In The Name of God The Most Compassionate, The Most Merciful

دانشگاه صنعتی شیراز

Electric Machines I

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Table of Contents

Chapter 2
Electromagnetic Circuits

Electromagnetic Quantities

H	: Magnetic field intensity	(A/m)
B	: Magnetic flux density	(T) or (Wb/m^2)
ϕ	: Magnetic flux	(Wb)
F	: Magnetomotive force, MMF	(A.turns)
E	: Electromotive force	(V)
μ	: Permeability	(H/m)
$\Re$	: Reluctance	(A.turns/Wb)
P	: Permeance	(Wb/(A.turns))
λ	: Flux linkage	(Wb.turns)
L	: Inductance	(H)

Different Unit Systems

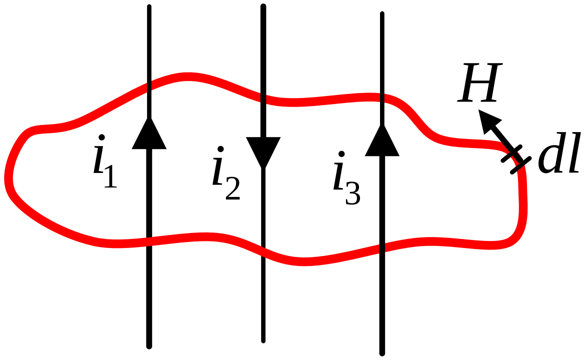
SI	CGS
1 Tesla	10^4 Gauss
1 Wb	10^8 Maxwell
1 A.t/m	10^{-3} Orsterd

Free space permeability $\mu_0 = 4\pi \times 10^{-7}$ H/m

Free space Permittivity $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

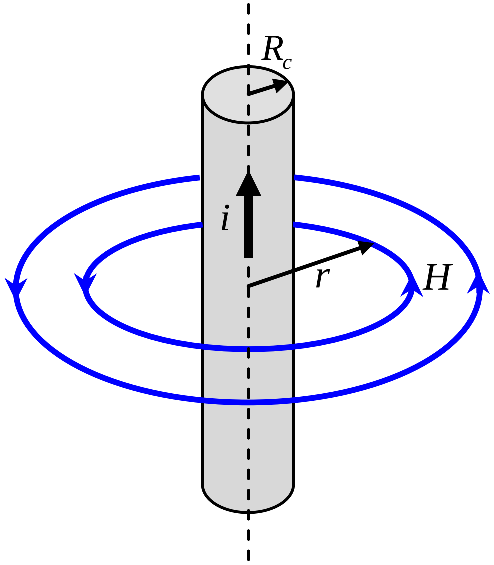
Relation Between *H* and *i*

The magnetic field intensity around a bundle of current carrying conductors is expressed as:



$$\oint H.dl = \sum i$$

Consider a current carrying conductor; the magnetic field intensity around the conductor is expressed as



$$\oint H.dl = \sum i \Rightarrow H = \frac{i}{2\pi r}$$

Relation Between *B* and *H*

The relation between the magnetic flux density and the magnetic field intensity in a medium without magnetization is expressed as:

$$B = \mu H$$

Or

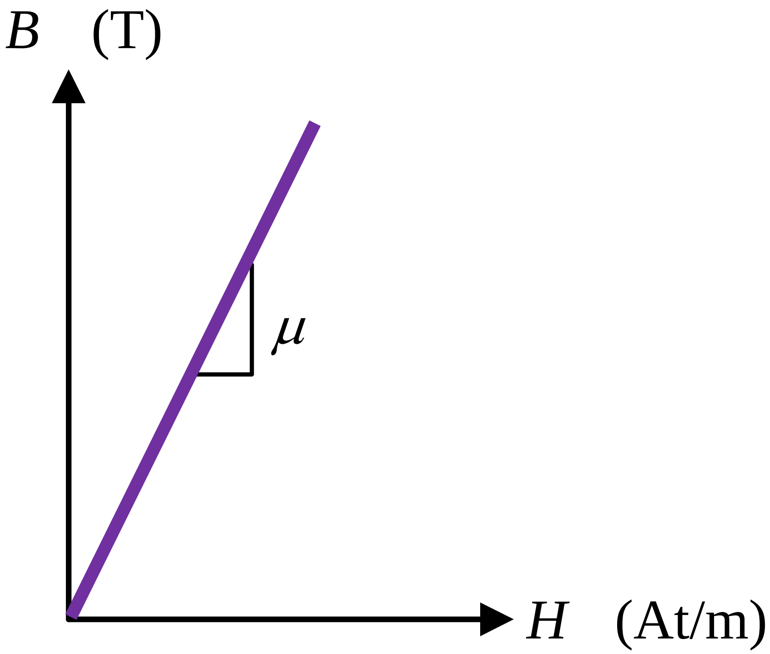
$$B = \mu_0 \mu_r H$$

where

μ_0 is the free space permeability

μ_r is the relative permeability

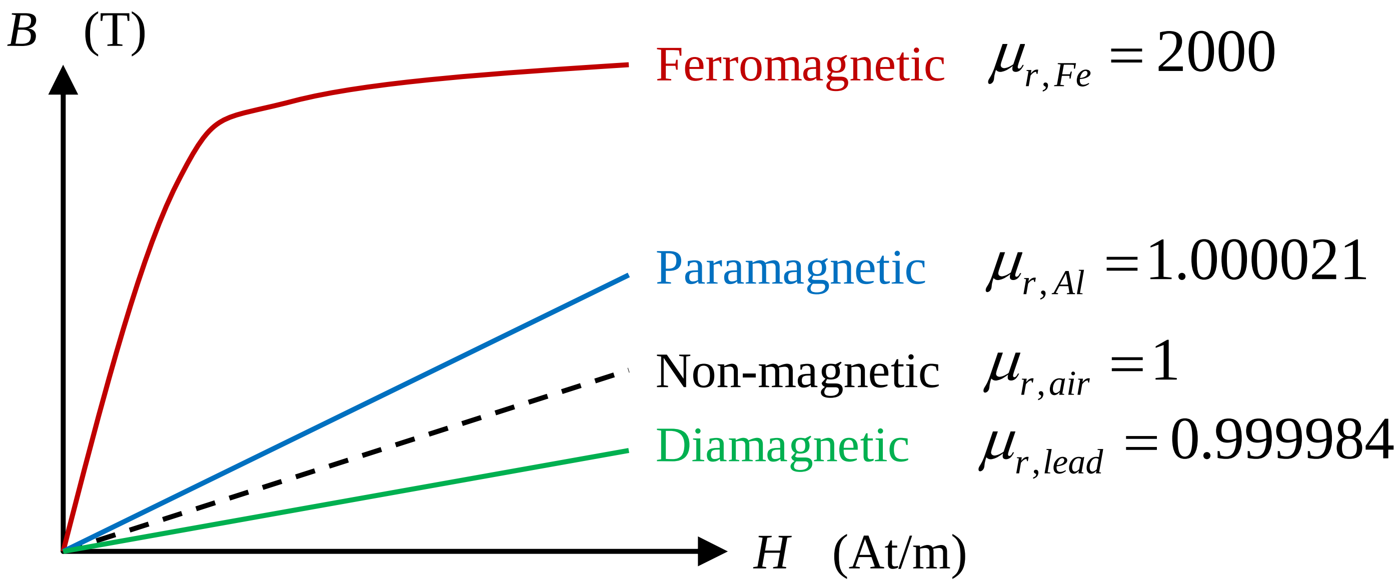
$$\mu_0 = 4\pi \times 10^{-7} \qquad \text{H/m}$$



Materials Classification in terms of magnetic property

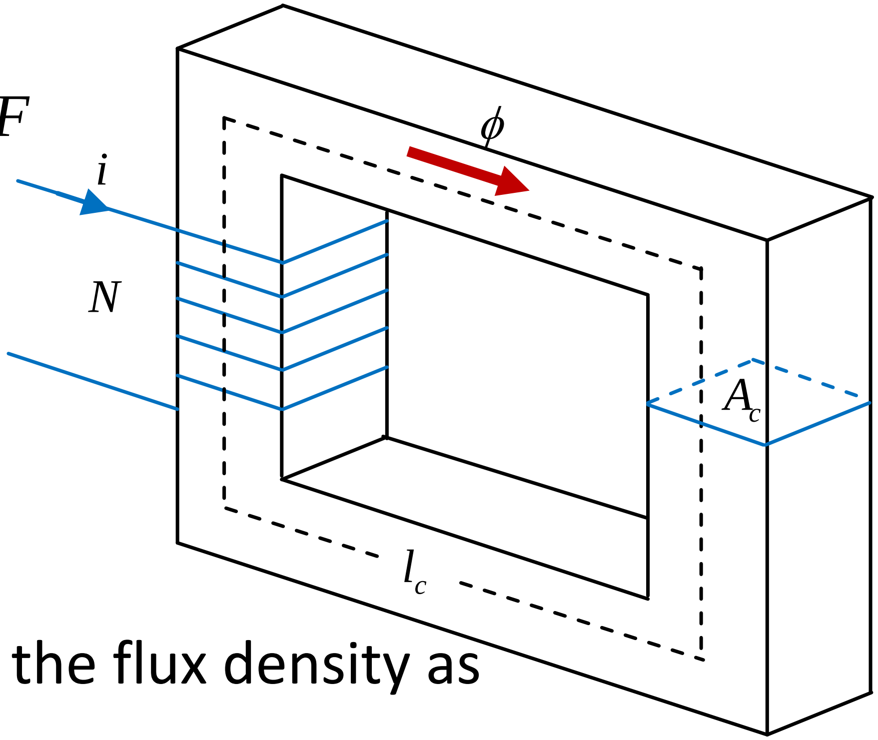
Depending on the ability of materials for magnetizing, materials are classified as

1. Ferromagnetic materials (e.g. iron, steel, cobalt, nickel) $\mu_r >> 1$
2. Paramagnetic materials (e.g. Aluminium) $\mu_r > 1$
3. Diamagnetic materials (e.g. Solid lead, copper) $\mu_r < 1$



Magnetic Equivalent Circuits

Consider the following electromagnetic system



- The relation between *H* and *i* is

$$\oint H.dl = Ni \;\Rightarrow\; H\,l_c = Ni = F$$

Which is expressed as

$$H = \frac{Ni}{l_c}$$

- The relation between *B* and *H* is

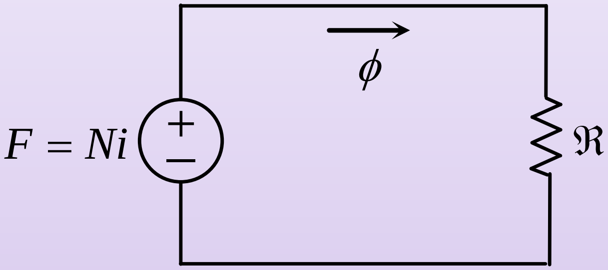
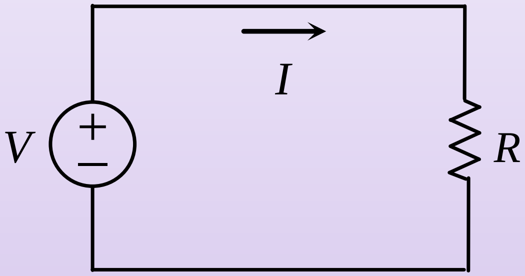
$$B = \mu H \;\Rightarrow\; B = \frac{\mu Ni}{l_c}$$

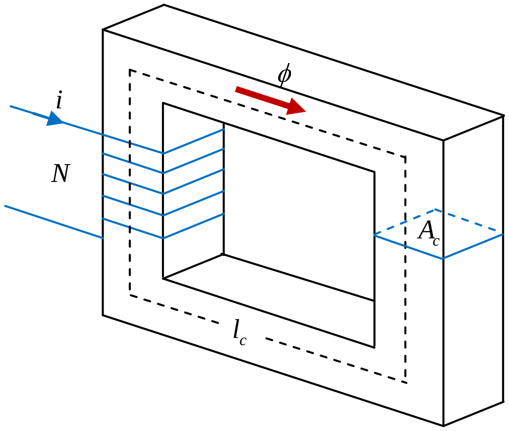
The magnetic flux is obtained from the flux density as

$$\phi = \int_A B \cdot dA \Rightarrow \phi = A_c B \Rightarrow \phi = \frac{Ni}{l_c/(\mu A_c)} = \frac{Ni}{\Re} \quad \text{where} \quad \Re = \frac{l_c}{\mu A_c} = \frac{1}{P}$$

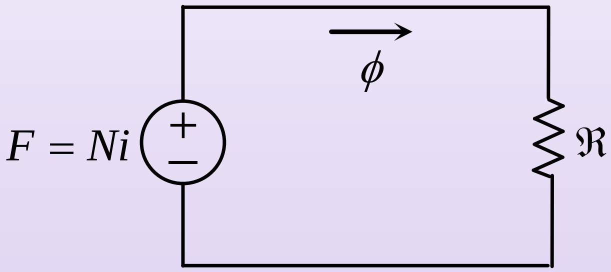
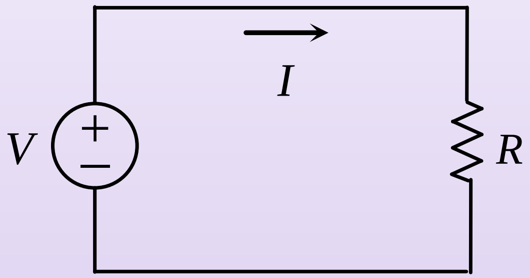
Magnetic Equivalent Circuits

For electromagnetic systems, an equivalent circuit can be obtained. For example the equivalent circuit of the following system is

Magnetic circuit	**Electric circuit**
	
$F = \Re\phi$	$V = RI$
$\Re = \frac{l_c}{\mu A_c} = \frac{1}{P}$	$R = \frac{\rho l_w}{A_w} = \frac{1}{G}$



Magnetic and Electric Circuits Analogy

Magnetic circuit	Electric circuit
	
Magnetomotive force (F)	Voltage (V)
Magnetic flux (f)	Electric current (I)
Reluctance ($\Re$)	Electric resistance (R)
$F = \Re\phi$	$V = RI$
$\Re = \frac{l_c}{\mu A_c} = \frac{1}{P}$	$R = \frac{\rho l_w}{A_w} = \frac{1}{G}$

Assumptions

The magnetic field calculation using magnetic equivalent circuit is an **approximated method** because:

1. The **flux leakage** has been neglected.
2. The cross sectional **area** of the core is not the same everywhere (at the corners the area is larger).
3. The **length** of the core is the average value (at the outer edge of the core the length is larger compared to that of the inner edge.
4. Relative **permeability** of the ferromagnetic core is not constant.
5. **Fringing effect** has been neglected.

To solve the problem accurately **Maxwell's equations** must be used.

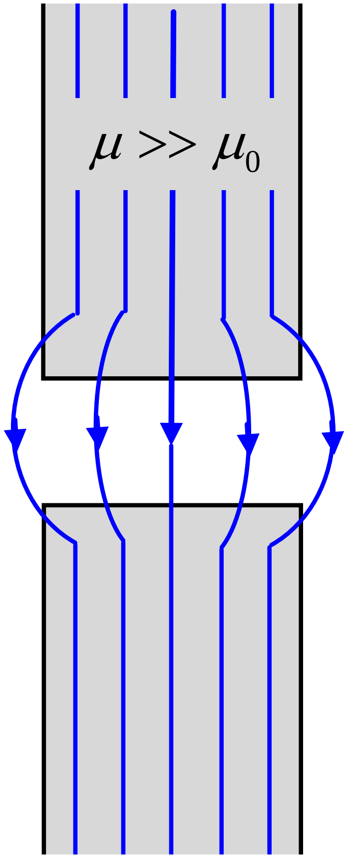
Fringing Effect

- If the magnetic flux passing through a ferromagnetic core faces an air-gap, the effective area of the air-gap will be larger than that of the core.

