In The Name of God The Most Compassionate, The Most Merciful

دانشگاه صنعتی شیراز

Electric Machines II

Table of Contents

Chapter 1
Single-Phase Transformers

Introduction

Transformers have the following characteristics

1. Transformers are **electromagnetic energy conversion** systems; as they receive electrical energy from the network; convert it to the magnetic energy; and then the magnetic energy is converted to the electrical energy with different voltage and current level.

2. A transformer has at least two windings: a primary and a secondary winding. **Primary winding** is the winding connected to the **power source** and the **secondary winding** is that connected to the **load**.

3. There is **no electrical connection** between the primary and secondary windings (except in auto-transformers); the connection is through a magnetic field.

Introduction

4. If the secondary voltage is lower than that of primary, the transformer is **step-down**; otherwise it is **step-up**.

5. **Swapping the primary and secondary windings** will change a step-down transformer to a step-up transformer and vice-versa.

6. In a **step-up** transformer, the **number of turns of the secondary winding** is **higher** than that of the **primary** winding.

7. In a **step-down** transformer, the **number of turns of the secondary winding** is **lower** than that of the **primary** winding.

8. Since transformers have **no mechanical part**, their **efficiency** is normally **very high**.

Applications of Transformers

1. Electric Power Transmission Systems.

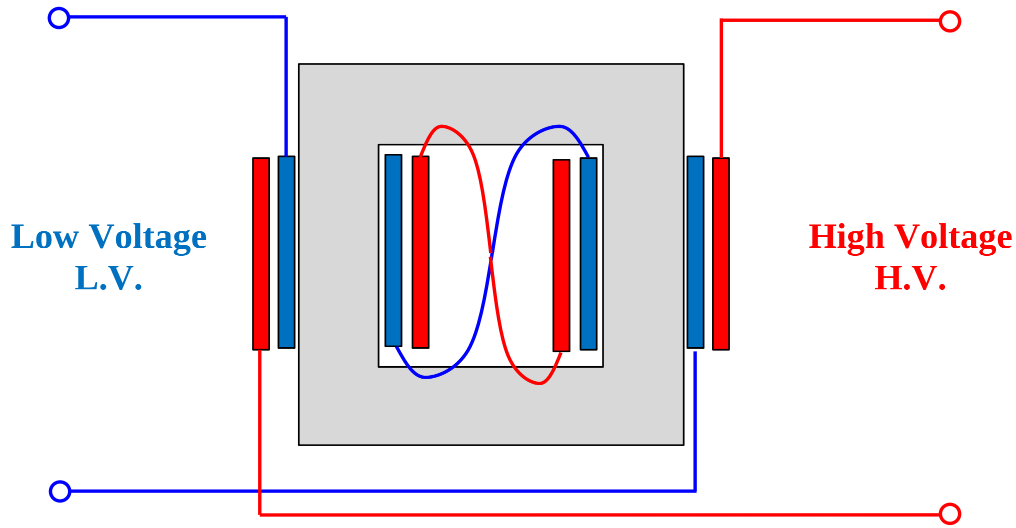
2. Impedance Matching (e.g. in speakers).

3. Blocking the dc component of an ac + dc signal or power.

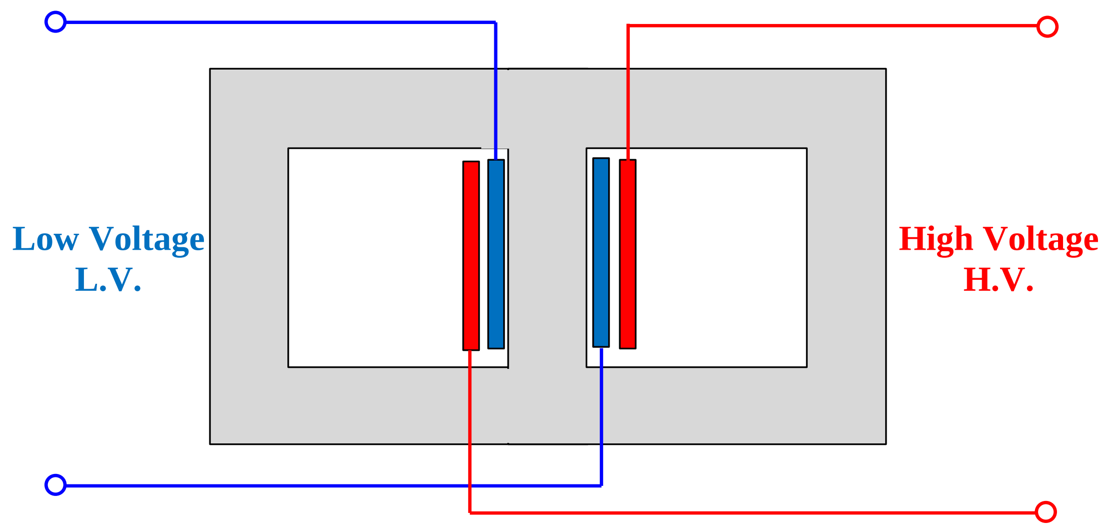
4. Voltage and current measurement: Voltage or potential transformers (VT) or (PT); Current transformers (CT).

