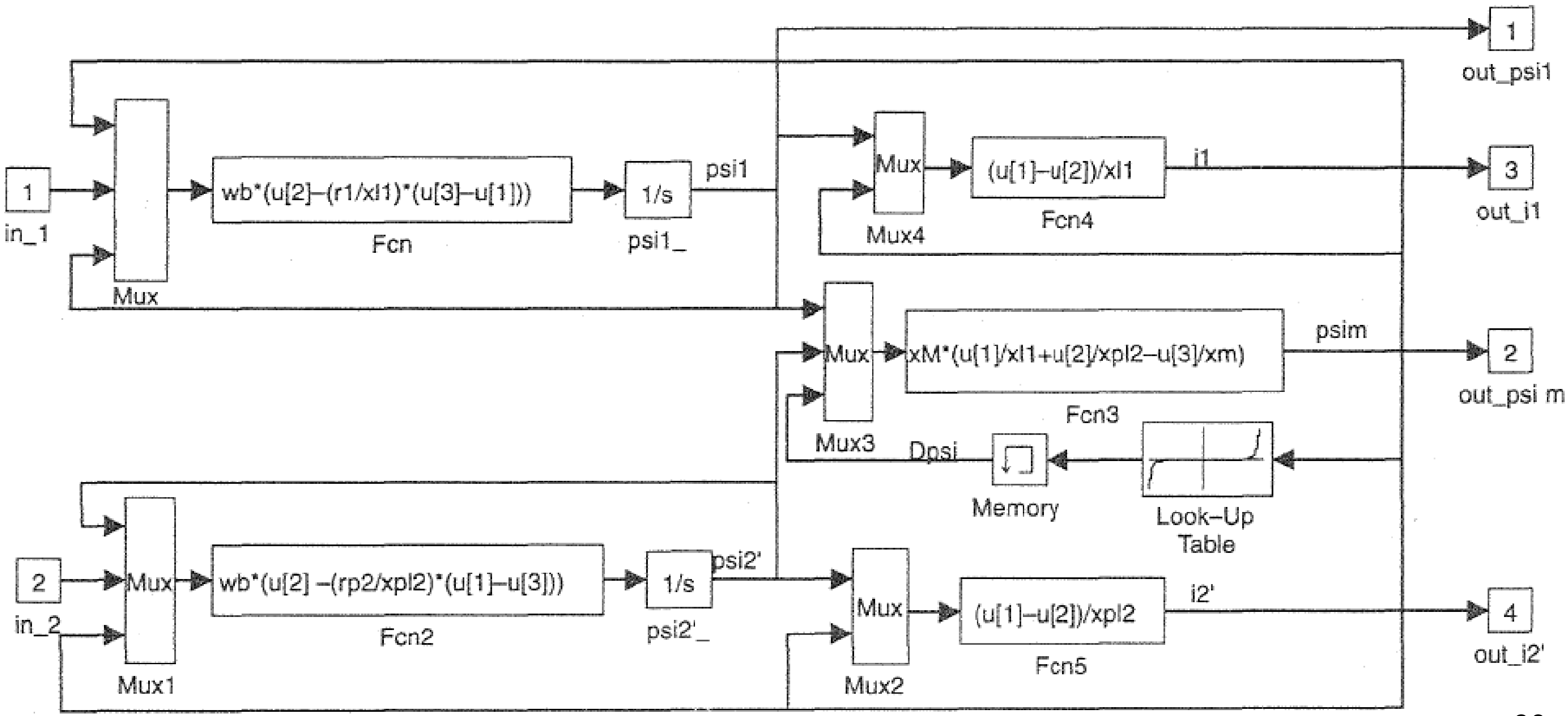
$$v_{CN} = v_{CG} - v_{NG}$$

$$v_{NG} = R_N\left(i_A + i_B + i_C\right)$$

Three-Phase Connections

Y-Y Connection

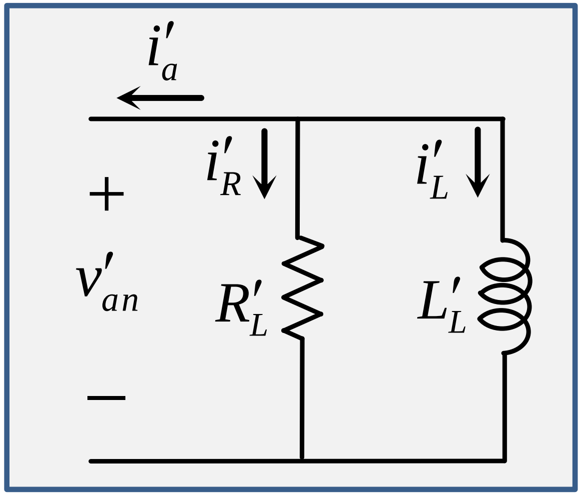
Inside the Aa_unit

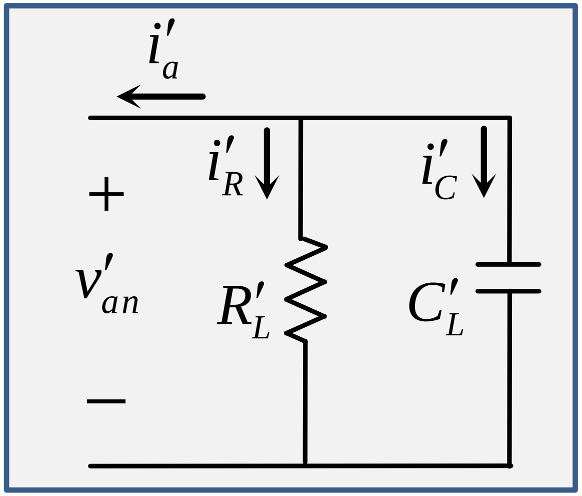


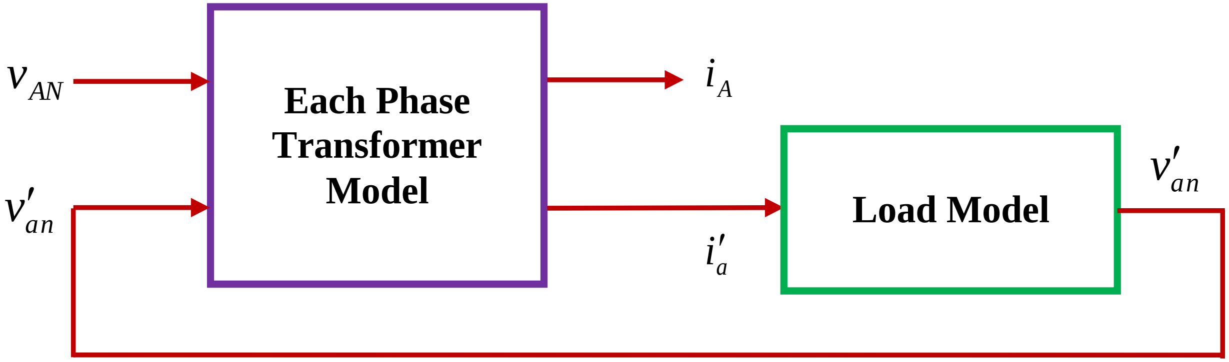