- To incorporate this effect, fringing coefficient is defined: C_f

- Using this coefficient, the effective air-gap area is calculated as

$$A_g = (1 + C_f)A_c$$



Maxwell equations

In general, the Maxwell equations are expressed as

Name	Microscopic equations	Macroscopic equations
Gauss's law	$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$	$\nabla \cdot \mathbf{D} = \rho_f$
Gauss's law for magnetism	$\nabla \cdot \mathbf{B} = 0$	$\nabla \cdot \mathbf{B} = 0$
Maxwell Faraday equation (Faraday's law of induction)	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$	$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$
Ampere's circuital law (with Maxwell's correction)	$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$	$\nabla \times \mathbf{H} = \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}$

Maxwell equations

Important quantities

$\nabla\cdot$: Divergence operator

$\nabla\times$: Curl operator

$\partial/\partial t$: Partial derivative with respect to time

$\mathbf{E}$: Electric field or electric field intensity (V/m)

$\mathbf{B}$: Magnetic field or magnetic flux density (T)

$\mathbf{D}$: Electric displacement field or electric flux density (C/m^2)

$\mathbf{H}$: Magnetic field intensity (A/m)

μ_0 : Permeability of free space (H/m)

ε_0 : Permittivity of free space (F/m)

σ : Conductivity (mhos/m)

Maxwell equations

Important quantities

ρ_f : Free charge density (C/m^3)

ρ : Total charge density (C/m^3)

$\mathbf{J}_f$: Free current density (A/m^2)

$\mathbf{J}$: Total current density (A/m^2)

Important relation

$$\mathbf{D} = \varepsilon \mathbf{E}$$

$$\mathbf{B} = \mu \mathbf{H}$$

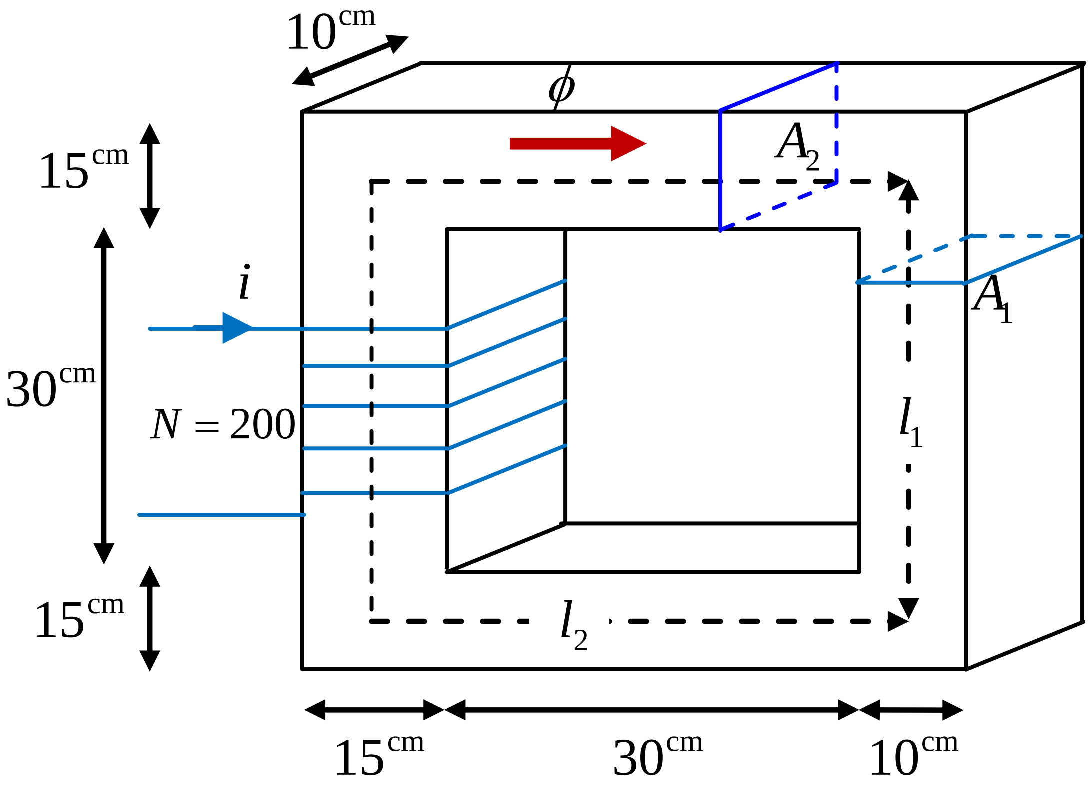
$$\mathbf{J} = \sigma \mathbf{E}$$

Magnetic Equivalent Circuits

Example 1: In the following electromagnetic system, calculate the magnetic flux in the ferromagnetic core if the current is 1 A and the relative permeability of the core is 2500.

$$\Re = \frac{l_c}{\mu A_c}$$

$$Ni = \Re \phi$$



Magnetic Equivalent Circuits

Solution 1: $i = 1\ \text{A}$ $\qquad \phi = ?$

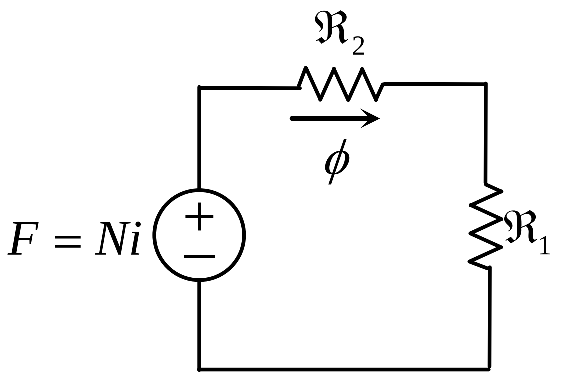
$$\Re_1 = \frac{l_1}{\mu_0 \mu_r A_1}$$

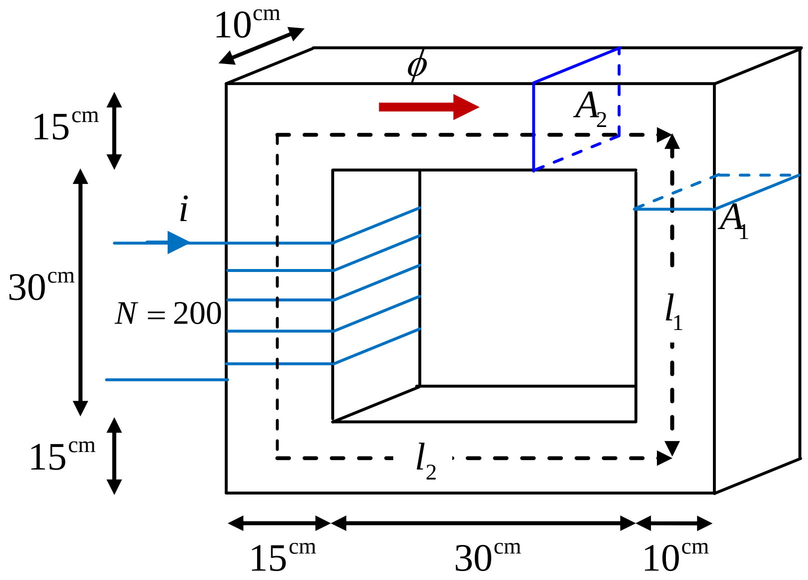
$$= \frac{0.45}{4\pi \times 10^{-7} \times 2500 \times 0.01} = 14300\ \text{At/Wb}$$

$$\Re_2 = \frac{l_2}{\mu_0 \mu_r A_2}$$

$$= \frac{1.3}{4\pi \times 10^{-7} \times 2500 \times 0.015} = 27600\ \text{At/Wb}$$

$$\phi = \frac{Ni}{\Re_1 + \Re_2} = \frac{200 \times 1}{14300 + 27600} = 4.8\ \text{mWb}$$



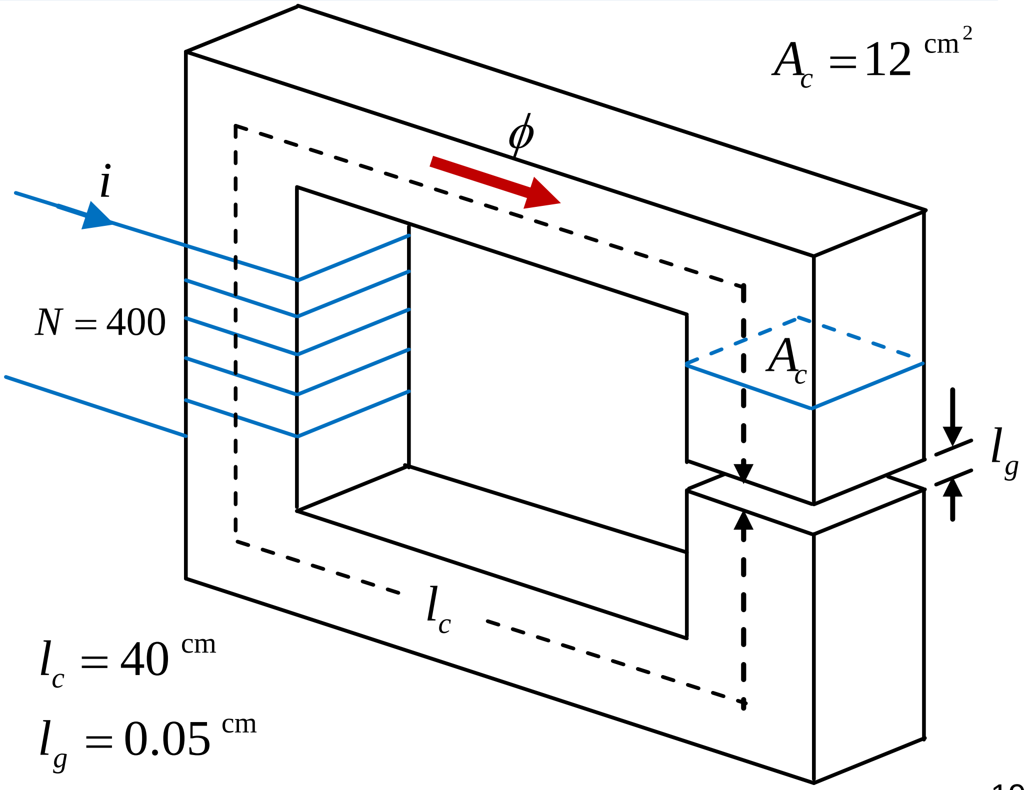


Magnetic Equivalent Circuits

Example 2: In the following electromagnetic system, calculate the current if the magnetic flux density in the air-gap is 0.5 T and the relative permeability of the core is 4000. The fringing effect is considered by a factor of 5%.

$$\Re = \frac{l}{\mu A}$$

$$Ni = \Re\phi$$



Magnetic Equivalent Circuits

Solution 2: $B_g = 0.5 \text{ T}$ $\qquad i = ?$

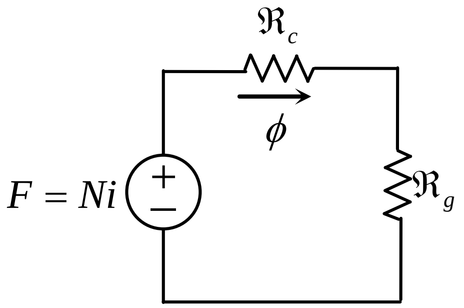
$$\Re_c = \frac{l_c}{\mu_0 \mu_r A_c}$$

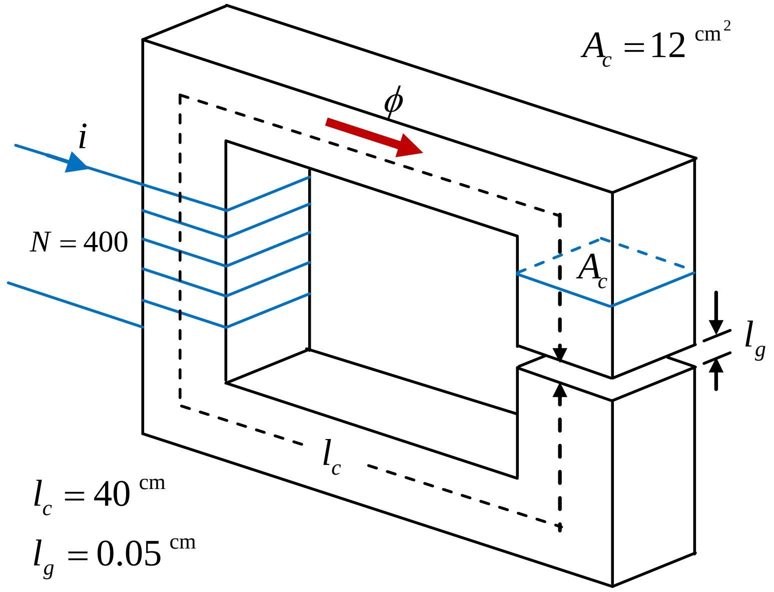
$$= \frac{0.4}{4\pi \times 10^{-7} \times 4000 \times 0.0012} = 66314 \text{ At/Wb}$$

$$\Re_g = \frac{l_g}{\mu_0 A_g} = \frac{l_g}{\mu_0 A_c (1 + C_f)}$$

$$= \frac{0.0005}{4\pi \times 10^{-7} \times 0.0012 \times 1.05} = 315783 \text{ At/Wb}$$

$$Ni = \phi(\Re_c + \Re_g)$$





Magnetic Equivalent Circuits

Solution 2:

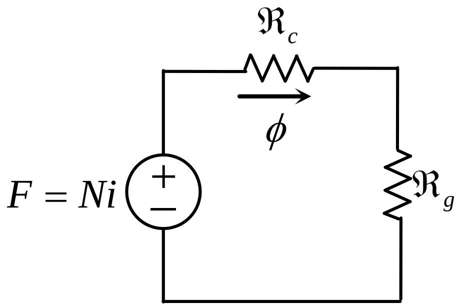
$$\phi = A_g B_g = A_c B_c$$

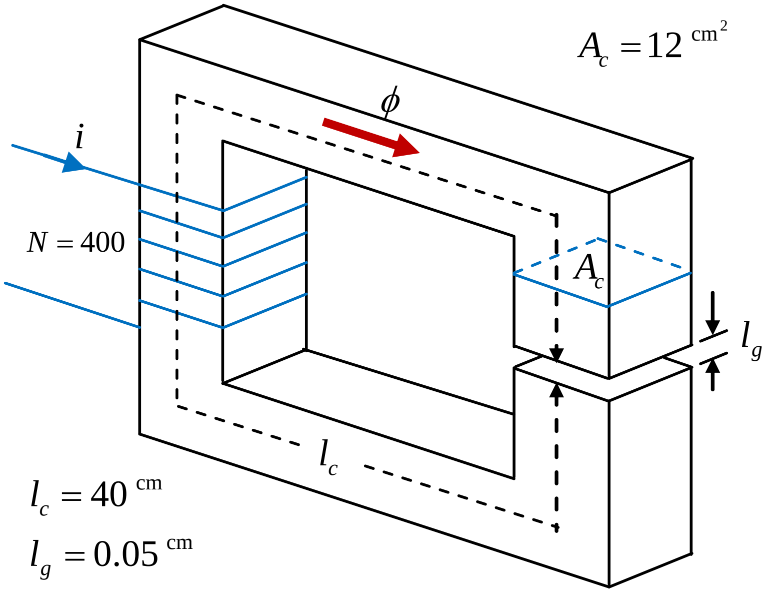
$$Ni = \phi\left(\Re_c + \Re_g\right) \Rightarrow Ni = A_g B_g\left(\Re_c + \Re_g\right)$$