Structures of Transformers

1. Core Type



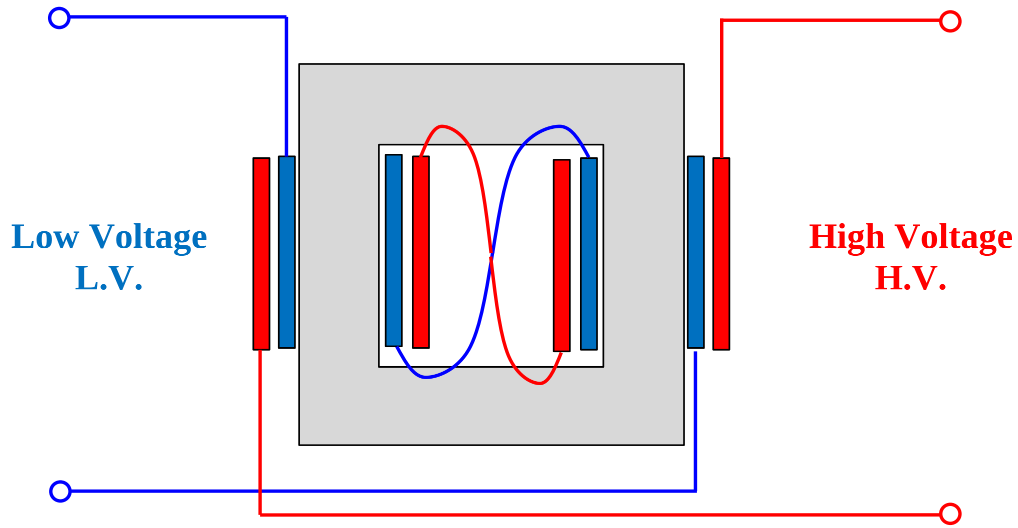
2. Shell Type



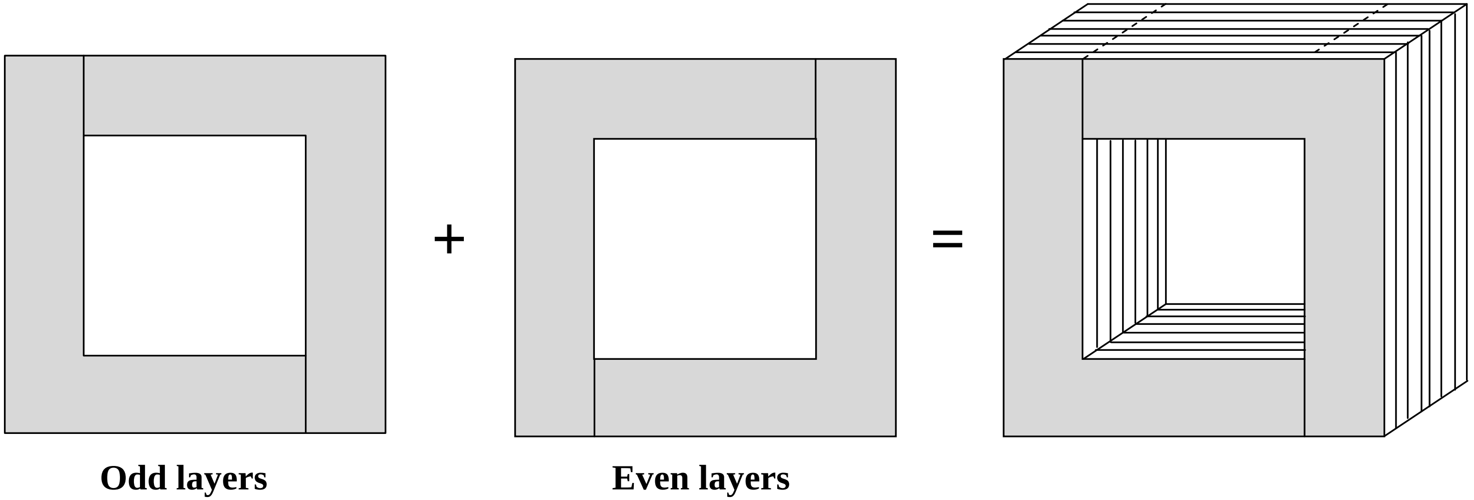
In both structures, to reduce the eddy current losses, the core is **laminated**