Three-Phase Connections

Y-Y Connection

Inside the Ref_Load an

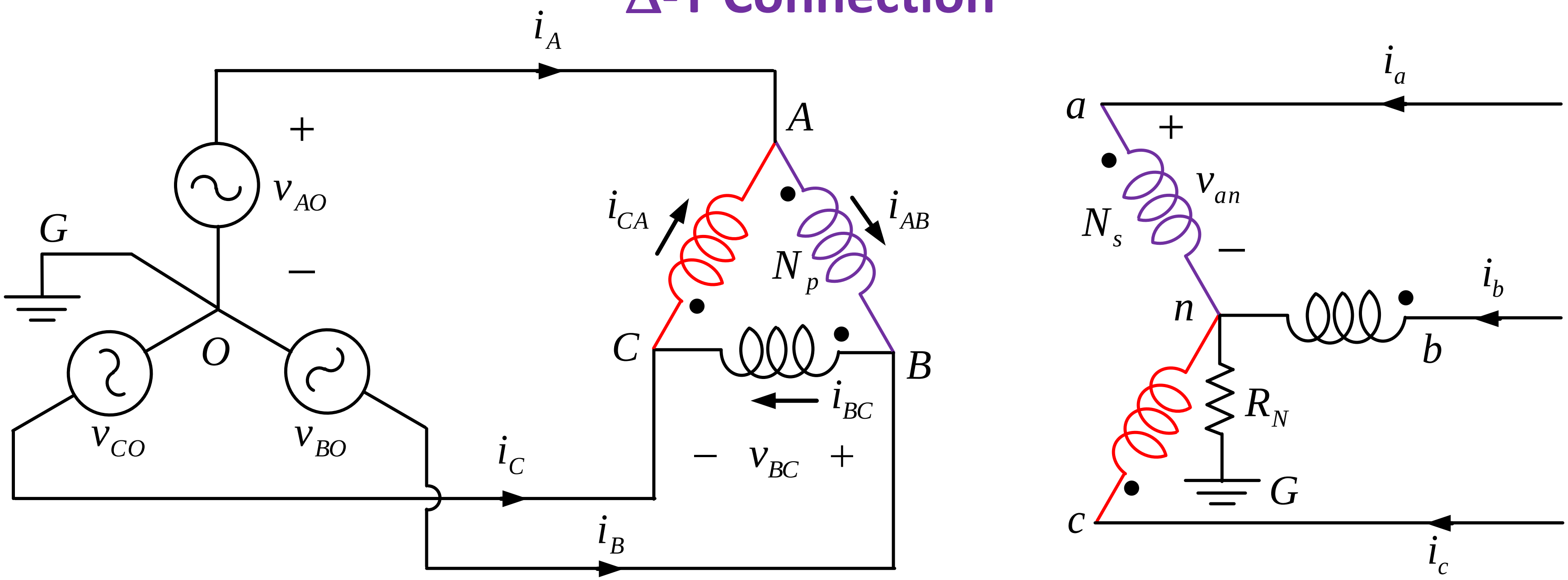






Three-Phase Connections

Δ-Y Connection



Voltage & current expressions

$$v_{AB} = v_{AO} - v_{BO}$$

$$v_{BC} = v_{BO} - v_{CO}$$

$$v_{CA} = v_{CO} - v_{AO}$$