$$i = \frac{A_g B_g\left(\Re_c + \Re_g\right)}{N}$$

$$i = \frac{(0.0012\times 1.05)\times 0.5\times(66314+315783)}{400}$$

$$i = 0.6018 \text{ A}$$



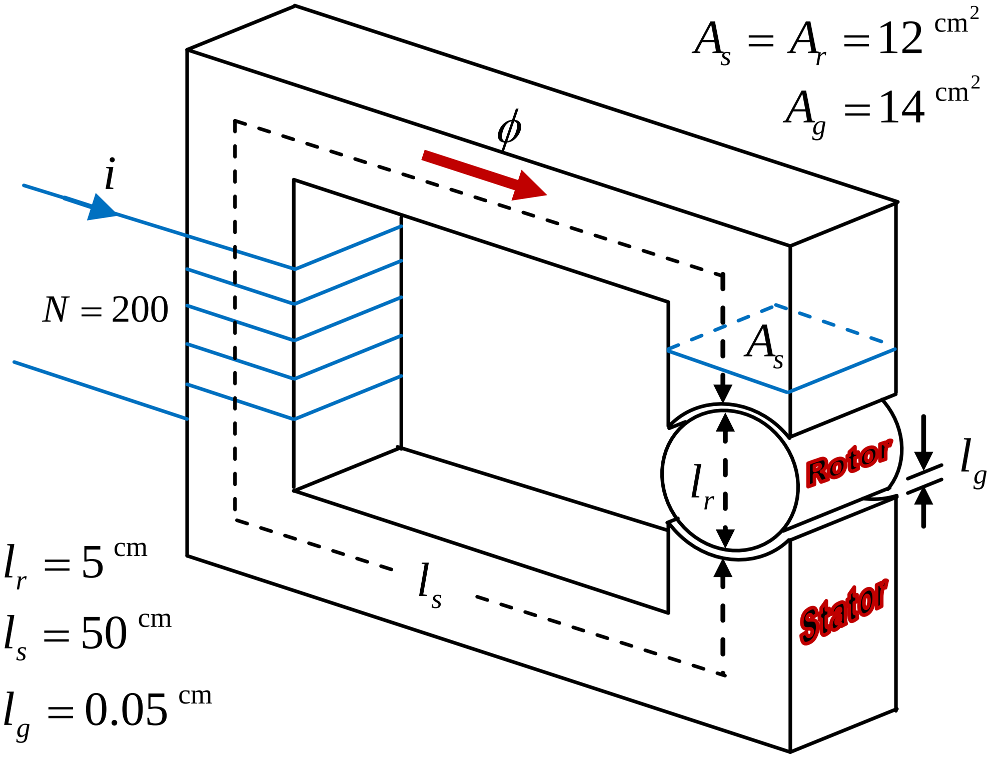


Magnetic Equivalent Circuits

Example 3: In the following electromagnetic system, calculate the magnetic flux density in the air-gap if the current is 1 A and the relative permeability of the core is 2000. The cross sectional area of the air-gap with fringing effect is 14 cm^2.

$$\Re = \frac{l_c}{\mu A_c}$$

$$Ni = \Re\phi$$



$$A_s = A_r = 12 \text{ cm}^2$$

$$A_g = 14 \text{ cm}^2$$

$$l_r = 5 \text{ cm}$$

$$l_s = 50 \text{ cm}$$

$$l_g = 0.05 \text{ cm}$$

Magnetic Equivalent Circuits

Solution 3: $i = 1\ \text{A}$ $\qquad B_g = ?$

$$\Re_s = \frac{l_s}{\mu_0 \mu_r A_s}$$

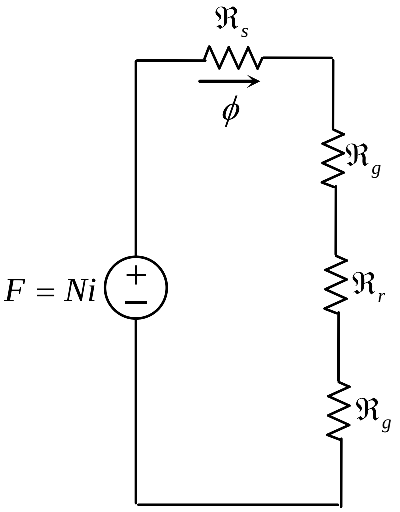
$$= \frac{0.5}{4\pi \times 10^{-7} \times 2000 \times 0.0012} = 166000\ \text{At/Wb}$$

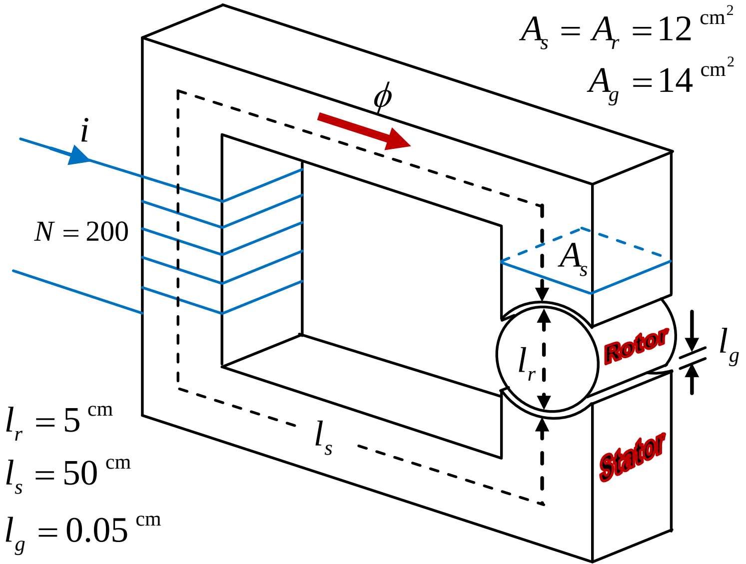
$$\Re_r = \frac{l_r}{\mu_0 \mu_r A_r}$$

$$= \frac{0.05}{4\pi \times 10^{-7} \times 2000 \times 0.0012} = 16600\ \text{At/Wb}$$

$$\Re_g = \frac{l_g}{\mu_0 A_g}$$

$$= \frac{0.0005}{4\pi \times 10^{-7} \times 0.0014} = 284000\ \text{At/Wb}$$





$A_s = A_r = 12\ \text{cm}^2$

$A_g = 14\ \text{cm}^2$

$l_r = 5\ \text{cm}$

$l_s = 50\ \text{cm}$

$l_g = 0.05\ \text{cm}$

Magnetic Equivalent Circuits

Solution 3:

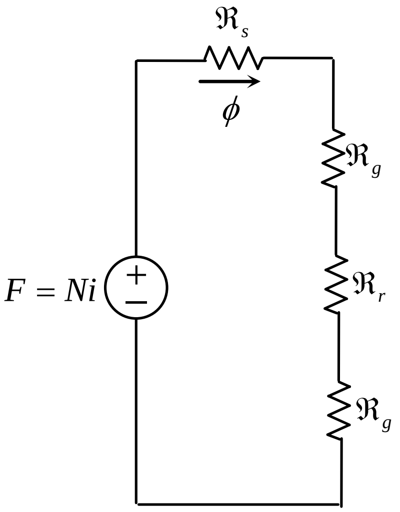
$$\phi = \frac{Ni}{\Re_s + 2\Re_g + \Re_r}$$

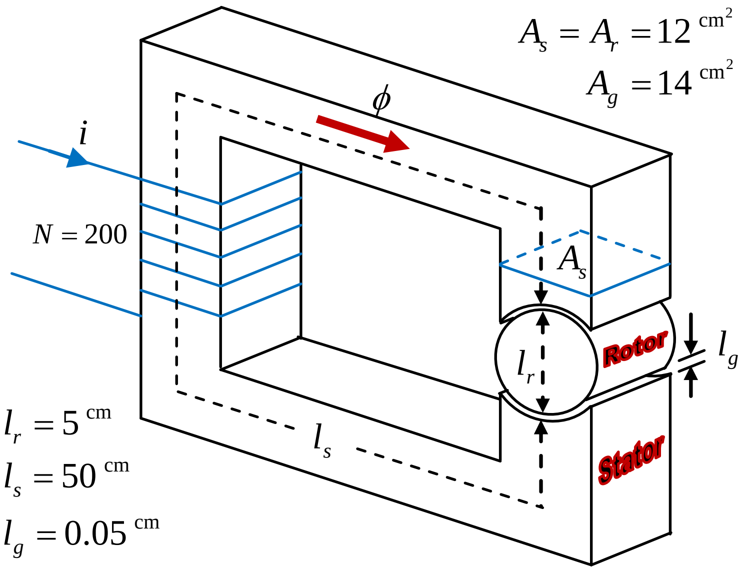
$$B_g = \frac{\phi}{A_g}$$

$$B_g = \frac{Ni}{A_g\left(\Re_s + 2\Re_g + \Re_r\right)}$$

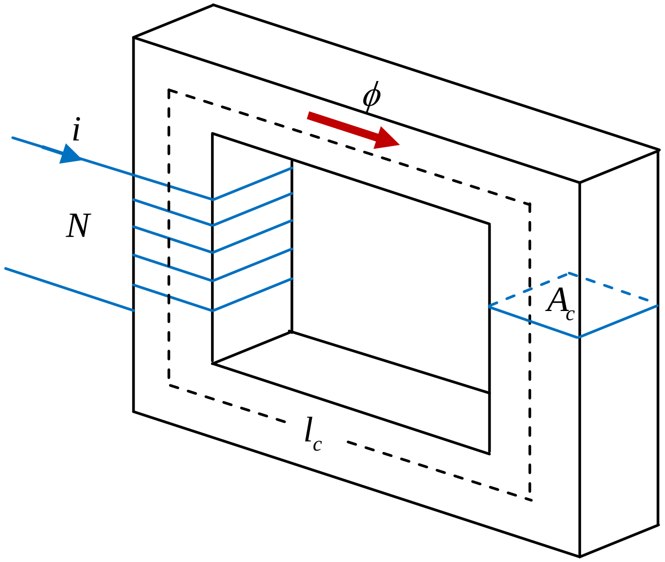
$$B_g = \frac{200\times 1}{0.0014\left(166000 + 2\times 284000 + 16600\right)}$$

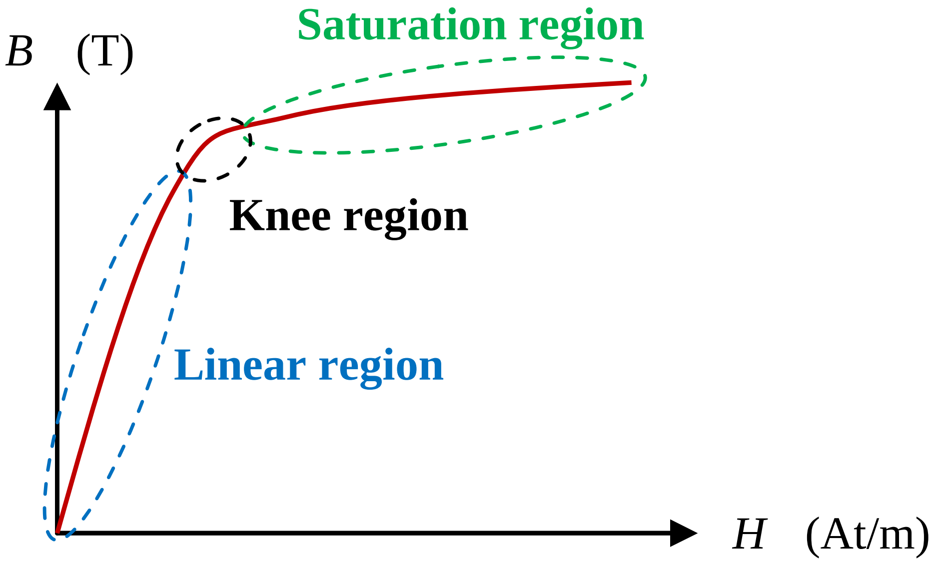
$$\boxed{B_g = 0.19 \quad \mathrm{T}}$$

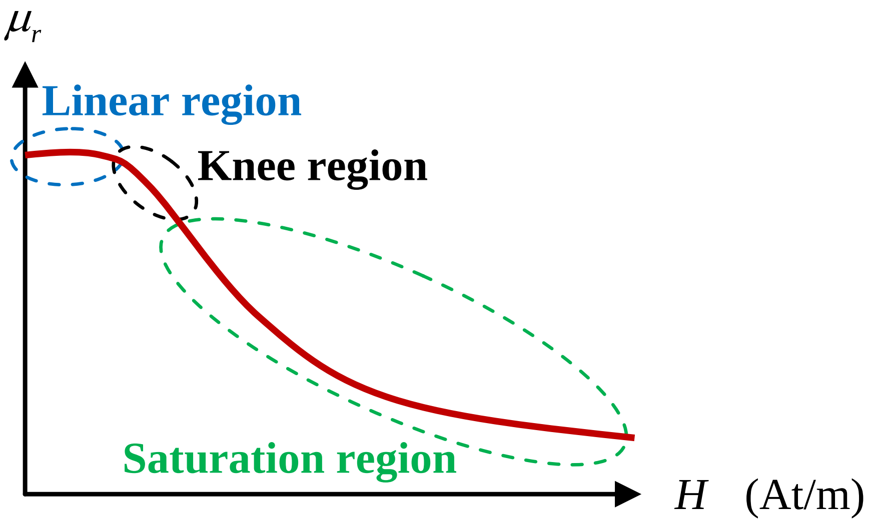




Nonlinear Behaviour of Ferromagnetic Materials







- μ_r is NOT constant.
- So reluctance is not constant.

Nonlinear Behaviour of Ferromagnetic Materials

Three cases in nonlinear problems

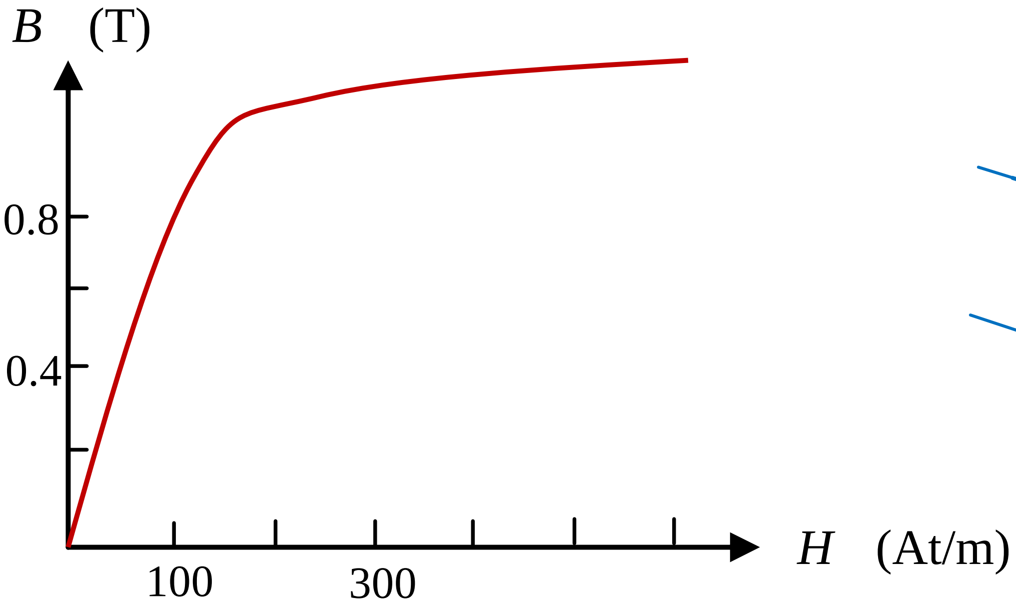
Case 1: If *B* is known, from the B-H curve, *H* is obtained and then permeability is found. Other quantities are also obtained.

