In The Name of God The Most Compassionate, The Most Merciful

Electric Machines II

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Chapter 2
Principle of AC Machines

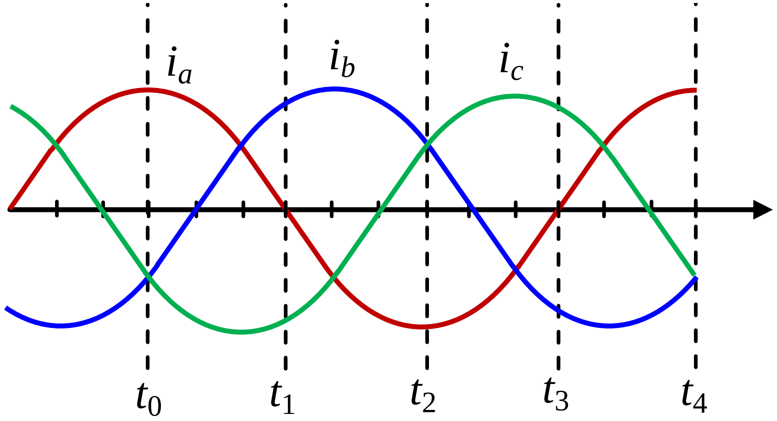
Rotating Magnetic Field

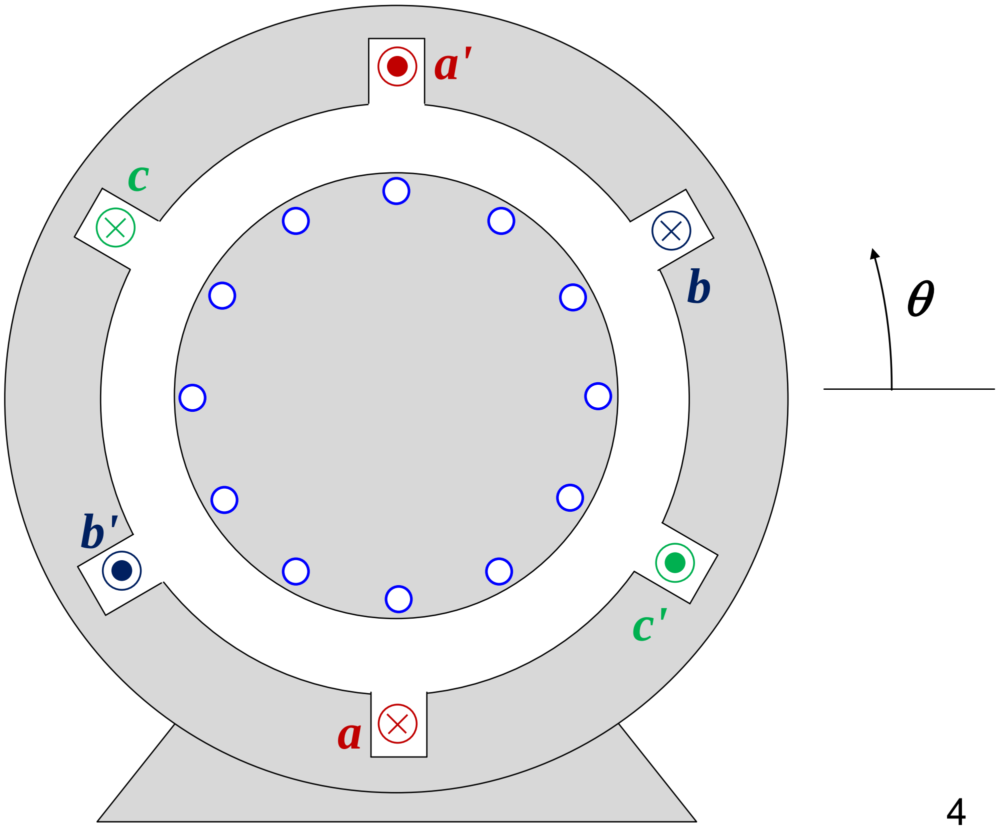
Consider the following **3-phase AC** machine with **2 poles** and **concentrated winding**. The 3-phase AC currents can be expressed as:

$$i_a(t) = I_m \cos \omega t$$

$$i_b(t) = I_m \cos(\omega t - 120^\circ)$$

$$i_c(t) = I_m \cos(\omega t + 120^\circ)$$



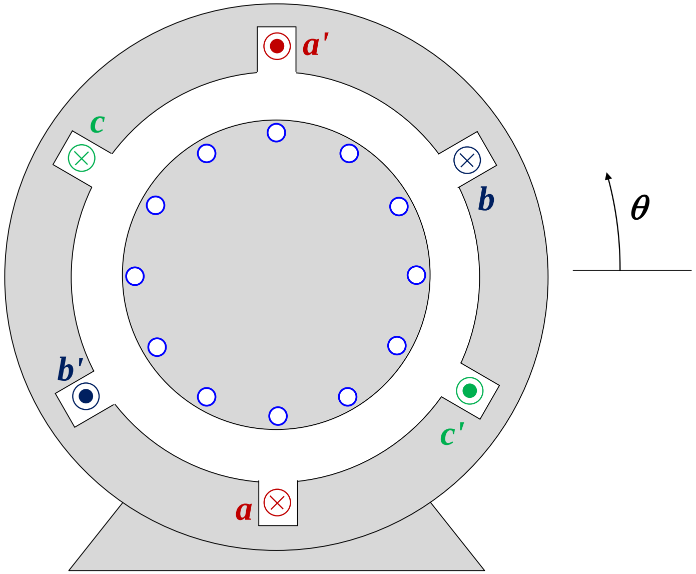


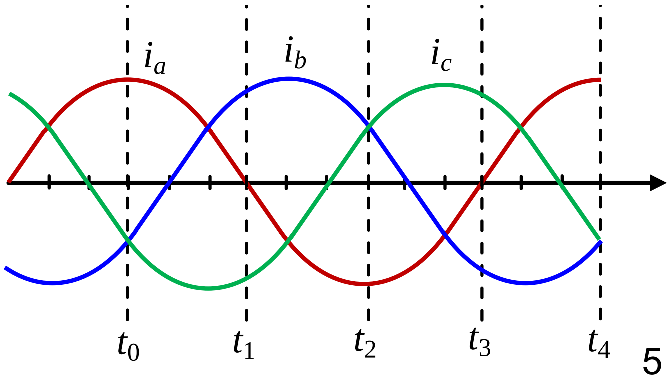
Rotating Magnetic Field

The rotating magnetomotive force (MMF) can be expressed as

$$\vec{F}(\theta,t)=\vec{F}_a(\theta,t)+\vec{F}_b(\theta,t)+\vec{F}_c(\theta,t)$$

$$\vec{F}(\theta,t)=Ni_a(t)\cos\theta+Ni_b(t)\cos(\theta-120^\circ)+Ni_c(t)\cos(\theta+120^\circ)$$





Rotating Magnetic Field

The rotating magnetomotive force (MMF) can be expressed as

$$\vec{F}(\theta,t) = Ni_a(t)\cos\theta + Ni_b(t)\cos(\theta - 120^\circ) + Ni_c(t)\cos(\theta + 120^\circ)$$

$$\begin{aligned}\vec{F}(\theta,t) = {} & NI_m \cos\omega t \cos\theta \\ & + NI_m \cos(\omega t - 120^\circ)\cos(\theta - 120^\circ) \\ & + NI_m \cos(\omega t + 120^\circ)\cos(\theta + 120^\circ)\end{aligned}$$

$$\boxed{\vec{F}(\theta,t) = \tfrac{3}{2} NI_m \cos(\omega t - \theta)}$$

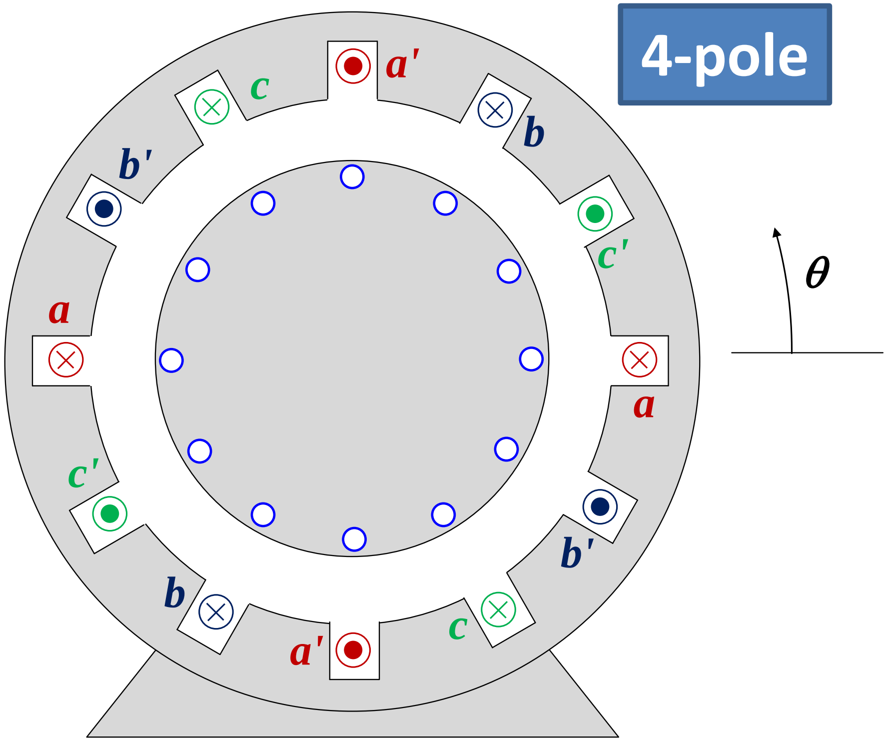
$$\cos a \cos b = \tfrac{1}{2}\cos(a-b) + \tfrac{1}{2}\cos(a+b)$$