$$i_A = i_{AB} - i_{CA}$$

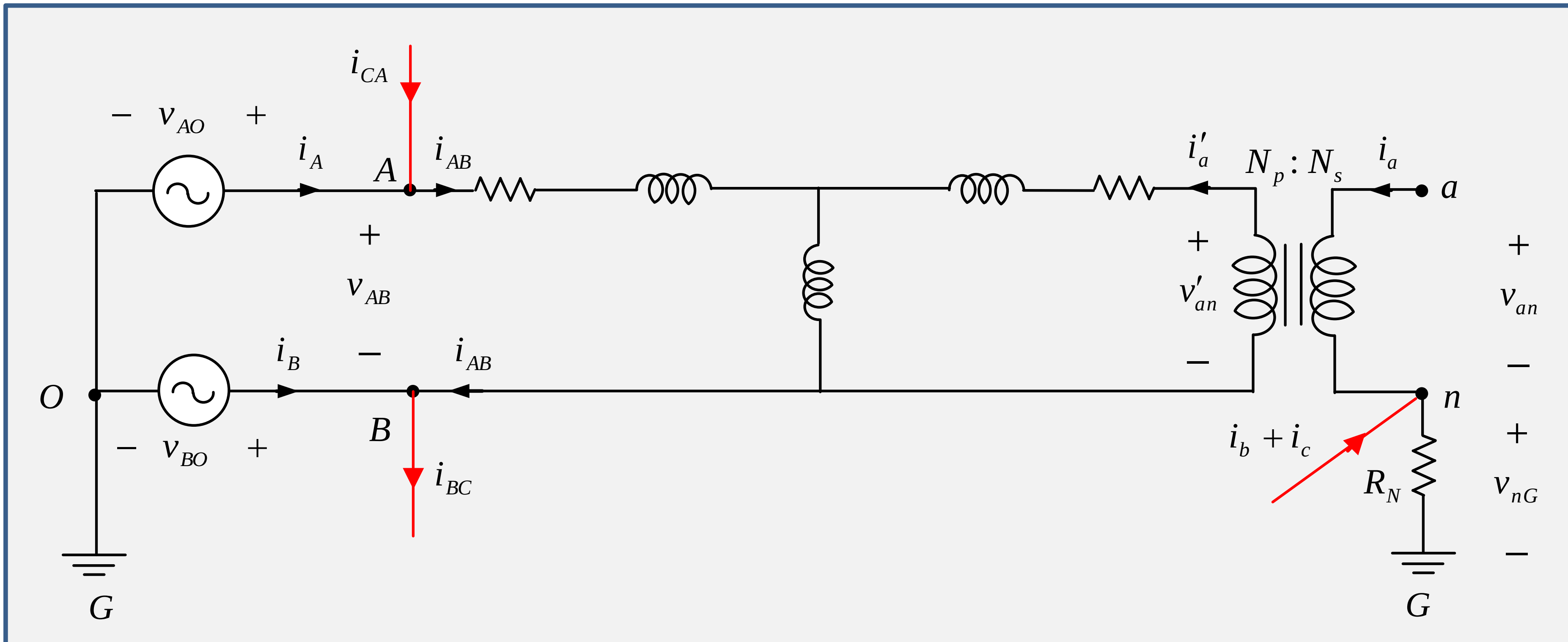
$$i_B = i_{BC} - i_{AB}$$

$$i_C = i_{CA} - i_{BC}$$

Three-Phase Connections

Δ-Y Connection

Equivalent circuit representation of one transformer in detail



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