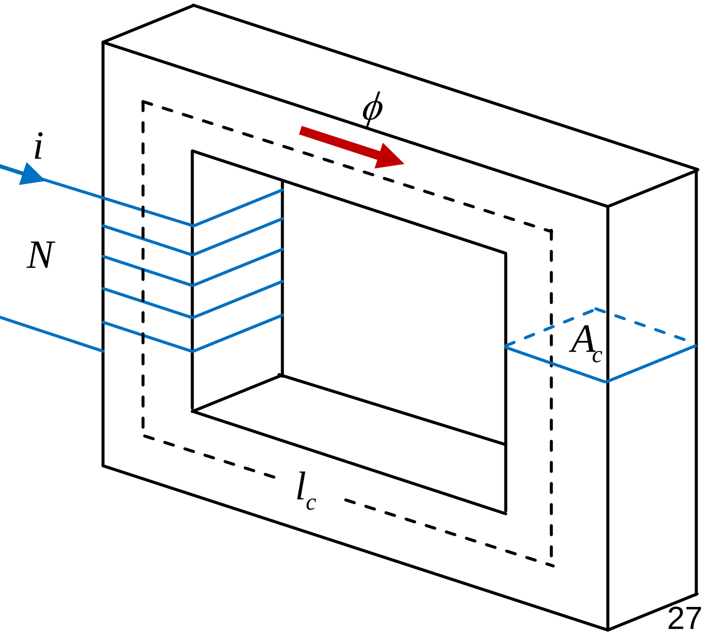
Case 2: If *H* is known, from the B-H curve, *B* is obtained and then permeability is found.

Case 3: If *i* is known, from the electromagnetic system a relation between B and H is obtained and by intersecting the relation with the B-H curve both B and H are found simultaneously.

Magnetic Equivalent Circuits

Example 4: In the following electromagnetic system, calculate the current if the magnetic flux in the core is 0.012 Wb. The average length of the core is 55 cm, the cross sectional area of the core is 150 cm^2, and the number of turns is 200. The B-H is given below. Also calculate the relative permeability and reluctance of the core.





Magnetic Equivalent Circuits

Solution 2: $\phi = 0.012 \text{ Wb} \qquad i = ? \qquad \mu_r = ? \qquad \Re = ?$

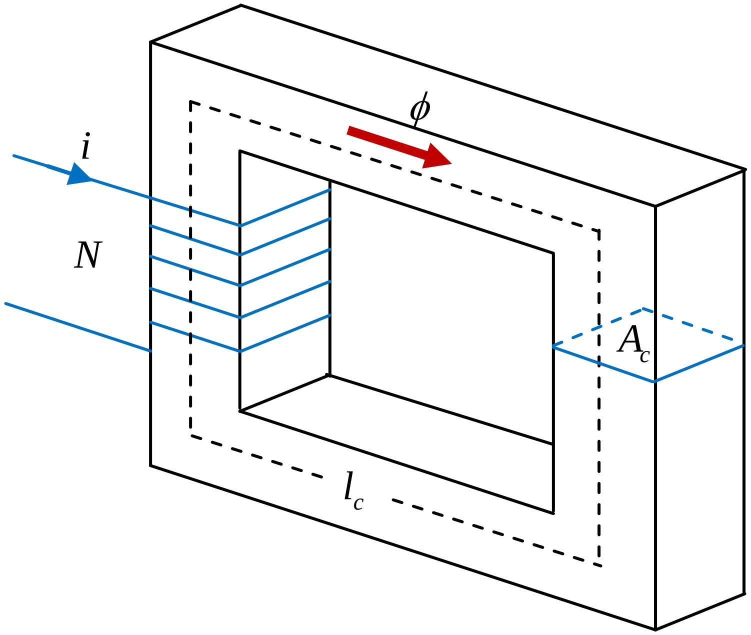
$$B = \frac{\phi}{A_c} = \frac{0.012}{0.015} = 0.8 \ \text{ T}$$

From the B-H curve $H = 115 \ \text{ At/m}$

$$Ni = Hl_c \Rightarrow i = \frac{Hl_c}{N} = \frac{115 \times 0.55}{200} = 0.316 \ A$$

$$\mu = \frac{B}{H} = \frac{0.8}{115} = 6.9 \times 10^{-3} \ \text{ H/m}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{6.9 \times 10^{-3}}{4\pi \times 10^{-7}} = 5540$$



$$\Re = \frac{F}{\phi} = \frac{200 \times 0.316}{0.012} = 5270 \ \text{ At/Wb}$$

Magnetic Equivalent Circuits

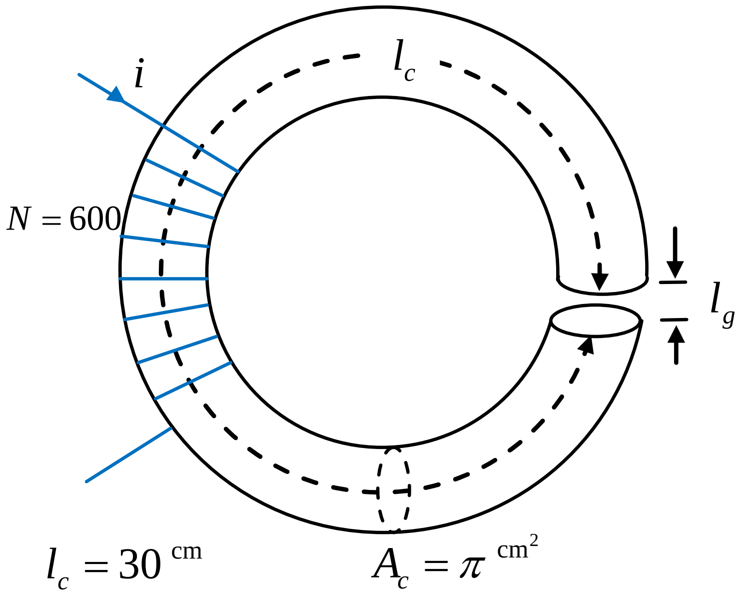
Example 5: Calculate the current if the magnetic flux is 0.5 mWb.

a) Without air-gap and relative permeability of 4000.

b) With an air-gap of 1 mm length and relative permeability of 4000.

c) With an air-gap of 1 mm length based on the following B-H table:

H (At/m)	2500	3000	3500	4000
B (T)	1.55	1.59	1.6	1.615



Magnetic Equivalent Circuits

Solution 5: $\phi = 0.5 \text{ mWb}$ $i = ?$

Part a) $l_g = 0$ $\mu_r = 4000$

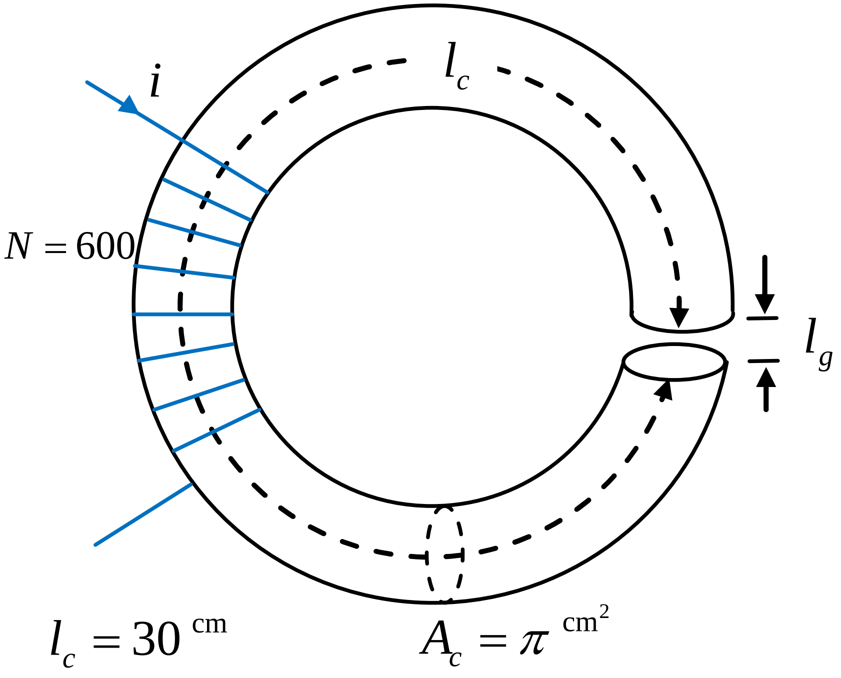
$$\Re_c = \frac{l_c}{\mu_0 \mu_r A_c}$$

$$= \frac{30\times10^{-2}}{4\pi\times10^{-7}\times4000\times\pi\times10^{-4}}$$

$$= 1.9\times10^5 \quad \text{At/Wb}$$

$$Ni = \Re_c\,\phi$$

$$i = \frac{\Re_c \phi}{N} = \frac{1.9\times10^5\times0.5\times10^{-3}}{600} = 0.158 \text{ A}$$



Magnetic Equivalent Circuits

Solution 5: $\phi = 0.5 \text{ mWb}$ $\quad i = ?$

Part b) $l_g = 1 \text{ mm}$ $\quad \mu_r = 4000$

$$\Re_c = 1.9\times10^5 \quad \text{At/Wb}$$

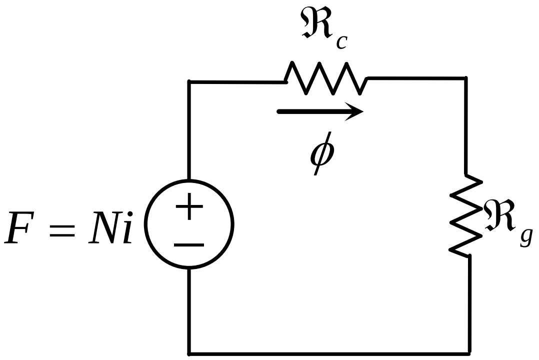
$$\Re_g = \frac{l_g}{\mu_0 A_g}$$

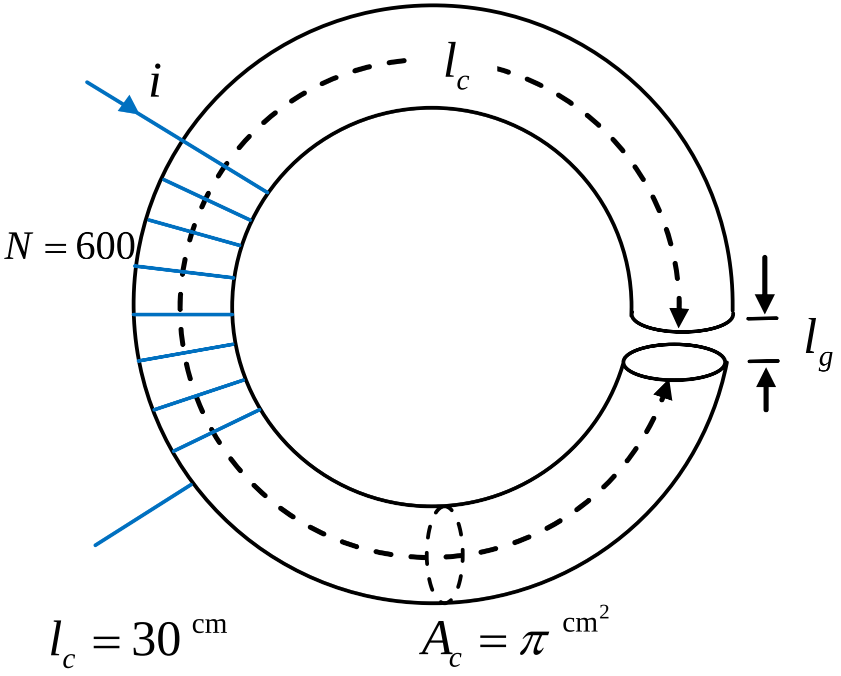
$$= \frac{1\times10^{-3}}{4\pi\times10^{-7}\times\pi\times10^{-4}}$$

$$= 25.2\times10^5 \quad \text{At/Wb}$$

$$Ni = \left(\Re_c + \Re_g\right)\phi$$

$$i = \frac{\left(\Re_c + \Re_g\right)\phi}{N} = 2.25 \ \text{A}$$





Magnetic Equivalent Circuits

H (At/m)	2500	3000	3500	4000
B (T)	1.55	1.59	1.6	1.615

Solution 5: $\phi = 0.5 \text{ mWb}$ $i = ?$

Part c) $l_g = 1 \text{ mm}$

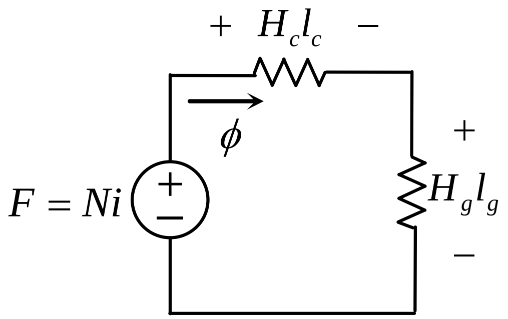
$$B_c = B_g = \frac{\phi}{A_c} = \frac{0.5\times10^{-3}}{\pi\times10^{-4}} = 1.59 \text{ T}$$

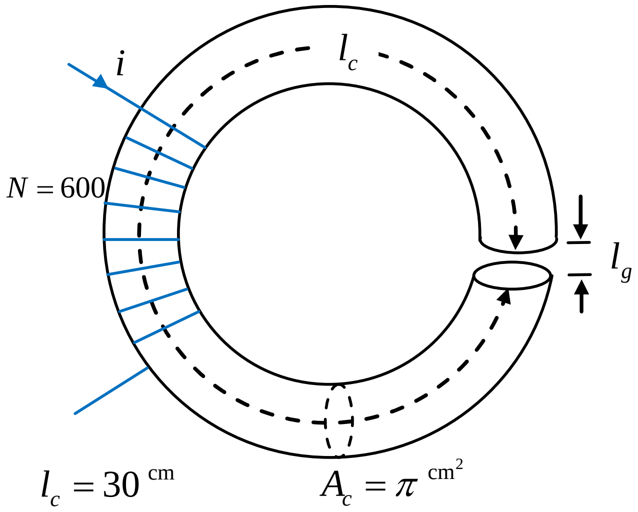
From the table $H_c = 3000 \text{ At/m}$

$$H_g = \frac{B_g}{\mu_0} = \frac{1.59}{4\pi\times10^{-7}} = 1.26\times10^{6} \text{ At/m}$$

$$Ni = H_c l_c + H_g l_g$$

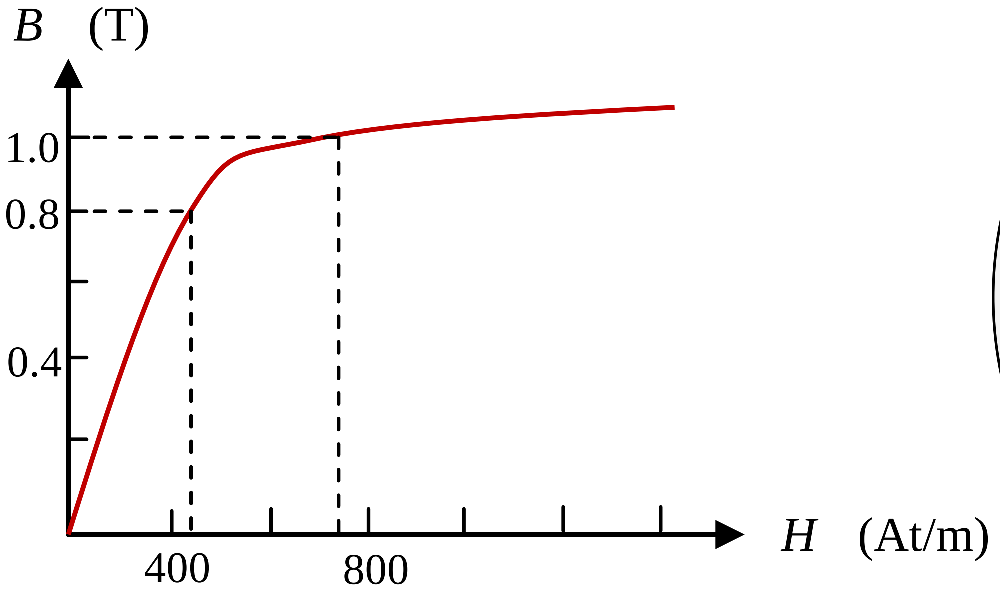
$$i = \frac{H_c l_c + H_g l_g}{N} = 3.6 \text{ A}$$

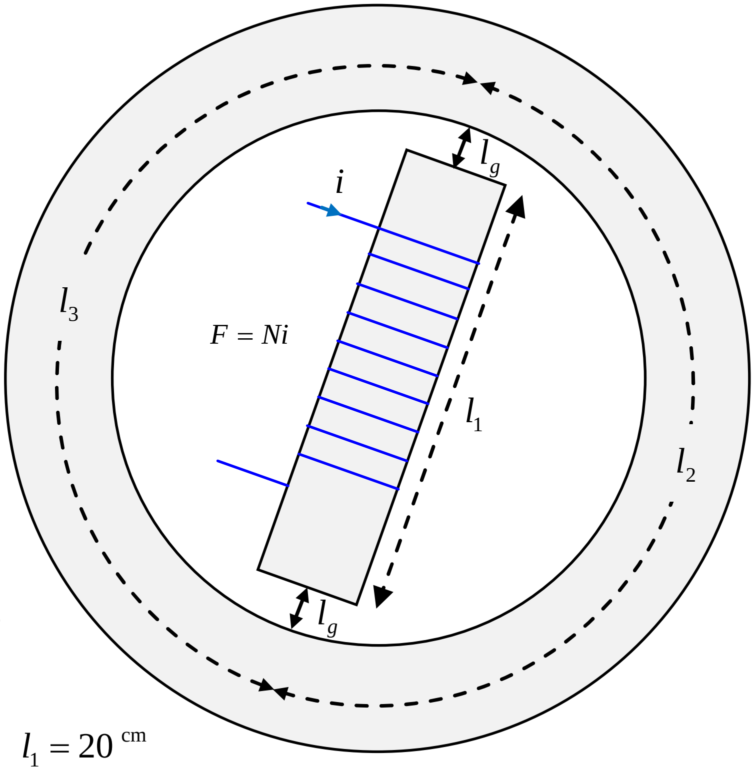




Magnetic Equivalent Circuits

Example 6: In the following electric motor calculate the MMF if the magnetic flux density in the air-gap is 0.8 T. B-H curve is given below.