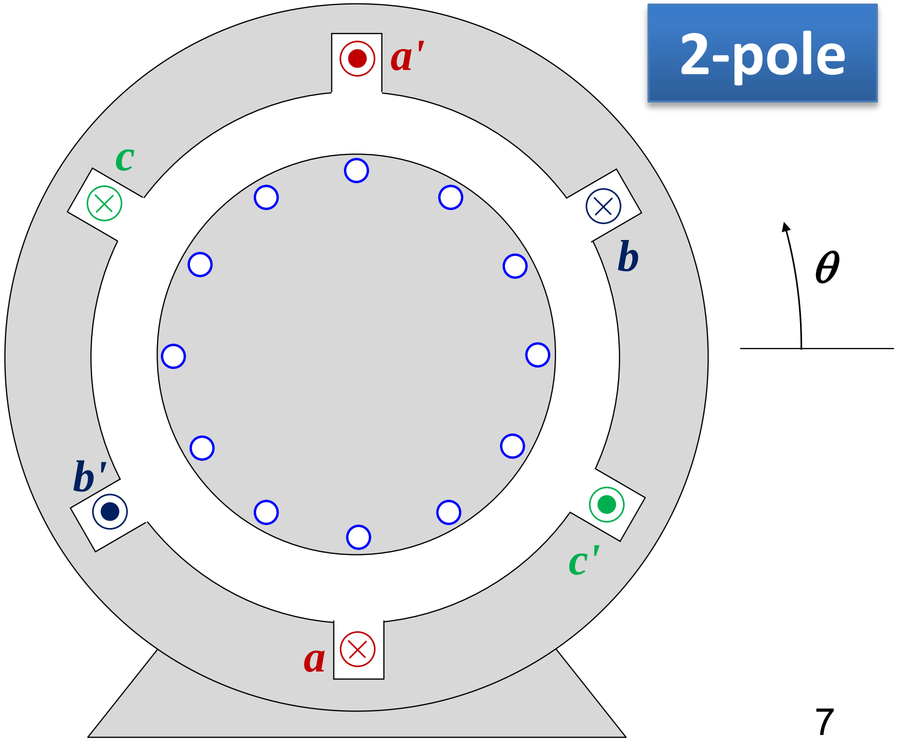
Electrical & Mechanical Angles

Consider an AC machine with p poles.

$$\theta_{elec} = \frac{p}{2}\theta_{mech}$$

$$\omega_{elec} = \frac{p}{2}\omega_{mech}$$

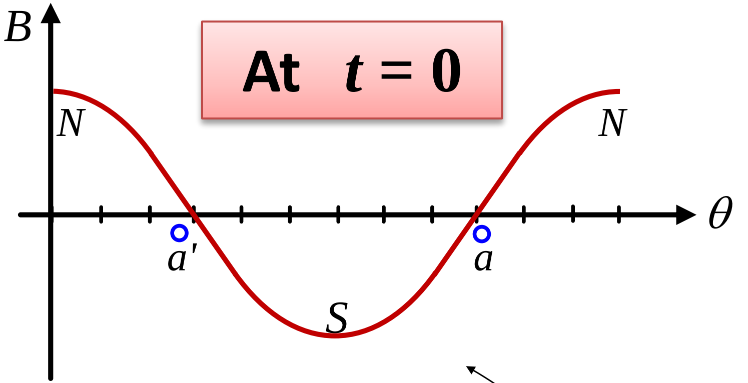




Induced Voltage

Consider an AC machine. Ideally the flux density seen by coil *aa'* is expressed as

$$B_a(t,\theta) = B_m \cos\theta \cos\omega t$$



At $t = 0$

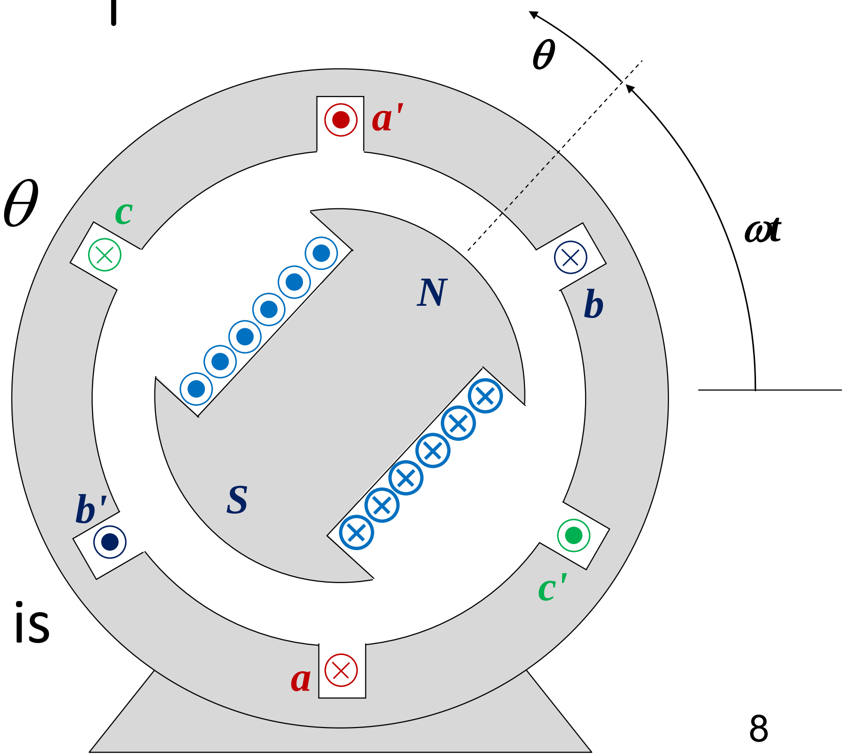
The corresponding air-gap flux is

$$\phi(t) = \int_A B \cdot dA \Rightarrow \phi_a(t) = \int_0^L \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} B_a \cdot r dl d\theta$$