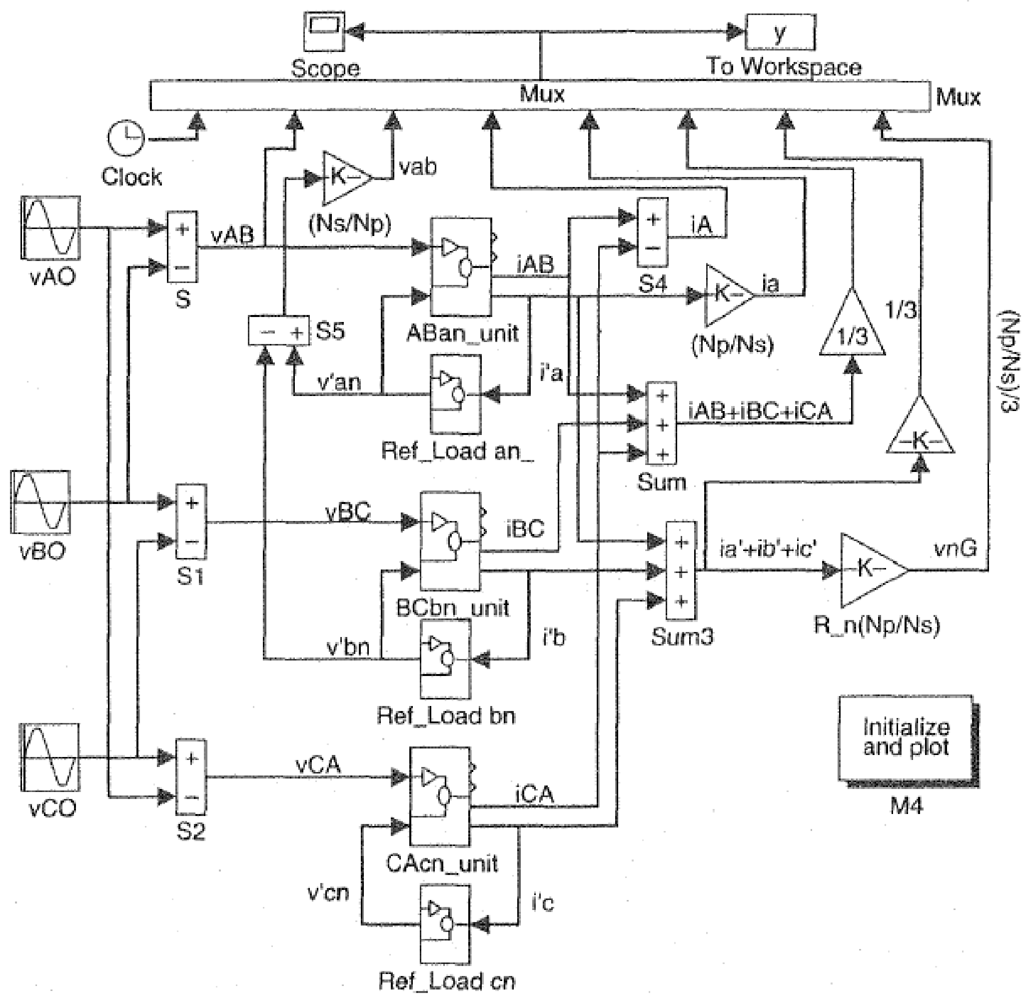
Three-Phase Connections

Δ-Y Connection



Simulation in MATLAB/SIMULINK

$$v_{AB} = v_{AO} - v_{BO}$$

$$v_{BC} = v_{BO} - v_{CO}$$

$$v_{CA} = v_{CO} - v_{AO}$$

$$i_A = i_{AB} - i_{CA}$$

$$i_B = i_{BC} - i_{AB}$$

$$i_C = i_{CA} - i_{BC}$$