$l_1 = 20$ cm

$l_2 = l_3 = 30$ cm

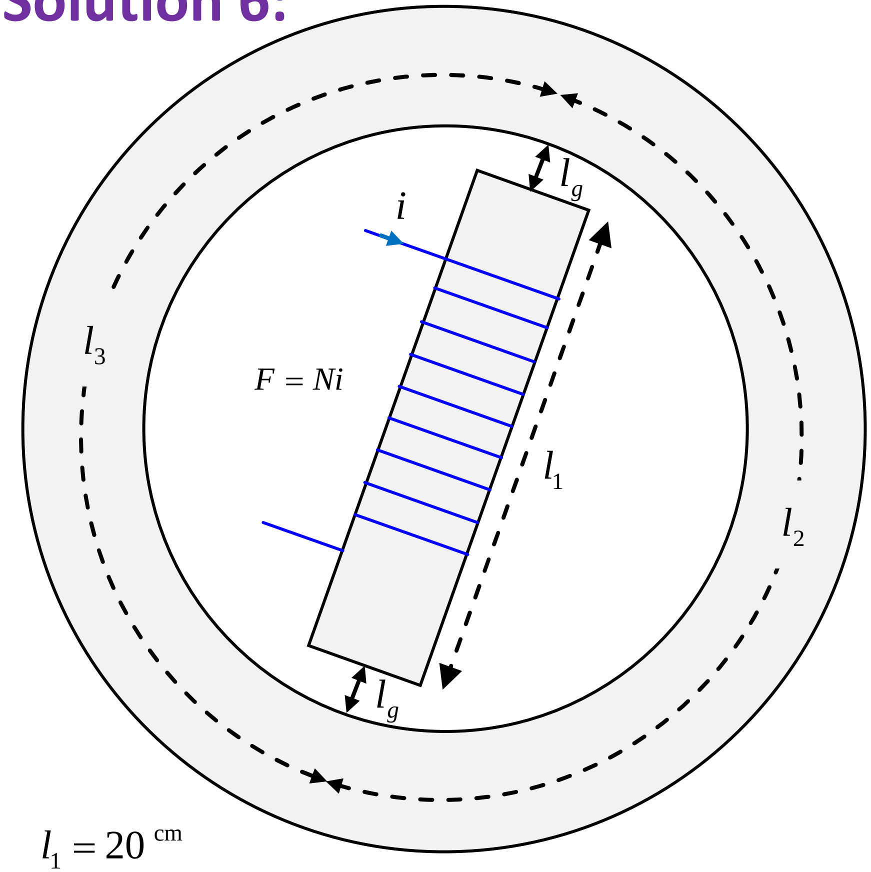
$l_g = 5$ mm

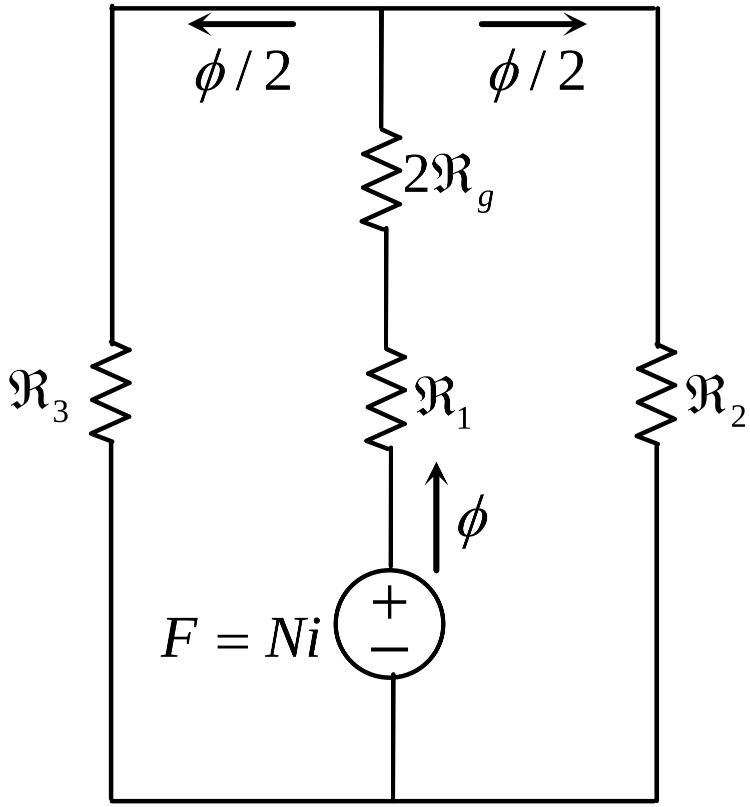
$A_1 = A_g = 200$ cm^2

$A_2 = A_3 = 80$ cm^2

Magnetic Equivalent Circuits

Solution 6:





$l_1 = 20\ \text{cm}$

$l_2 = l_3 = 30\ \text{cm}$

$l_g = 5\ \text{mm}$

$A_1 = A_g = 200\ \text{cm}^2$

$A_2 = A_3 = 80\ \text{cm}^2$

Magnetic Equivalent Circuits

Solution 6: $B_g = 0.8\ \mathrm{T}$ $F = ?$

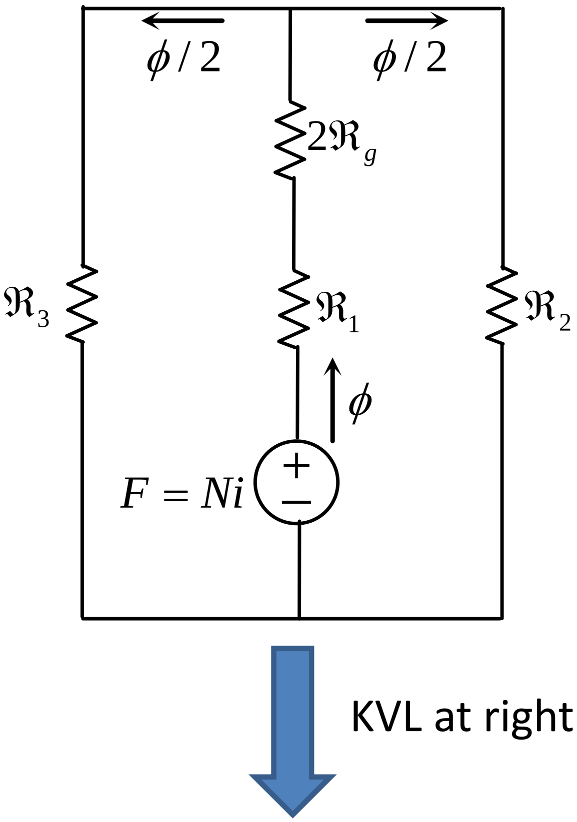
$$\phi_1 = \phi_g$$

$A_1 = A_g$ (Fringing effect is neglected)

Therefore $B_1 = B_g = 0.8\ \mathrm{T}$

From the curve $H_1 = 450\ \mathrm{At/m}$

$$H_g = \frac{B_g}{\mu_0} = \frac{0.8}{4\pi \times 10^{-7}} = 6.37 \times 10^5\ \mathrm{At/m}$$



KVL at right loop

$$F = 2H_g l_g + H_1 l_1 + H_2 l_2$$

Magnetic Equivalent Circuits

Solution 6: $B_g = 0.8 \text{ T}$ $F = ?$

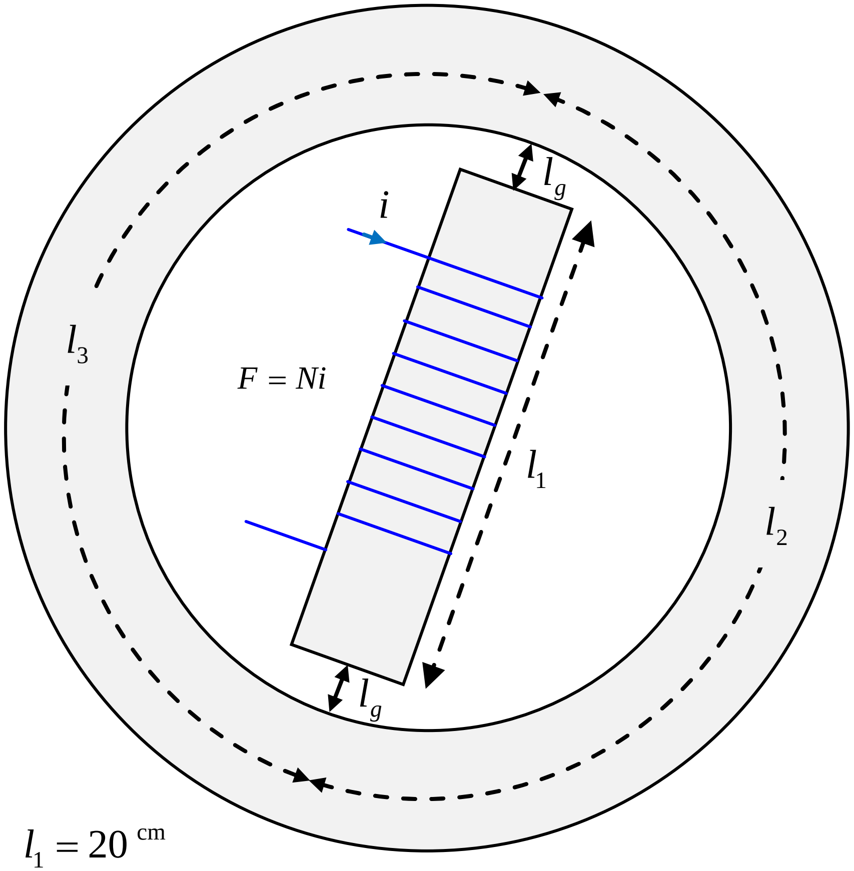
$$\phi_1 = \phi_g = B_g A_g$$
$$= 0.8 \times 200 \times 10^{-4}$$
$$= 1.6 \times 10^{-2} \text{ Wb}$$

$$\phi_2 = \tfrac{1}{2}\phi_1 = 0.8 \times 10^{-2} \text{ Wb}$$

$$B_2 = \frac{\phi_2}{A_2} = \frac{0.8 \times 10^{-2}}{80 \times 10^{-4}} = 1 \text{ T}$$

From the curve $H_2 = 720 \text{ At/m}$

$$F = 2H_g l_g + H_1 l_1 + H_2 l_2 = 6621 \text{ At}$$



$l_1 = 20 \text{ cm}$

$l_2 = l_3 = 30 \text{ cm}$

$l_g = 5 \text{ mm}$

$A_1 = A_g = 200 \text{ cm}^2$

$A_2 = A_3 = 80 \text{ cm}^2$

Magnetic Equivalent Circuits

If the B-H curve is given but both B and H are unknown

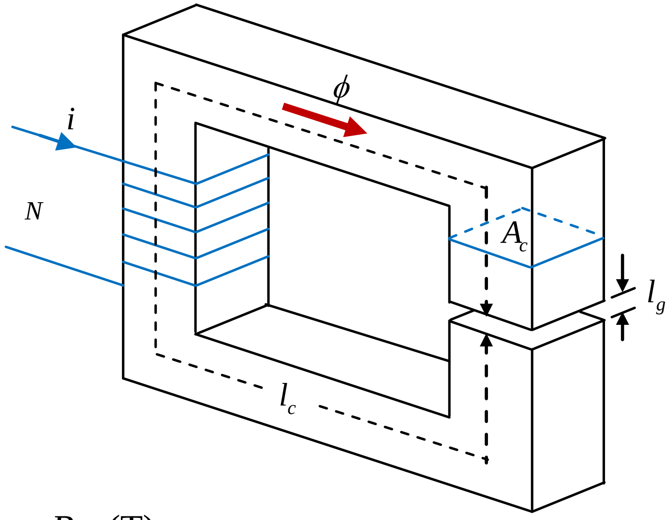
$$F = Ni = H_c l_c + H_g l_g$$

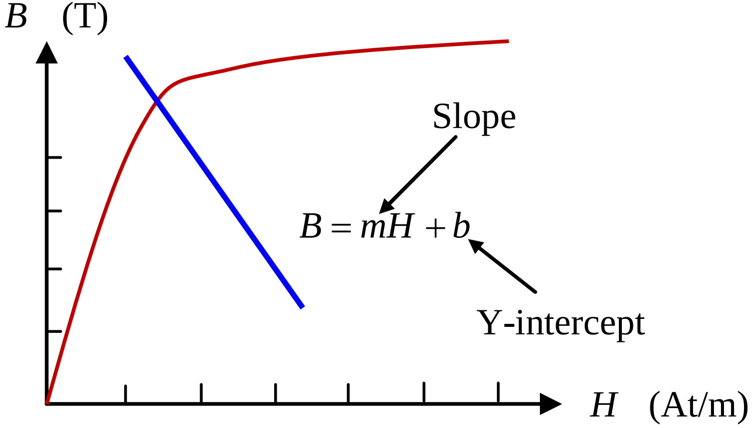
$$Ni = H_c l_c + \frac{B_g}{\mu_0} l_g$$

$$\phi_c = \phi_g \Rightarrow A_c B_c = A_g B_g \Rightarrow B_g = \frac{A_c}{A_g} B_c$$

$$Ni = H_c l_c + \frac{l_g A_c}{\mu_0 A_g} B_c$$

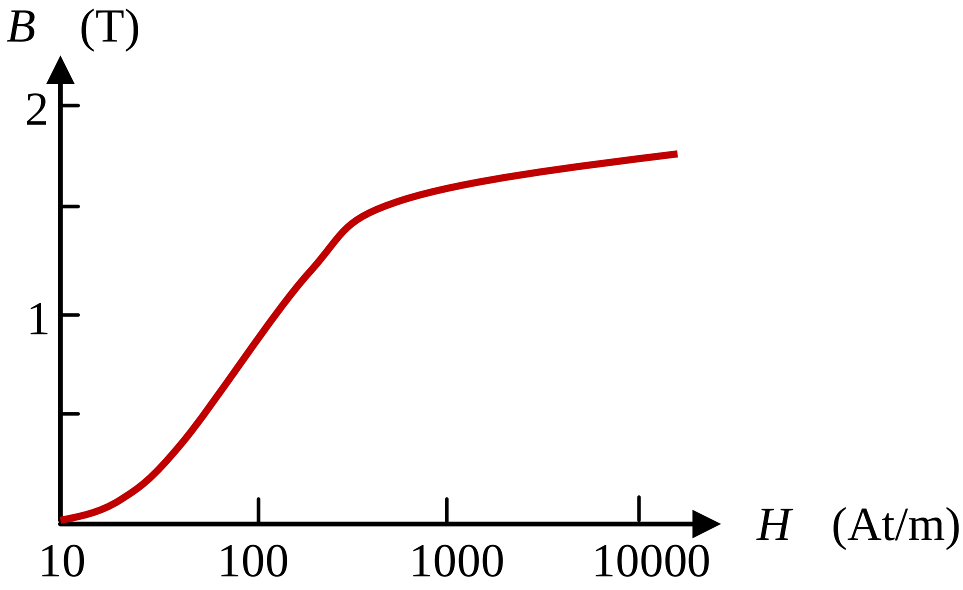
$$B_c = \frac{\mu_0 A_g}{l_g A_c} Ni - \frac{\mu_0 l_c A_g}{l_g A_c} H_c$$