$$\boxed{\phi_a(t) = 2LrB_m \cos\omega t}$$

$$\phi_a(t) = \phi_m \cos\omega t \qquad \phi_m = 2LrB_m$$

where L is the stator axial length and r is the stator inner radius



Induced Voltage

$$\phi_a(t) = \phi_m \cos\omega t \qquad \phi_m = 2LrB_m$$

The induced voltage in coil *aa'* with N turns is

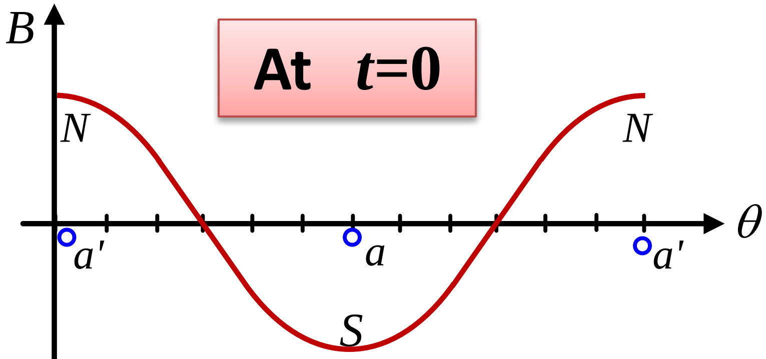
$$e_a(t) = -N\frac{d\phi_a}{dt}$$

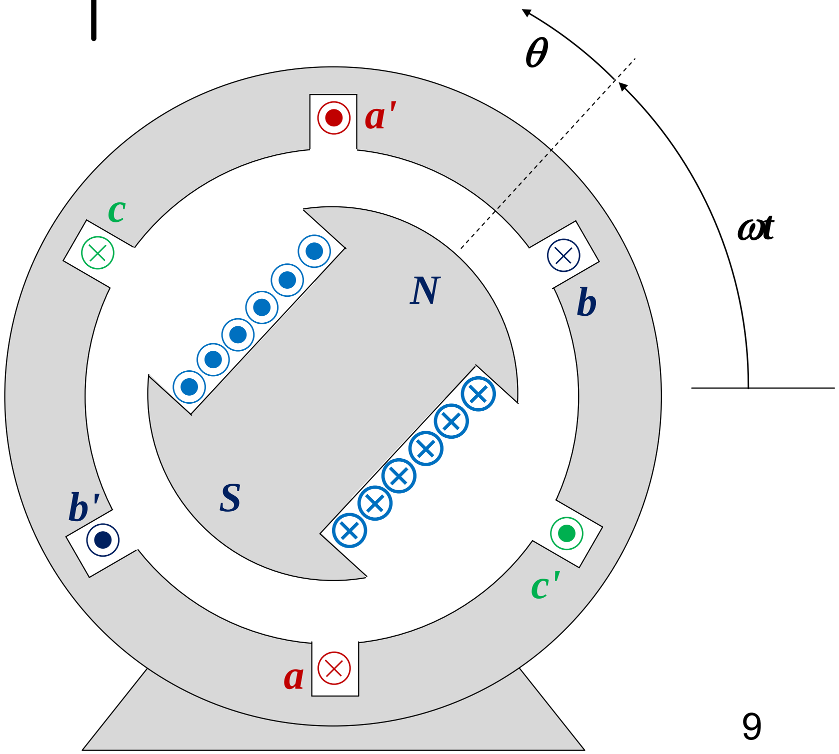
$$\Rightarrow \quad e_a(t) = N\phi_m\omega\sin\omega t$$

$$e_b(t) = N\phi_m\omega\sin(\omega t - 120^\circ)$$

$$e_c(t) = N\phi_m\omega\sin(\omega t + 120^\circ)$$

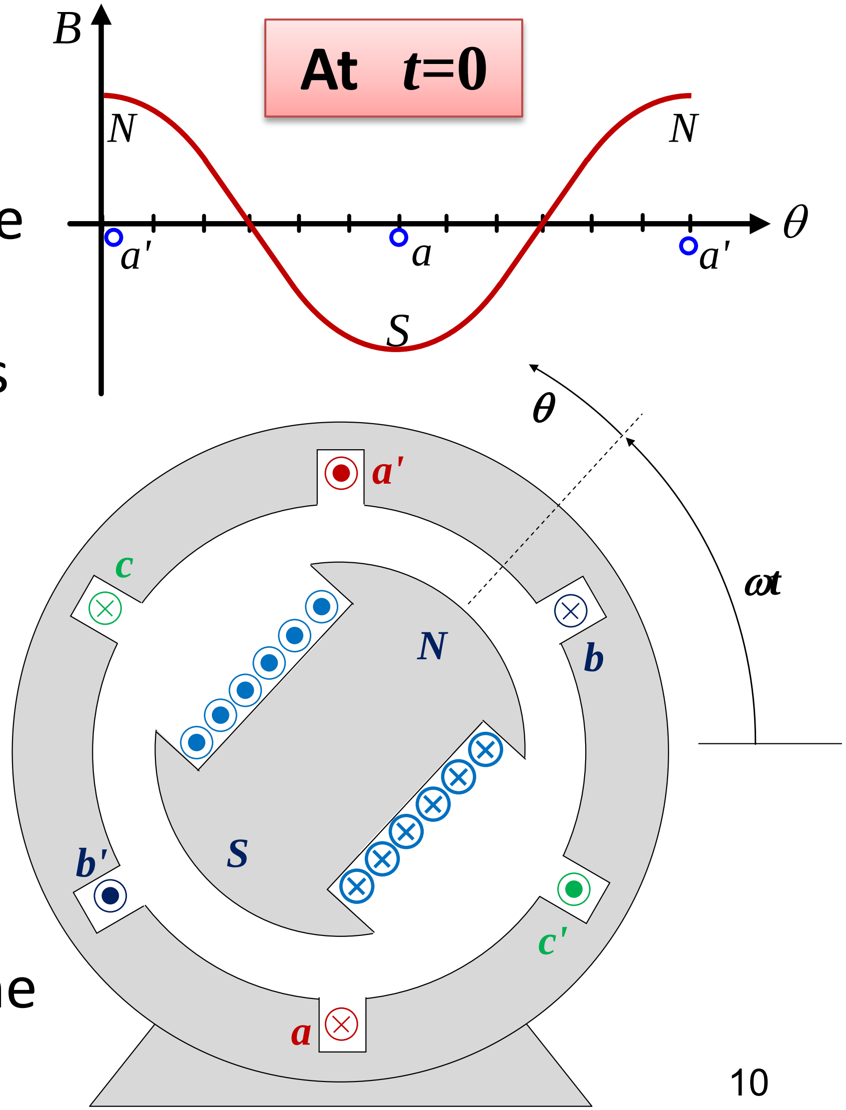
$$\boxed{E_{rms} = \frac{N\phi_m\omega}{\sqrt{2}}}$$


At t=0



Induced Voltage

$$E_{rms} = \frac{N\phi_m \omega}{\sqrt{2}} \Rightarrow E_{rms} = \frac{N\phi_m 2\pi f}{\sqrt{2}}$$



The RMS induced voltage in each phase having N turns in series with **concentrated** and **full pitch** windings is

$$E_{rms} = 4.44 N \phi_m f$$

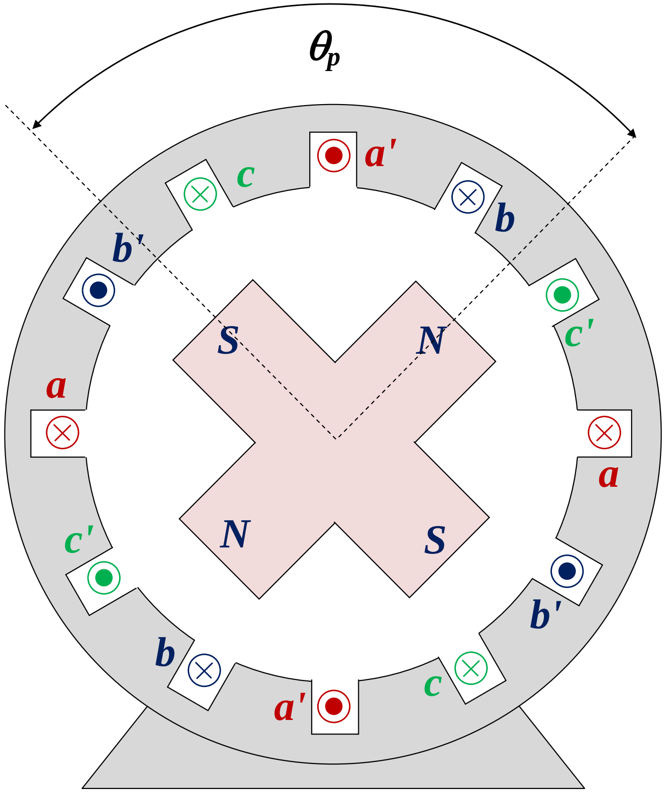
For machines with **distributed** and/or **short pitch** windings the RMS induced voltage is

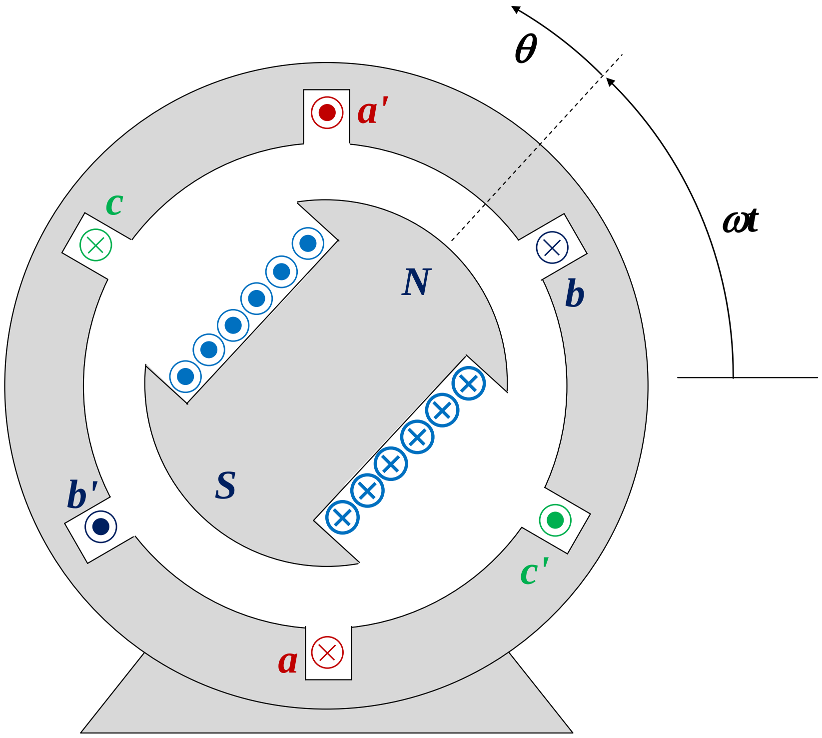
$$E_{rms} = 4.44 N \phi_m f \, k_p k_d$$

Where k_p is the pitch factor and k_d is the distribution factor.

Pole Pitch

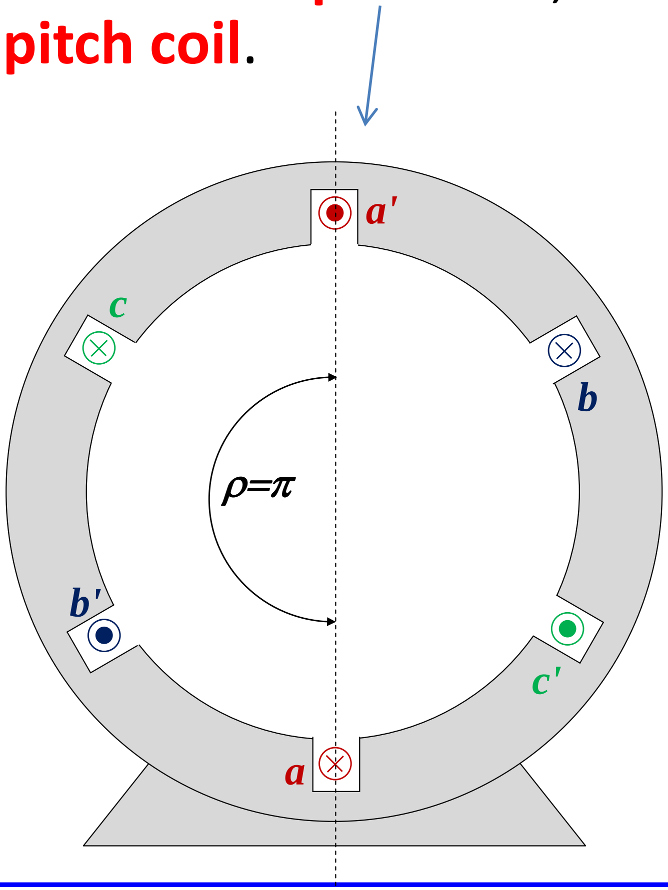
- **Pole pitch**: is the angle between two adjacent poles in electrical angle. Pole pitch is always 180 electrical degrees regardless of the number of poles.

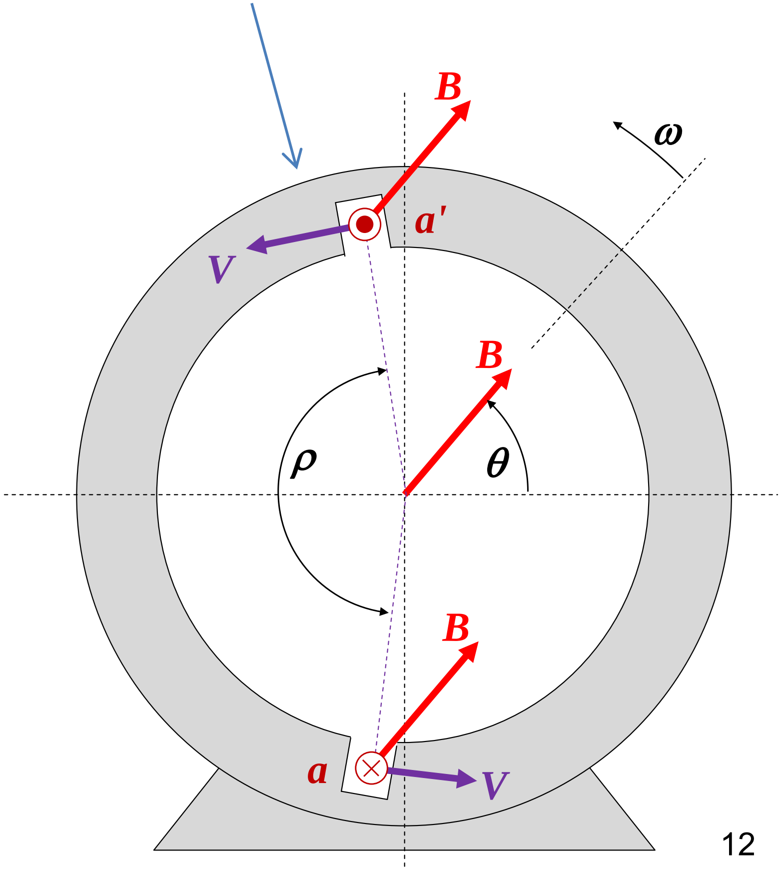




Coil Pitch

- **Coil pitch**: is the angle between two sides of one armature coil in electrical angle. If the coil pitch is 180 electrical degrees, the coil is a **full-pitch coil**; otherwise it is called **short**- or **chorded-pitch coil**.





Pitch Factor

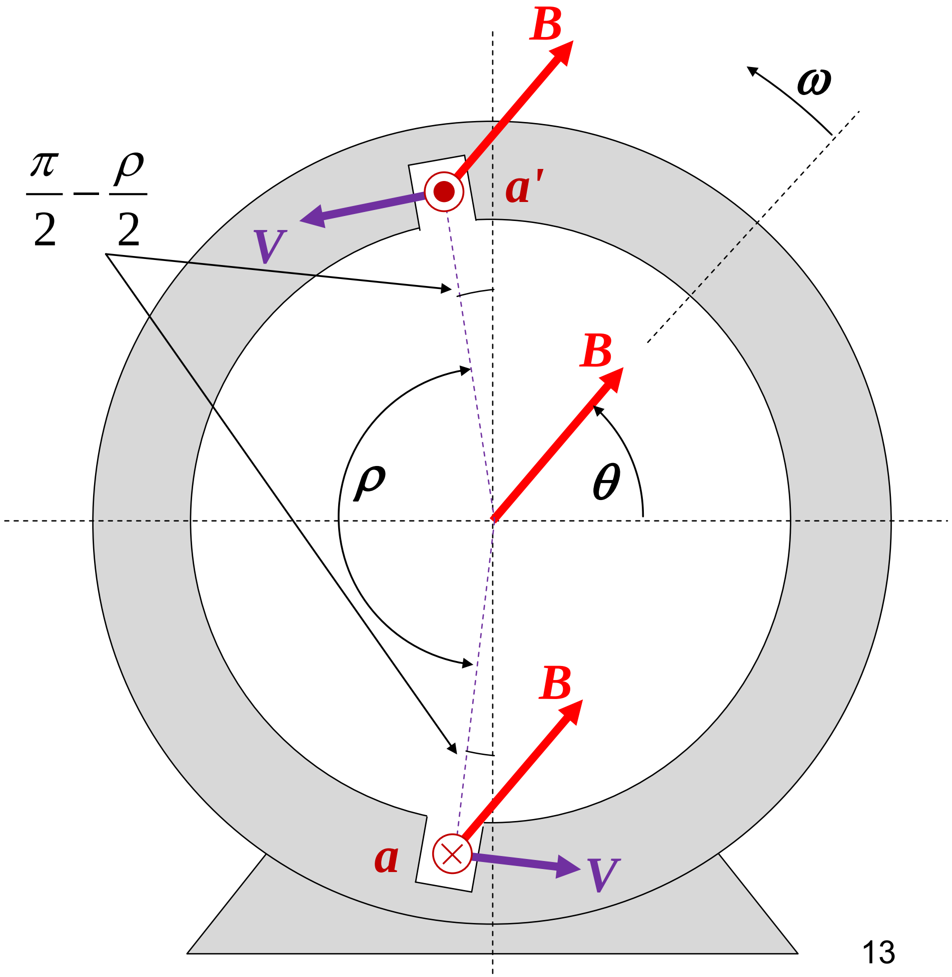
- Assume the axial length of the stator is l and the coil pitch is ρ as shown in the figure

$$e = (\vec{V}\times\vec{B})\cdot\vec{l}$$

$$e = VBl\sin(\vec{V},\vec{B})$$

$$\begin{cases} e_a = VBl\sin\left(\theta + \frac{\pi}{2} - \frac{\rho}{2}\right) \\ e_{a'} = VBl\sin\left(\pi + \frac{\pi}{2} - \frac{\rho}{2} - \theta\right) \end{cases}$$

$$\begin{cases} e_a = VBl\cos\left(\theta - \frac{\rho}{2}\right) \\ e_{a'} = -VBl\cos\left(\theta + \frac{\rho}{2}\right) \end{cases}$$