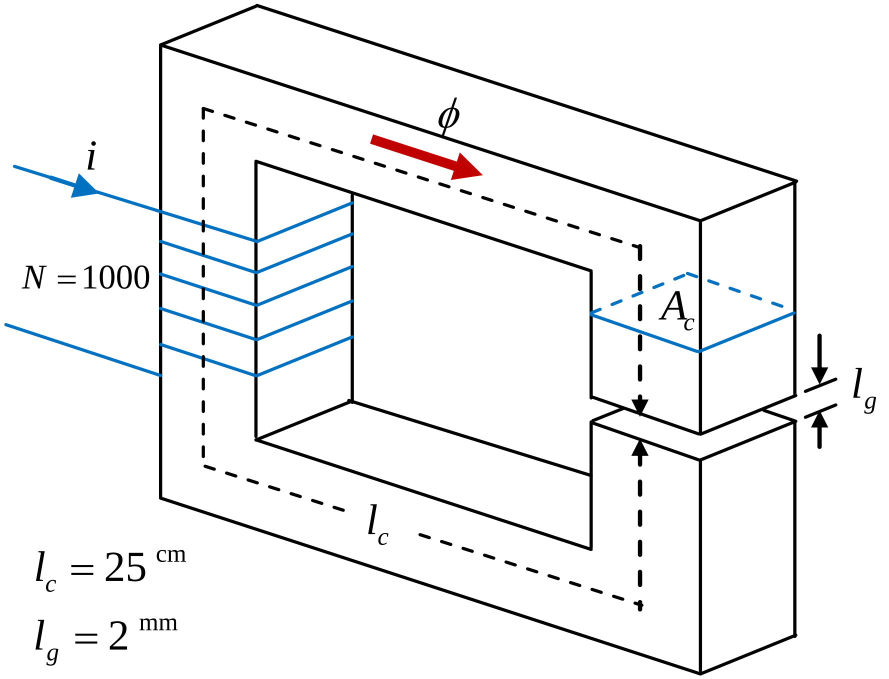




Magnetic Equivalent Circuits

Example 7: In the following electromagnetic system calculate the magnetic flux density if the current is 4 A. B-H curve is given below.



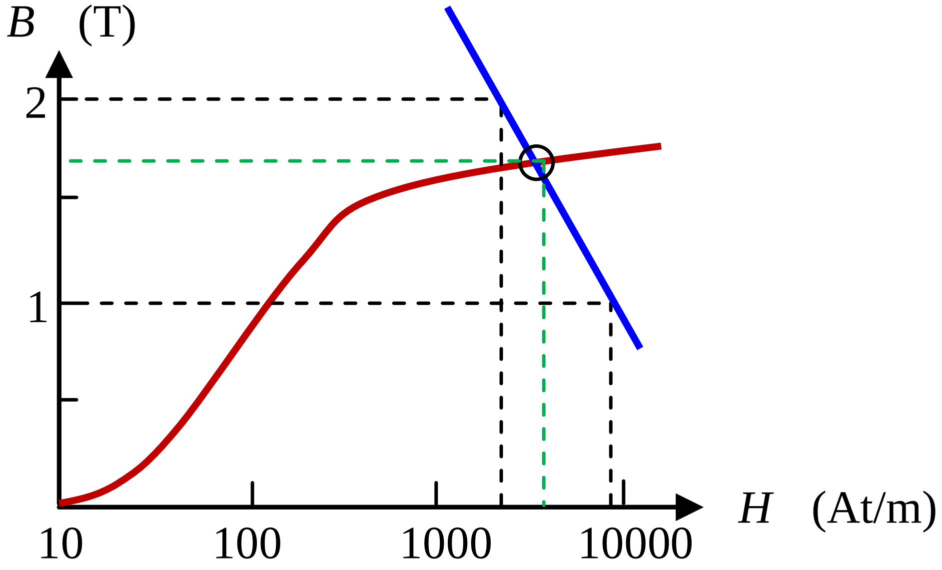


Magnetic Equivalent Circuits

Solution 7:

Fringing effect is neglected

so $B_g = B_c$



$$Ni = H_c l_c + H_g l_g$$

$$Ni = H_c l_c + \frac{B_g}{\mu_0} l_g$$

$$H_c = \frac{Ni}{l_c} - \frac{l_g}{\mu_0 l_c} B_c$$

$$H_c = 16000 - 6366 B_c$$

$$\begin{cases} if \;\; B_c = 1 \;\; \Rightarrow \;\; H_c = 9634 \\ if \;\; B_c = 2 \;\; \Rightarrow \;\; H_c = 3268 \end{cases}$$

$$B_c = 1.6 \text{ T} \quad \text{and} \quad H_c = 5800 \text{ At/m}$$

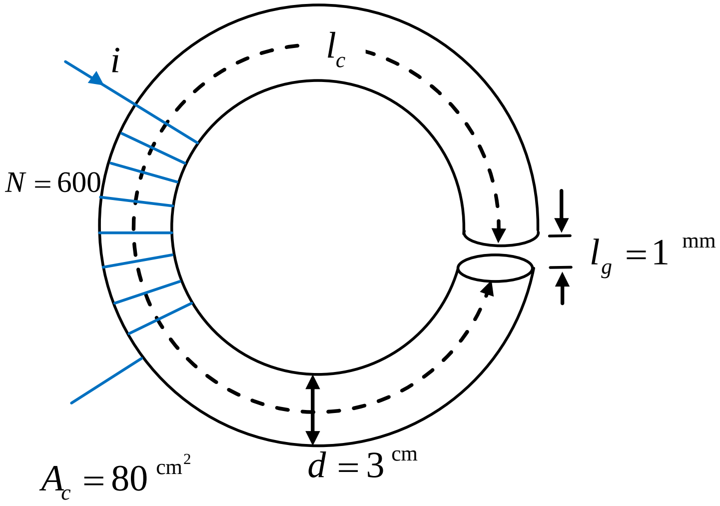
Magnetic Equivalent Circuits

Example 8: A steel loop with diameter of 3 cm and average length of 80 cm is shown below. If the air-gap length is 1 mm and N=600

a) Calculate the current if the magnetic flux in the air-gap is 0.75 mWb

b) Calculate the magnetic flux if the current is 2 A.

H (At/m)	200	400	600	800	1000	1200	1400	1600	1800	2020
B (T)	0.1	0.32	0.6	0.9	1.08	1.18	1.27	1.32	1.36	1.4



Fringing effect is neglected.

Magnetic Equivalent Circuits

Solution 8:

Part a) $\phi = 0.75 \text{ mWb}$ $i = ?$

$$\phi = B_g A_g \Rightarrow B_g = \frac{0.75\times10^{-3}}{\pi\left(1.5\times10^{-2}\right)^2} = 1.06 \text{ T}$$

Fringing effect is neglected, so $B_c = B_g = 1.06 \text{ T}$

From the curve $H_c = 900 \text{ At/m}$

$$Ni = H_c l_c + H_g l_g$$

$$Ni = H_c l_c + \frac{B_g}{\mu_0} l_g$$

$$i = \frac{1}{N}\left(H_c l_c + \frac{B_g}{\mu_0} l_g\right) = 2.6 \text{ A}$$

Magnetic Equivalent Circuits

Solution 8:

Part b) $i = 2 \ \text{A}$ $\phi = ?$

$$Ni = H_c l_c + \frac{B_g}{\mu_0} l_g$$

Fringing effect is neglected, so $B_c = B_g$

$$Ni = H_c l_c + \frac{B_c}{\mu_0} l_g$$

$$H_c = 1500 - 995 B_c \qquad \begin{cases} if \ \ B_c = 0 \ \Rightarrow \ H_c = 1500 \\ if \ \ H_c = 0 \ \Rightarrow \ B_c = 1.5 \end{cases}$$

From the curve $B_c = 0.78 \ \text{T}$ $\phi = B_c A_c = 0.55 \ \text{mWb}$

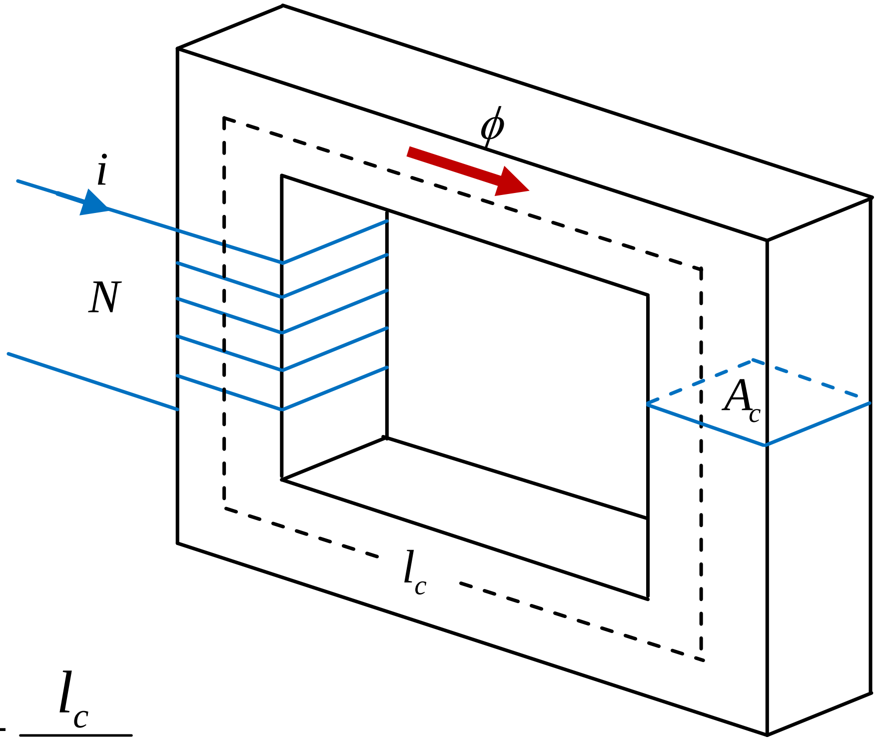
Inductance & Flux Linkage

Consider the following electromagnetic system

- The flux linkage is defined as

$$\lambda = N\phi$$

- The inductance can be found as



$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{NA_cB}{i}$$

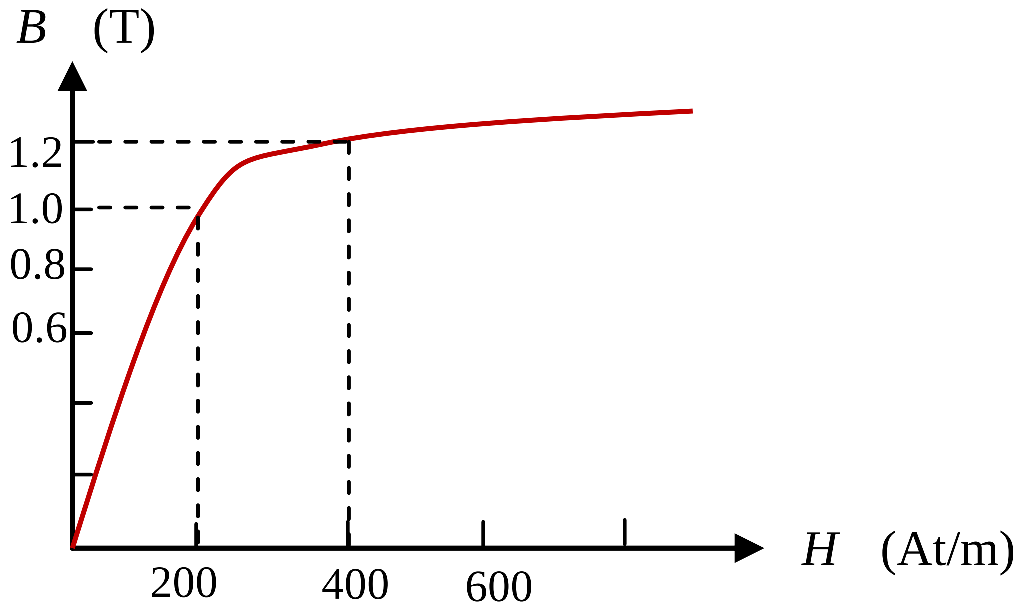
$$= \frac{NA_c\mu H}{i} = \frac{NA_c\mu H}{Hl_c / N}$$

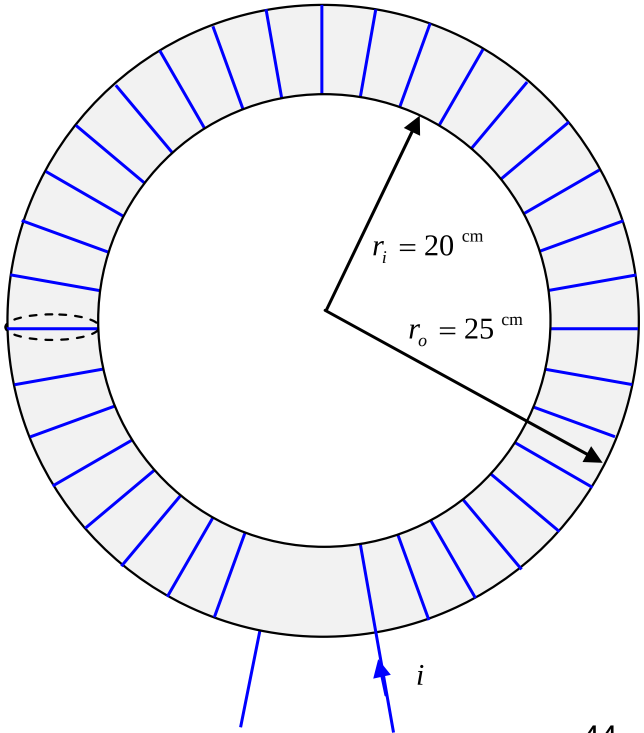
$$= \frac{N^2}{\dfrac{l_c}{A_c\mu}} = \frac{N^2}{\Re}$$

$$\Re = \frac{l_c}{\mu A_c}$$

Magnetic Equivalent Circuits

Example 9: In the following Electromagnetic circuit N=250, the inner and outer radii of the toroid are 20 and 25 cm respectively. The cross sectional area is circular. If the current is 2.5 A, calculate the flux density and the inductance.