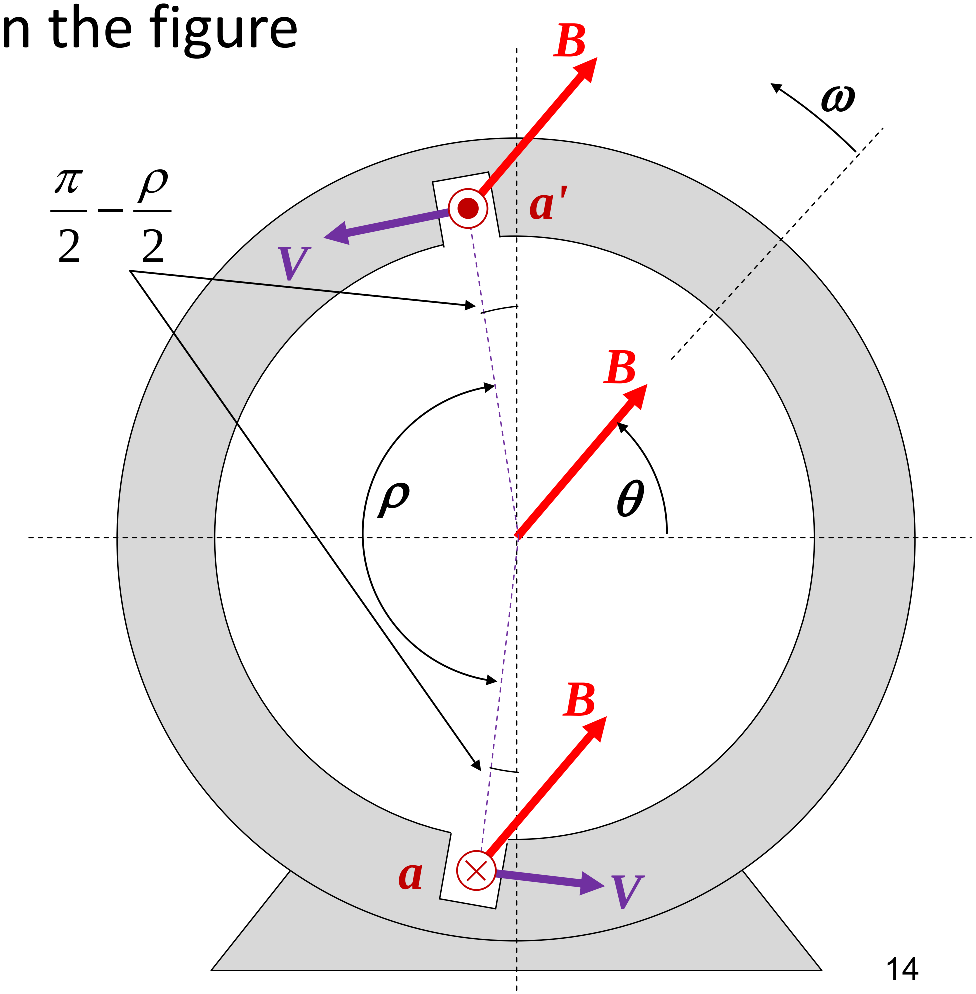


Pitch Factor

- Assume the axial length of the stator is l and the **coil pitch** is ρ in **electrical angle** as shown in the figure

$$\begin{cases} e_a = VBl\cos\left(\theta - \frac{\rho}{2}\right) \\ e_{a'} = -VBl\cos\left(\theta + \frac{\rho}{2}\right) \end{cases}$$

$$\left|e_{aa'}\right| = 2VBl\sin\theta\sin\left(\frac{\rho}{2}\right)$$



Pitch Factor

$$\left|e_{aa'}\right|_{Full-pitch} = 2VBl\sin\theta$$

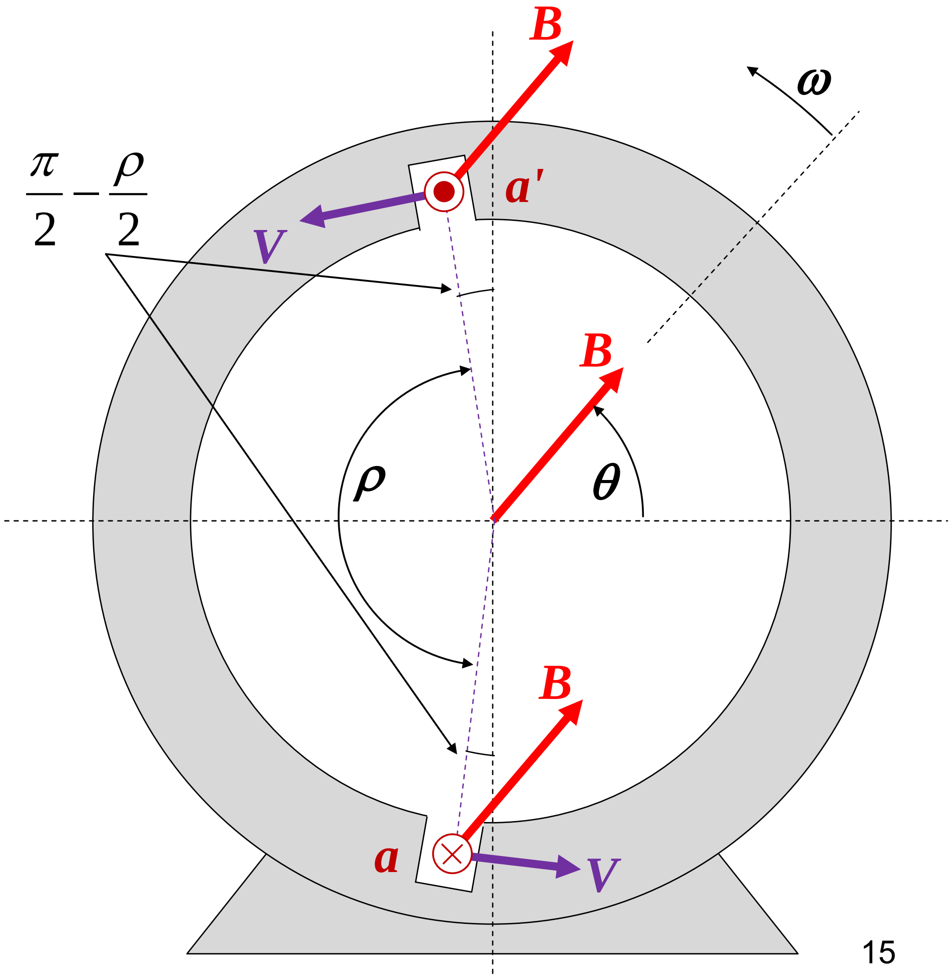
$$\left|e_{aa'}\right|_{Short-pitch} = 2VBl\sin\theta\sin\left(\tfrac{\rho}{2}\right)$$

The pitch factor is defined as

$$k_p = \frac{\left|e_{aa'}\right|_{\text{Short-pitch}}}{\left|e_{aa'}\right|_{\text{Full-pitch}}}$$

Therefore the pitch factor is

$$k_p = \sin\left(\tfrac{\rho}{2}\right)$$



Pitch Factor

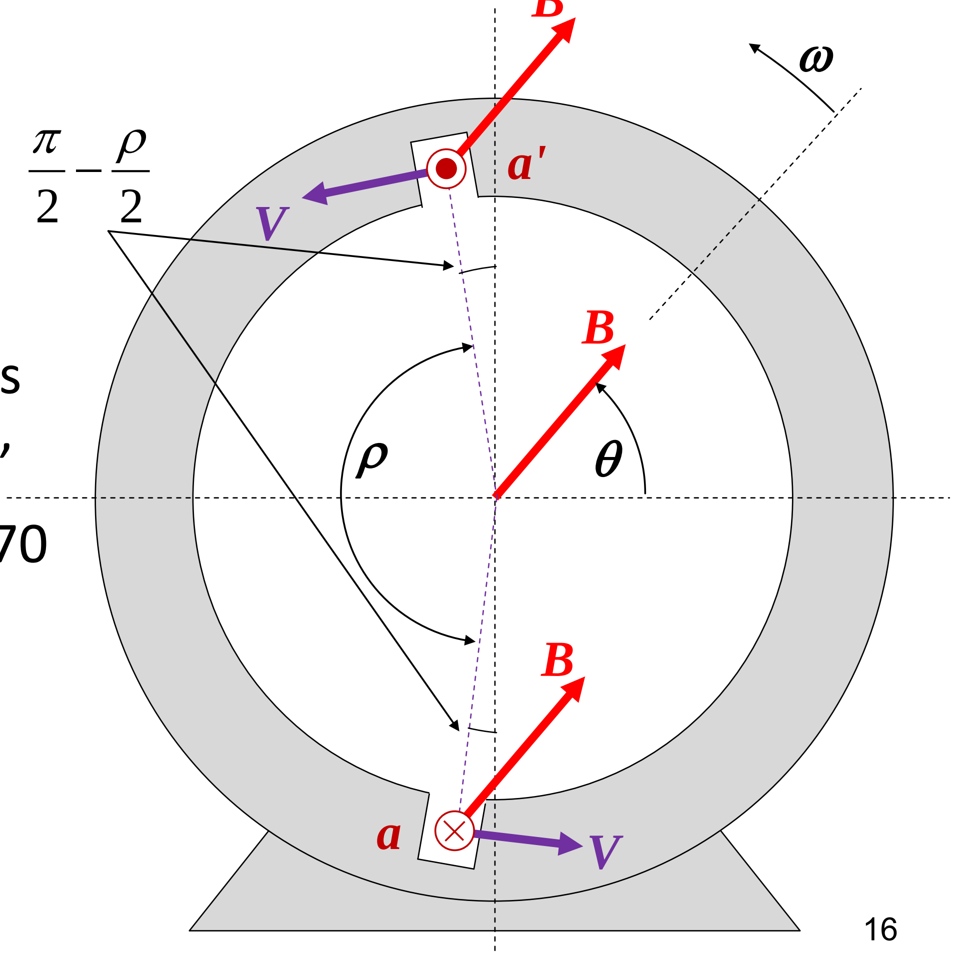
- Since the induced voltage may not be ideal sinusoidal, the pitch factor is expressed for all harmonics

$$k_{pn} = \sin\left(\frac{n\rho}{2}\right)$$

- **Why is short-pitch coil used?**

- Although short-pitch coil decreases the induced voltage (by about 3%), it **decreases** the **disturbing harmonics** significantly (by about 70 to 80%).

- Disturbing harmonics in electric machines are 5 and 7;



Pitch Factor

- **Example**: Consider a 3-phase, 2-pole machine having pitch coil ratio of 5/6. Calculate the pitch factor for odd harmonics 1 to 7.

$$\text{Pich factor ratio} = \frac{\rho}{\pi} \quad \Rightarrow \quad \rho = \frac{5\pi}{6}$$

Electrical rad

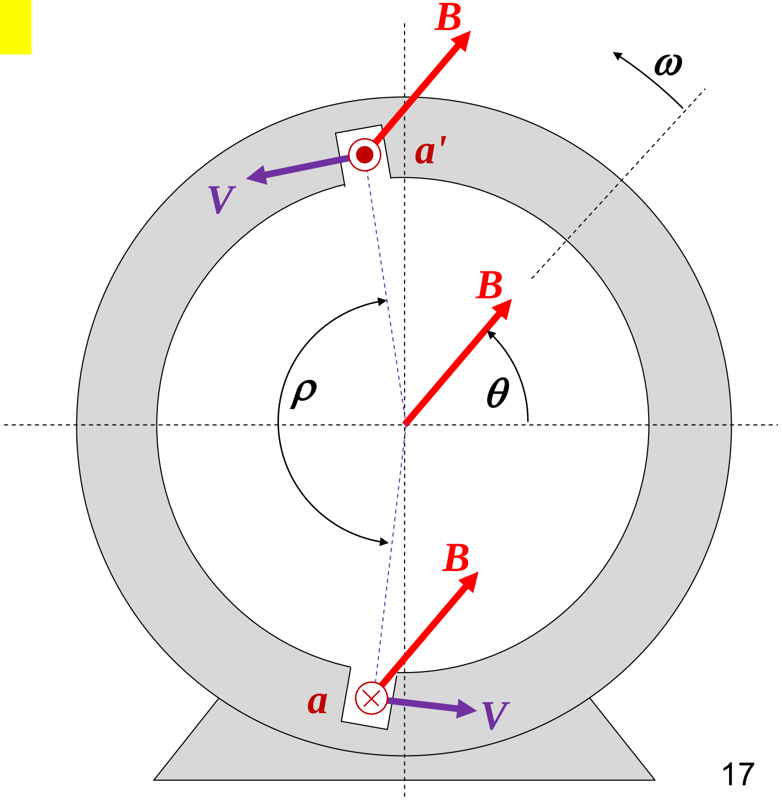
$$k_{pn} = \sin\left(\tfrac{n\rho}{2}\right)$$

$$k_{p1} = \sin\left(\tfrac{5\pi}{2\times 6}\right) = 0.966$$

$$k_{p3} = \sin\left(\tfrac{3\times 5\pi}{2\times 6}\right) = -0.707$$

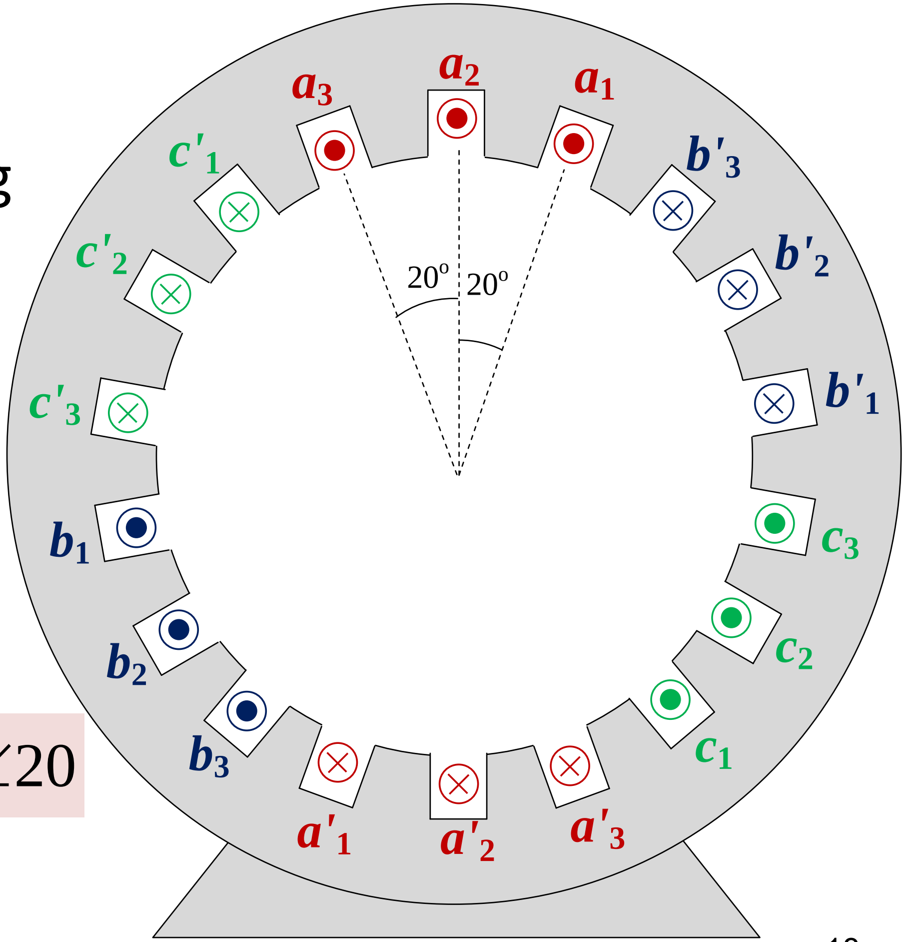
$$k_{p5} = \sin\left(\tfrac{5\times 5\pi}{2\times 6}\right) = 0.259$$

$$k_{p7} = \sin\left(\tfrac{7\times 5\pi}{2\times 6}\right) = 0.259$$



Distributed Windings

- To make the **MMF more sinusoidal**, distributed windings are used.
- Consider the following 2-pole single-layer distributed winding AC machine:
- The induced voltage in phase *a* is:



$$\vec{E}_{aa'} = \vec{E}_{aa'1} + \vec{E}_{aa'2} + \vec{E}_{aa'3}$$

$$\vec{E}_{aa'} = E_{rms}\angle -20 + E_{rms}\angle 0 + E_{rms}\angle 20$$

$$\vec{E}_{aa'} = 2.88 E_{rms}\angle 0$$

Distribution Factor

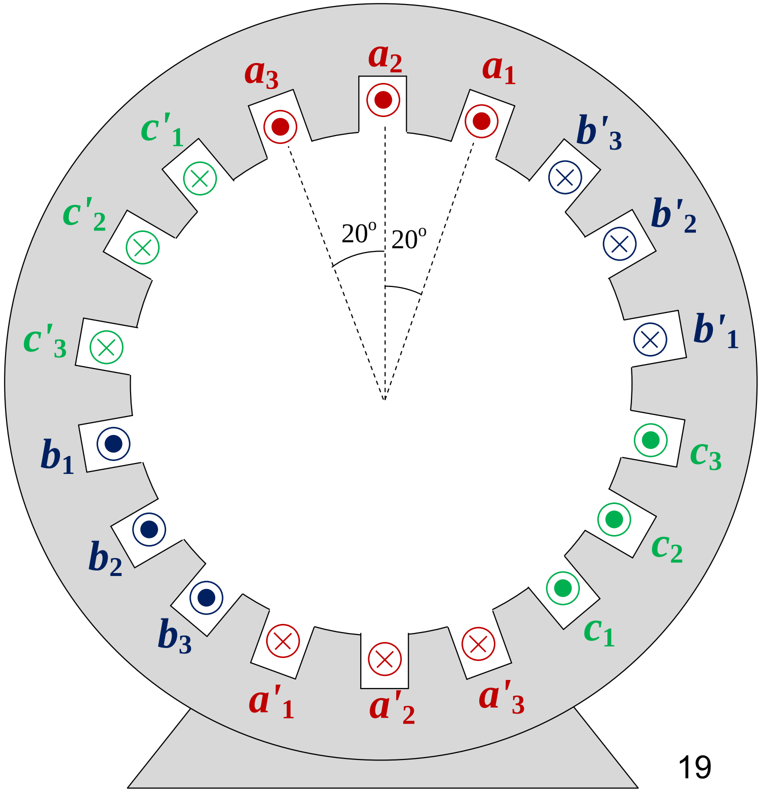
- Distribution factor is defined as:

$$k_d = \frac{\text{Induced voltage for distributed winding}}{\text{Induced voltage for concentrated winding}}$$

- In previous example

$$k_d = \frac{2.88E_{rms}}{3E_{rms}}$$

$$k_d = 0.96$$

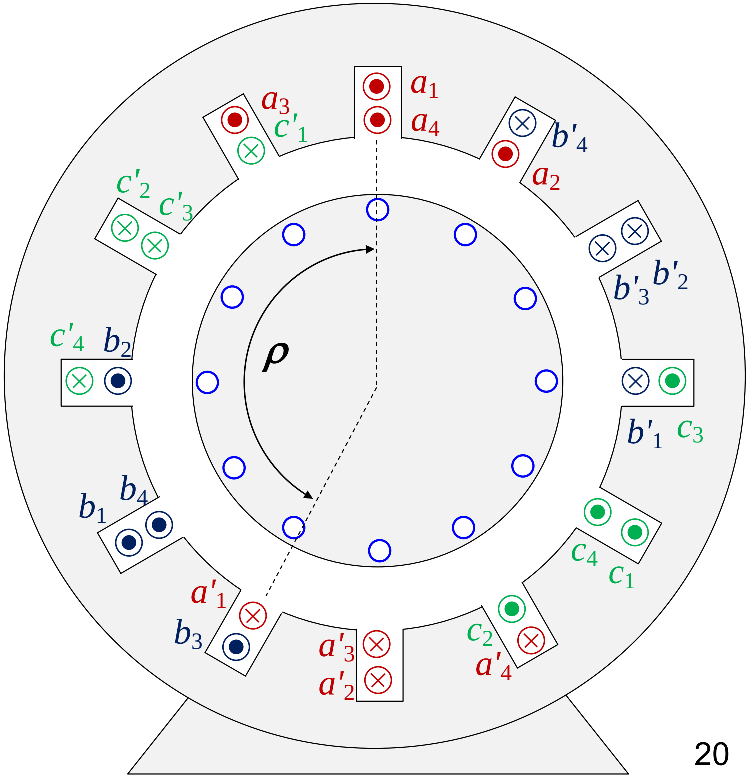


Multi-layer Winding

- Most of AC machines, especially large machines, have **2-layer** winding.

- 2-layer winding has the following **advantages** compared to single-layer winding:

1. Simpler winding
2. Simpler connection
3. Higher mechanical strength
4. Optimal use of slots
5. Lower cost



Distribution Factor

- Distribution factor is defined as:

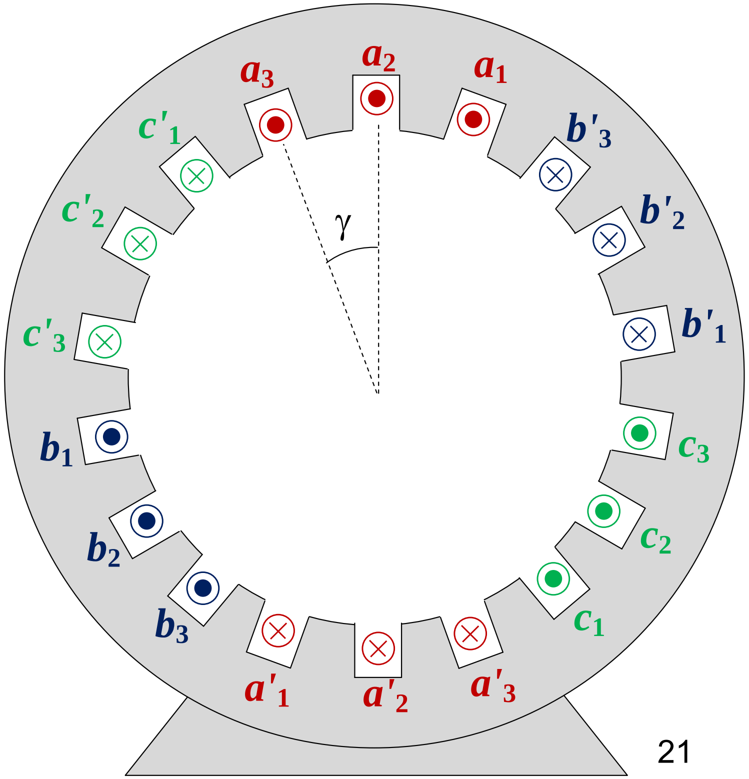
$$k_d = \frac{\text{Induced voltage for distributed winding}}{\text{Induced voltage for concentrated winding}}$$

- Assume the angle between two adjacent slots is γ in electrical angle and the number of slots per pole per phase is m, therefore the distribution factor is written as

$$k_d = \frac{\sin \frac{m\gamma}{2}}{m \sin \frac{\gamma}{2}}$$

- In n-th harmonic it is

$$k_{dn} = \frac{\sin \frac{m n \gamma}{2}}{m \sin \frac{n \gamma}{2}}$$



Winding Factor

- Winding factor is defined as

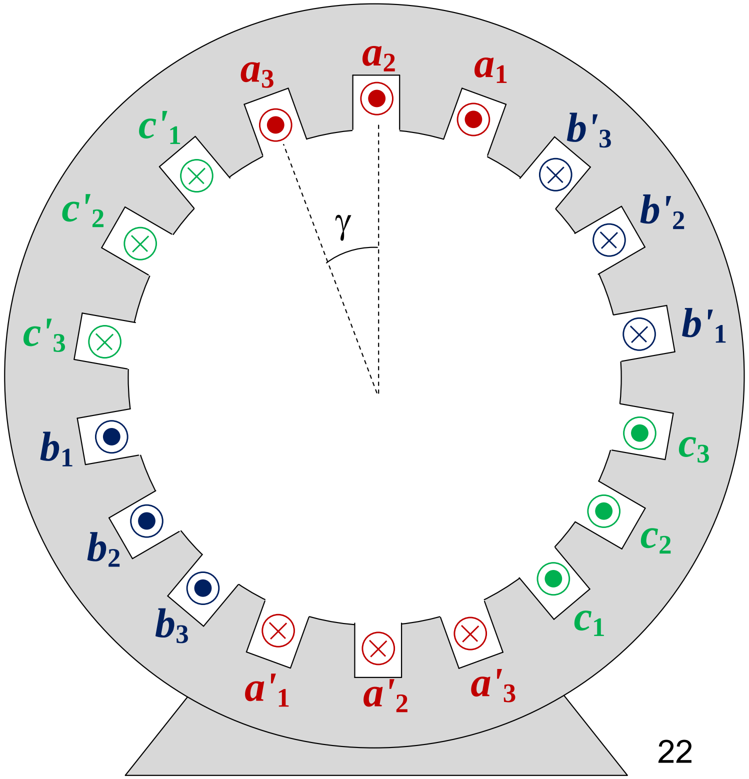
$$k_w = k_p k_d$$

$$k_p = \sin\left(\frac{\rho}{2}\right)$$

$$k_d = \frac{\sin\frac{m\gamma}{2}}{m\sin\frac{\gamma}{2}}$$

$$k_{pn} = \sin\left(\frac{n\rho}{2}\right)$$

$$k_{dn} = \frac{\sin\frac{mn\gamma}{2}}{m\sin\frac{n\gamma}{2}}$$



Winding Factor

- **Example**: Consider a 3-phase, **2-pole** generator where its stator winding connected in star form. The machine has 2-layer, short-pitch, distributed winding with 4 coils per phase. If each coil has 10 turns, rotor is rotated with 3000 rpm and the flux of each pole is 0.019 Wb, calculate the phase induced voltage.