Magnetic Equivalent Circuits

Solution 9:

Part a) $i = 2.5\ \text{A}$ $B = ?$

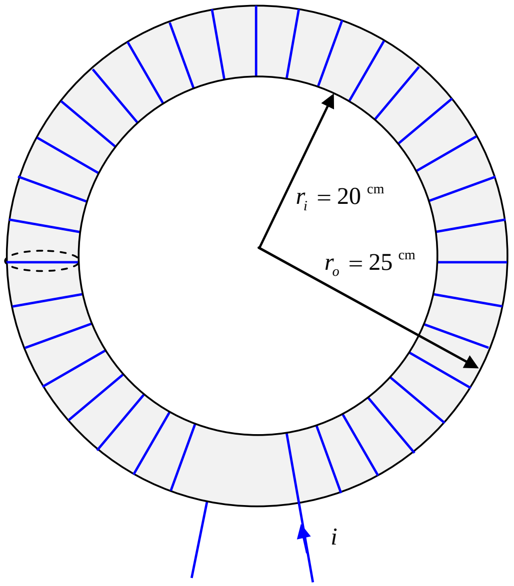
The average radius is calculated as:

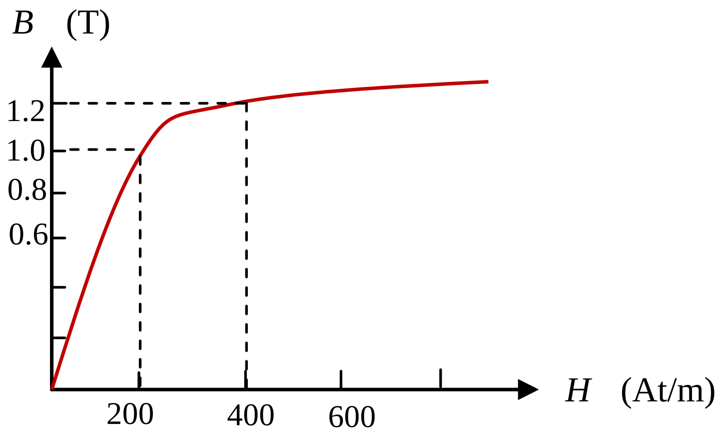
$$r_{ave} = \frac{r_i + r_o}{2} = \frac{20+25}{2} = 22.5\ \text{cm}$$

The magnetic field intensity is obtained

$$H = \frac{Ni}{l} = \frac{250\times 2.5}{2\pi\times 22.5\times 10^{-2}} = 442\ \ \text{At/m}$$

From the curve $B = 1.22\ \ \text{T}$





Magnetic Equivalent Circuits

Solution 9: $N = 250$

Part b) $i = 2.5\ \text{A}$ $L = ?$

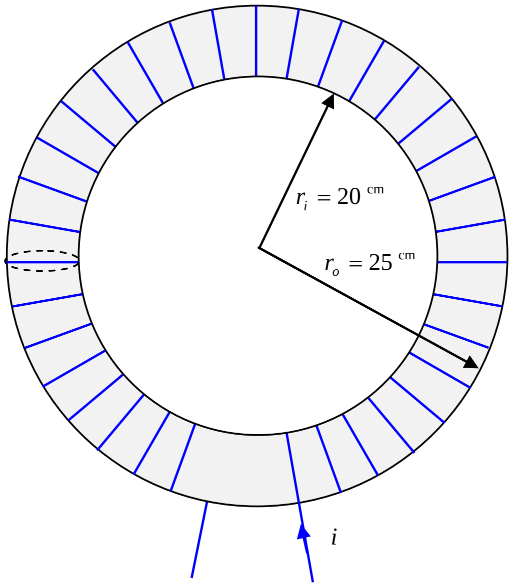
1st method

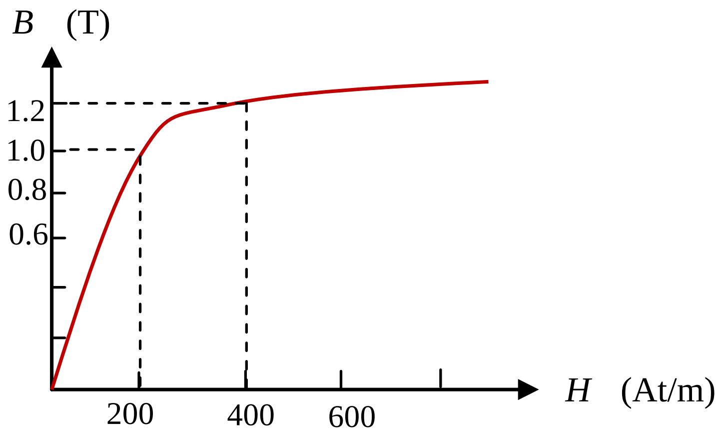
$$B = 1.22\ \text{T}$$

$$A = \pi\left(\frac{25-20}{2}\times10^{-2}\right)^2\ \ \text{m}^2$$

$$L = \frac{\lambda}{i} = \frac{N\phi}{i} = \frac{NAB}{i}$$

$$L = 240\ \text{mH}$$





Magnetic Equivalent Circuits

Solution 9:

Part b)

2nd method

$$N = 250$$

$$i = 2.5\ \text{A} \qquad L = ?$$

$$B = 1.22\ \text{T}$$

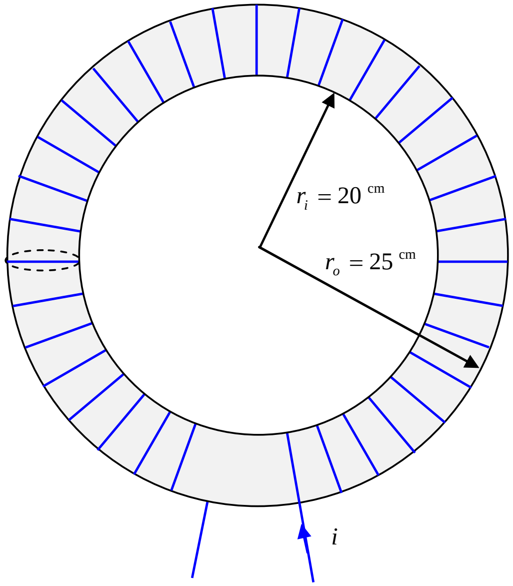
$$A = \pi\left(\frac{25-20}{2}\times 10^{-2}\right)^2\ \ \text{m}^2$$

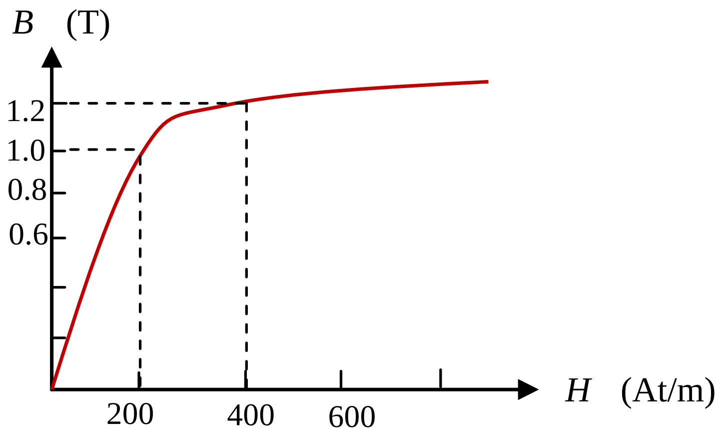
$$\mu = \frac{B}{H} = \frac{1.22}{442} = 2.76\times 10^{-3}\ \ \text{H/m}$$

$$\Re = \frac{l}{\mu A} = 259900\ \text{At/Wb}$$

$$L = \frac{N^2}{\Re}$$

$$L = 240\ \text{mH}$$



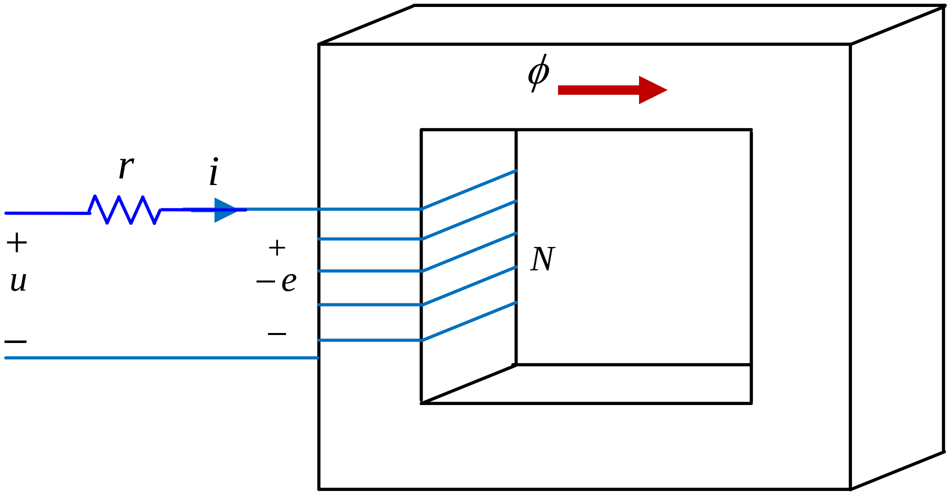


Stored Energy in Electromagnetic Systems

Consider the following electromagnetic system

- Based on Faraday's law

$$e_{ind} = -N\frac{d\phi}{dt}$$



- Writing KVL yields

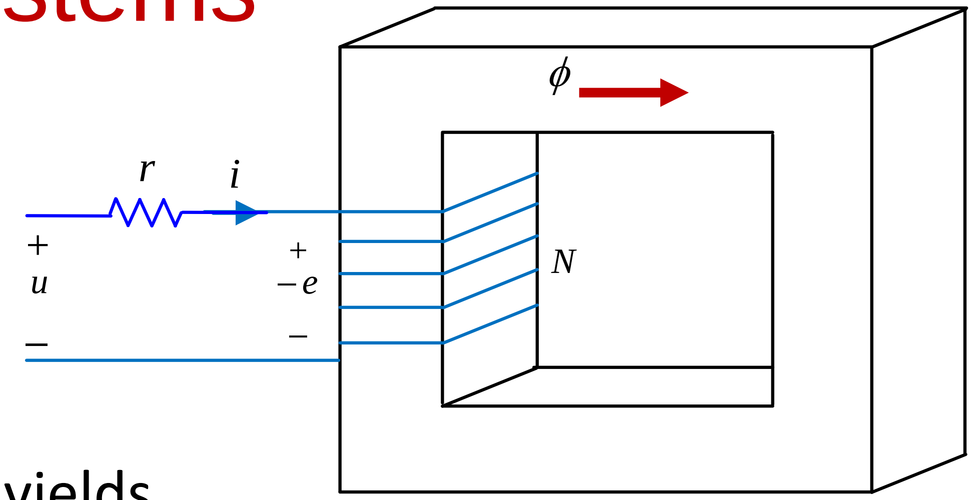
$$u = ri + N\frac{d\phi}{dt}$$

- Multiplying both sides by i results

$$ui = ri^2 + Ni\frac{d\phi}{dt}$$

Stored Energy in Electromagnetic Systems

$$P = ui = ri^2 + Ni\frac{d\phi}{dt}$$

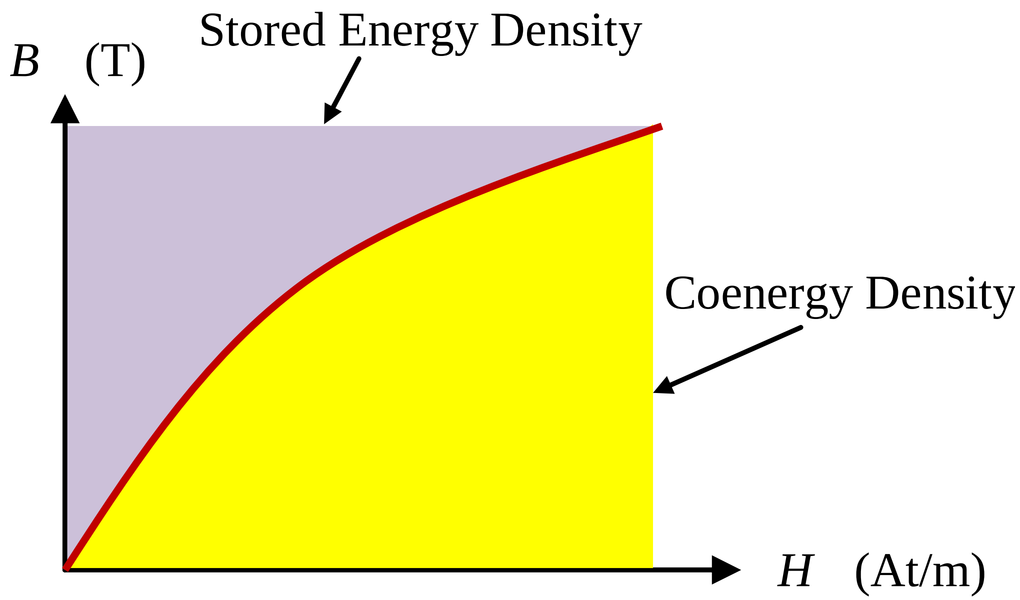


- Multiplying both sides by *dt* yields

$$dW = Pdt = \underbrace{uidt}_{\text{Electrical input energy}} = \underbrace{ri^2dt}_{\text{Ohmic losses}} + \underbrace{Ni\,d\phi}_{\text{Magnetic stored energy}}$$

Stored Energy in Electromagnetic Systems

$$\underbrace{uidt}_{\text{Electrical input energy}} = \underbrace{ri^2dt}_{\text{Ohmic losses}} + \underbrace{Ni\,d\phi}_{\text{Magnetic stored energy}}$$



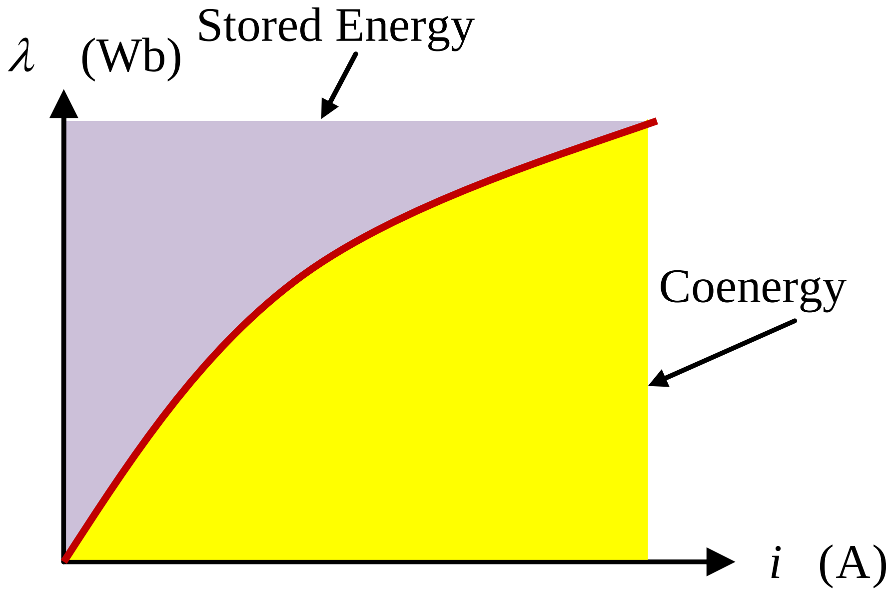
$$\boxed{dW_f = Ni\,d\phi}$$

$$\begin{cases} Ni = Hl \\ \phi = AB \end{cases} \Rightarrow dW_f = HlAdB \Rightarrow \boxed{W_f = V\int_{B_1}^{B_2} H.dB}$$

where V is the volume of the core.

Stored Energy in Electromagnetic Systems

$$\underbrace{uidt}_{\text{Electrical input energy}} = \underbrace{ri^2dt}_{\text{Ohmic losses}} + \underbrace{Ni\,d\phi}_{\text{Magnetic stored energy}}$$



$$dW_f = Ni\,d\phi$$

$$\{\lambda = N\phi \;\Rightarrow\; dW_f = id\lambda \;\Rightarrow$$

$$W_f = \int_{\lambda_1}^{\lambda_2} i.d\lambda$$

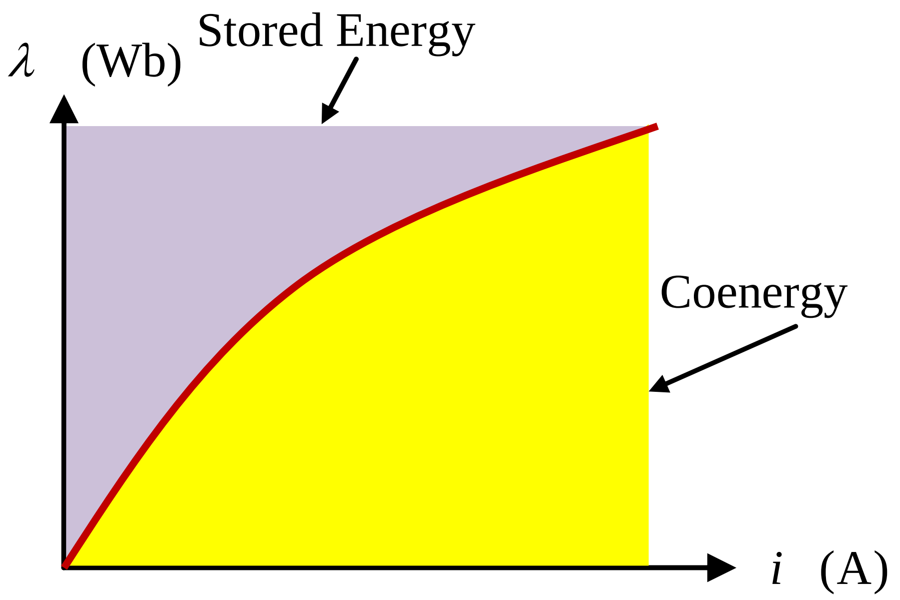
Stored Energy in Electromagnetic Systems

$$uidt = ri^2dt + Ni\,d\phi$$

Electrical input energy

Ohmic losses

Magnetic stored energy



$$\boxed{dW_f = Ni\,d\phi}$$

$$\{Ni = \Re\phi \;\Rightarrow\; dW_f = \Re\phi\,d\phi \;\Rightarrow$$

$$\boxed{W_f = \int_{\phi_1}^{\phi_2} \Re\phi\,d\phi}$$

Stored Energy in Electromagnetic Systems

If the system is **linear** or working in the linear region $\mu = \text{constant}$

$$\left\{\begin{array}{l} W_f = V\int_0^B H.dB \\ H = \dfrac{B}{\mu} \end{array}\right. \Longrightarrow W_f = V\int_0^B \frac{B}{\mu}dB \Longrightarrow \boxed{W_f = \tfrac{1}{2}\frac{VB^2}{\mu}}$$

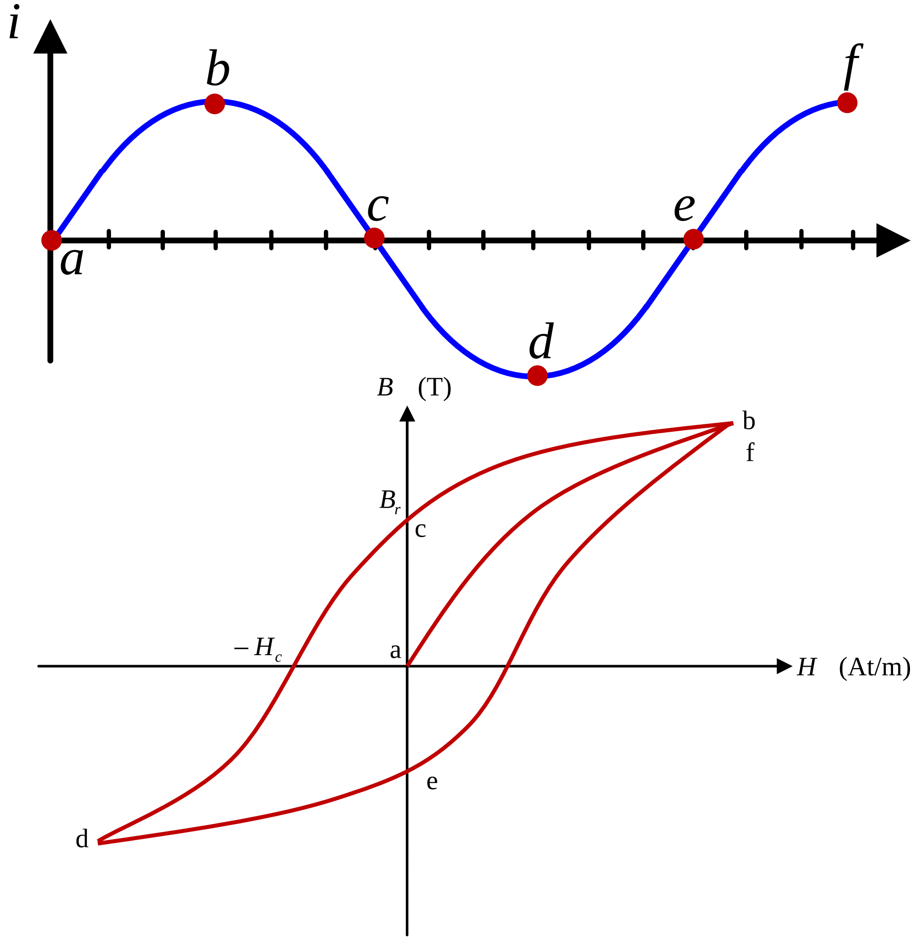
$$\left\{\begin{array}{l} W_f = \int_0^\lambda i\,d\lambda \\ \lambda = Li \end{array}\right. \Longrightarrow W_f = L\int_0^i i\,di \Longrightarrow \boxed{W_f = \tfrac{1}{2}Li^2}$$

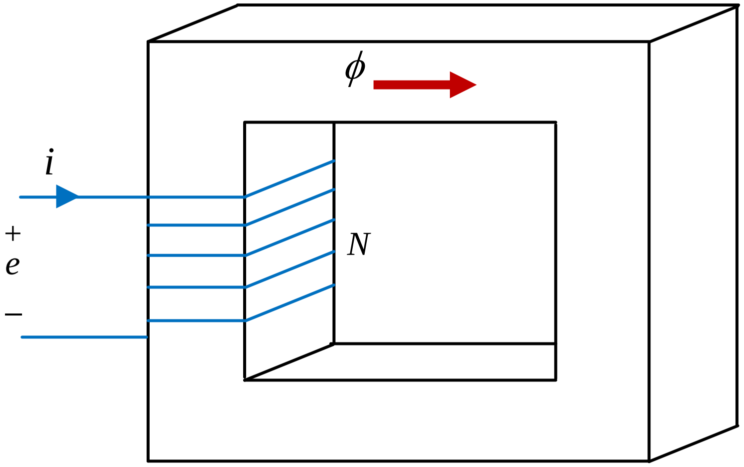
$$\left\{\begin{array}{l} W_f = \int_0^\phi \Re\phi\,d\phi \\ \Re = \text{constant} \end{array}\right. \Longrightarrow W_f = \Re\int_0^\phi \phi\,d\phi \Longrightarrow \boxed{W_f = \tfrac{1}{2}\Re\phi^2}$$

Magnetic Losses

Consider the following electromagnetic system

- Assume instead of dc current an ac current is applied





B_r	Residual flux density
H_c	Coercive force

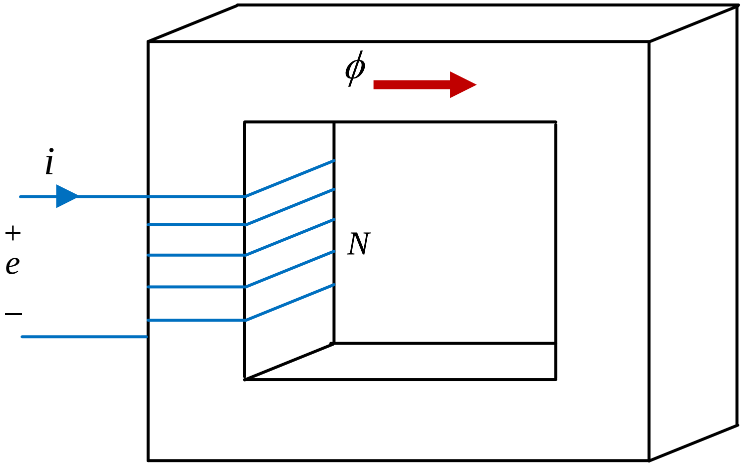
Hysteresis Losses

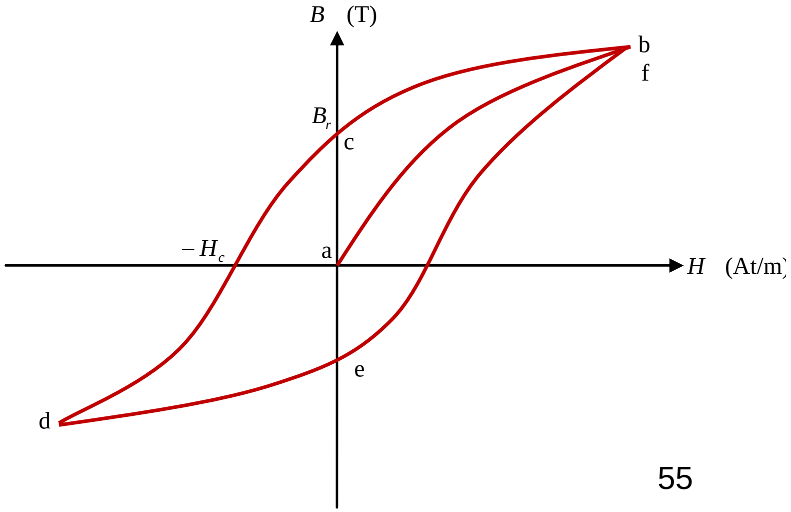
Hysteresis losses are due to residual flux in the ferromagnetic core and defined as:

$$P_h = k_h f B_{\max}^n \qquad 1.5 \le n \le 2.5$$

where

n	Steinmetz constant
f	frequency
$B_{\max}$	maximum flux density
k_h	constant depends on the type and volume of the core

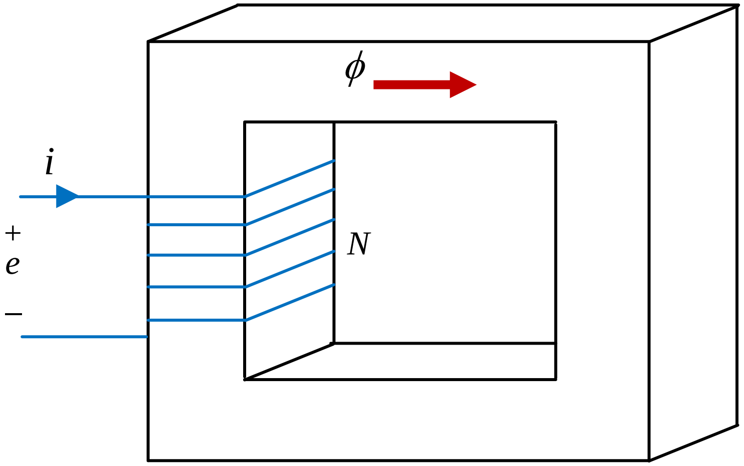




Eddy Current Losses

Eddy current losses are due to current circulating in the ferromagnetic core and defined as:

$$P_e = k_e f^2 B_{\max}^2$$



where

f frequency

$B_{\max}$ maximum flux density

k_e constant depends of the type and thickness of the core

The total core (magnetic) losses are defined as

$$P_c = P_h + P_e$$

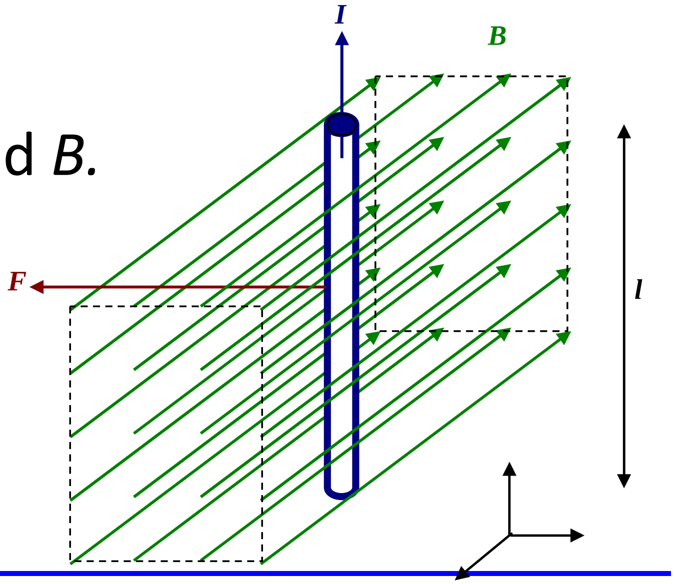
Electric Motor Principle

- Based on the principle theory of electromechanical systems, if a current-carrying conductor is located in a magnetic field, a force is exerted on the conductor.
- The force, F, is directly proportional to the current I, magnetic field density B and the length of the conductor l and according to the Lorentz law, the force can be expressed as follow,

$$F = Il \times B \Longrightarrow |F| = IlB\sin(\theta)$$

θ is the angle between the conductor and *B.*

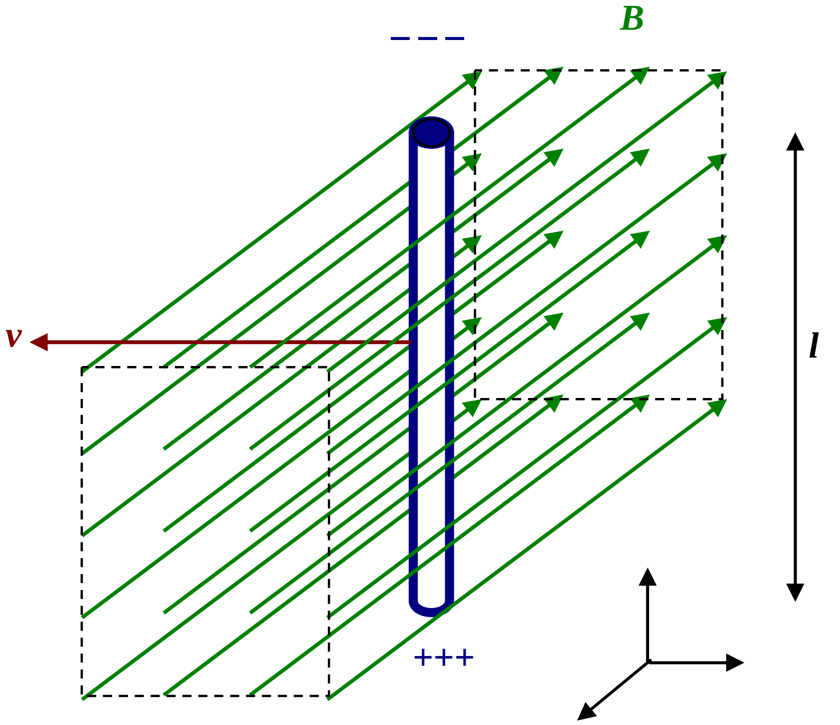
	Right hand	Left hand
Index finger	I	B
Middle finger	B	I
Thumb	F	F



Electric Generator Principle

- Another fundamental theory of electromechanical systems says, if a conductor moves with speed *v* inside a magnetic field with density *B*, a voltage *E* will be induced across the conductor which is expressed as,

$$E = (v \times B) \cdot l$$



Electric Generator Principle

- Example: In the following figure if *v*=0.5 m/s, *B*=0.5 T and *l*=10 m, calculate the induced voltage

$$E = \left(v \times B\right) \cdot l = vBl \sin(90) \cos(0) = 2.5 \quad \text{V}$$