$N_l = 2$

$N_{cp} = 4$

$N_{tc} = 10$

$n_r = 3000 \quad \text{rpm}$

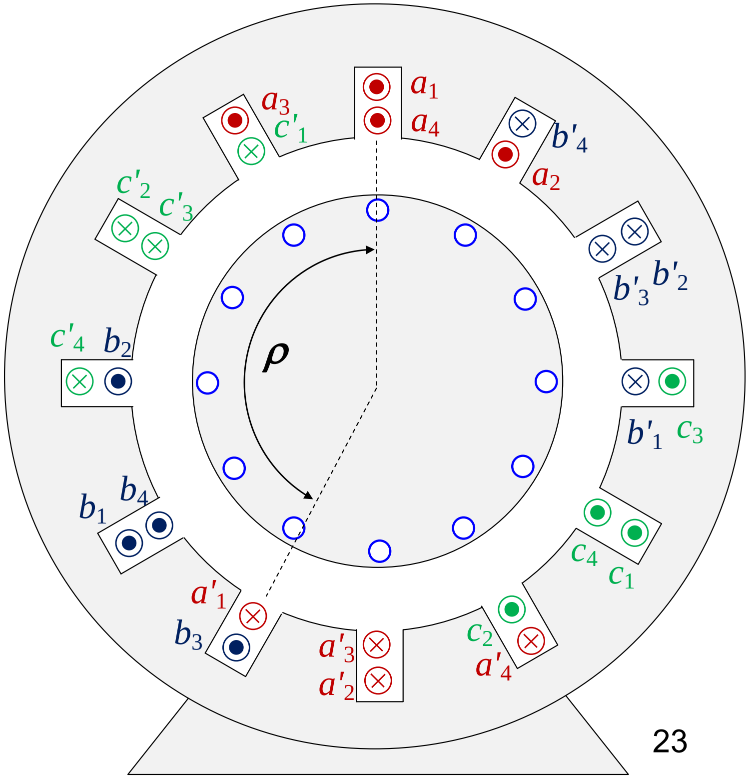
$\phi = 0.019 \ \text{Wb}$

$p = 2$

$E_{aa'} = ?$

$E_{rms} = 4.44 N_t \, f \, \phi \, k_w$

$k_w = k_p k_d$



Winding Factor

- **Solution**: $N_l = 2$ $N_{cp} = 4$ $N_{tc} = 10$ $n_r = 3000 \quad \text{rpm}$

$\phi = 0.019 \text{ Wb}$ $p = 2$ $E_{aa'} = ?$ $E_{rms} = 4.44 N_t\, f\, \phi\, k_w$ $k_w = k_p k_d$

$$f = \frac{p}{2}\frac{n_r}{60} = 50\,\text{Hz}$$

Number of turns per phase is

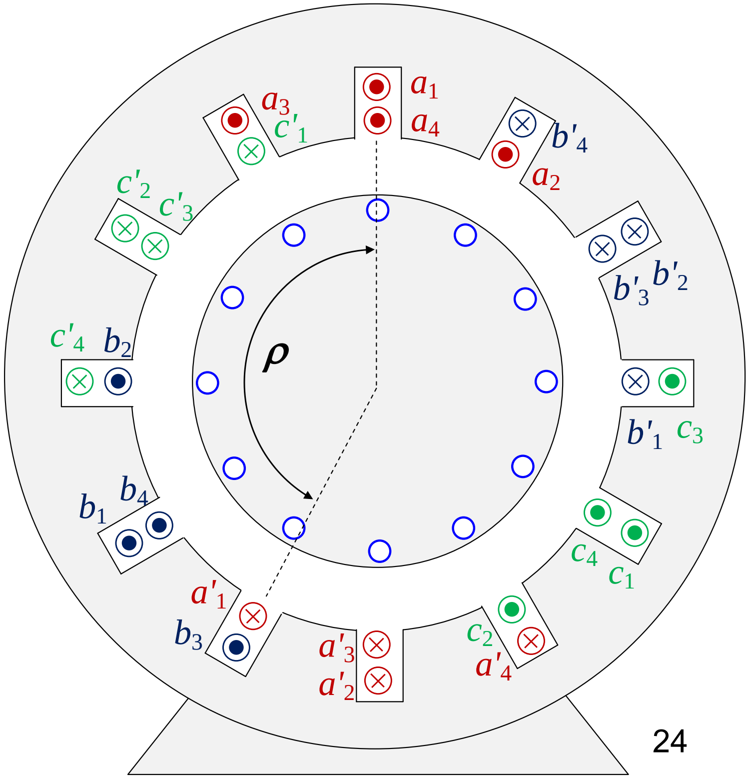
$$N_t = N_{tc} N_{cp} = 40$$

Coil pitch is

$$\rho = 150 \text{ elec.deg.}$$

Pitch factor is found as

$$k_p = \sin\left(\tfrac{\rho}{2}\right) = \sin\left(\tfrac{150}{2}\right) = 0.966$$



Winding Factor

- **Solution:** $N_l = 2$ $N_{cp} = 4$ $N_{tc} = 10$ $n_r = 3000 \quad \text{rpm}$

$\phi = 0.019 \ \text{Wb}$ $p = 2$ $E_{aa'} = ?$ $E_{rms} = 4.44 N_t \, f \, \phi \, k_w$ $k_w = k_p k_d$

$f = 50 \, \text{Hz}$ $N_t = 40$ $k_p = 0.966$

- Number of slots per pole per phase

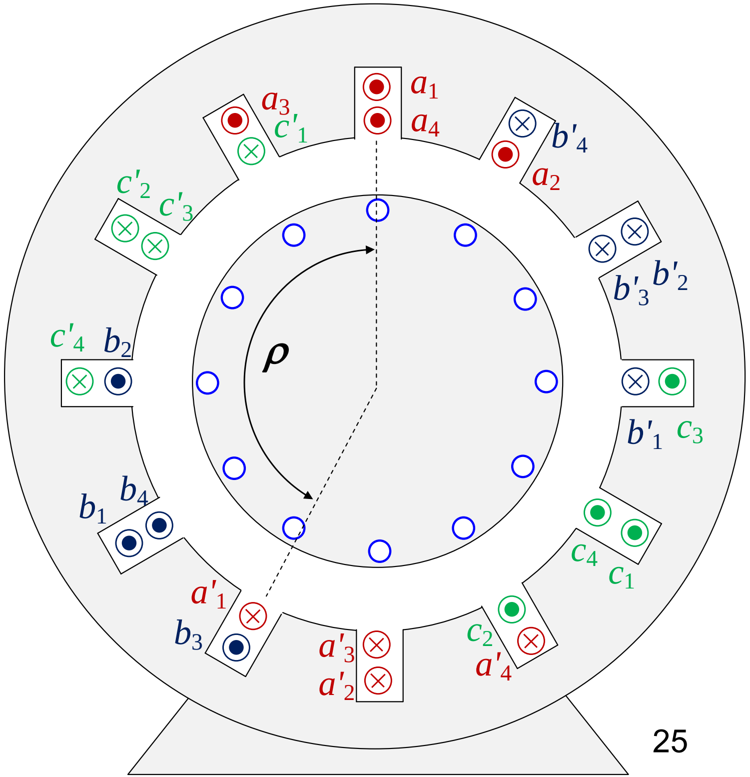
$$m = N_s / 3p = 2$$

- Slot-pitch is

$$\gamma = \frac{p}{2}\frac{360}{N_s} = \frac{360}{12} = 30 \ \text{elec.deg.}$$

- Distribution factor

$$k_d = \frac{\sin \frac{m\gamma}{2}}{m \sin \frac{\gamma}{2}} = \frac{\sin \frac{2\times 30}{2}}{2 \sin \frac{30}{2}} = 0.966$$



Winding Factor

- **Solution:** $N_l = 2$ $N_{cp} = 4$ $N_{tc} = 10$ $n_r = 3000 \quad \text{rpm}$

$\phi = 0.019 \ \text{Wb}$ $p = 2$ $E_{aa'} = ?$ $E_{rms} = 4.44 N_t \, f \, \phi \, k_w$ $k_w = k_p k_d$

$f = 50 \, \text{Hz}$ $N_t = 40$ $k_p = 0.966$ $k_d = 0.966$

- The RMS value of the phase induced voltage is

$$E_{rms} = 4.44 \times 40 \times 50 \times 0.019 \times 0.966 \times 0.966$$

$$E_{rms} = 157 \ \ \text{V}$$