Structures of Transformers

1. Core Type

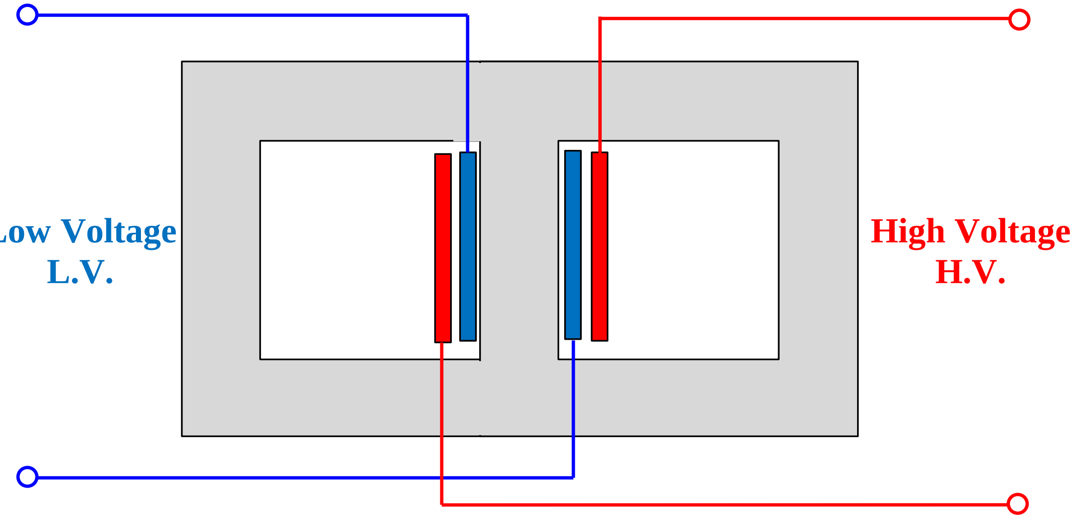


L layers

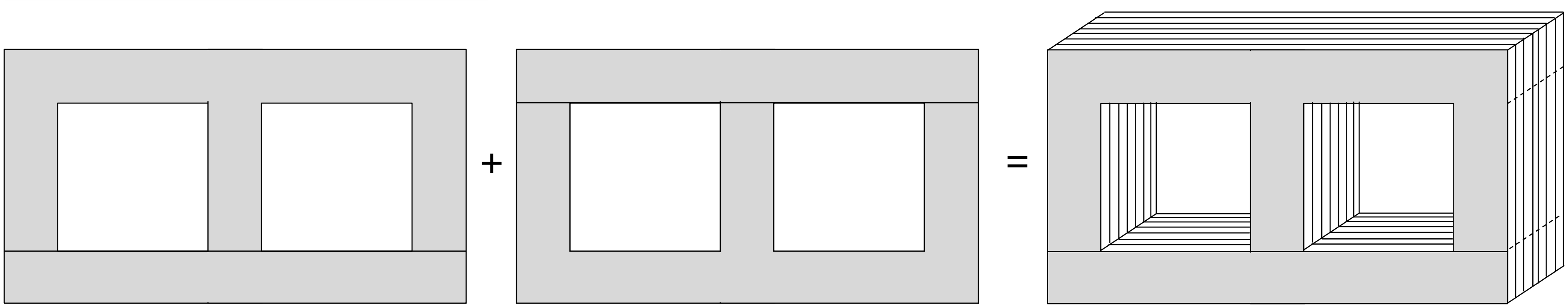


Structures of Transformers

2. Shell Type



E & I layers



Odd layers   Even layers

Some Points

- To reduce the eddy current losses, the core is **laminated**.

- Lamination material is **silicon steel** and the thickness of the laminations can be around 0.35 mm for 400 Hz.

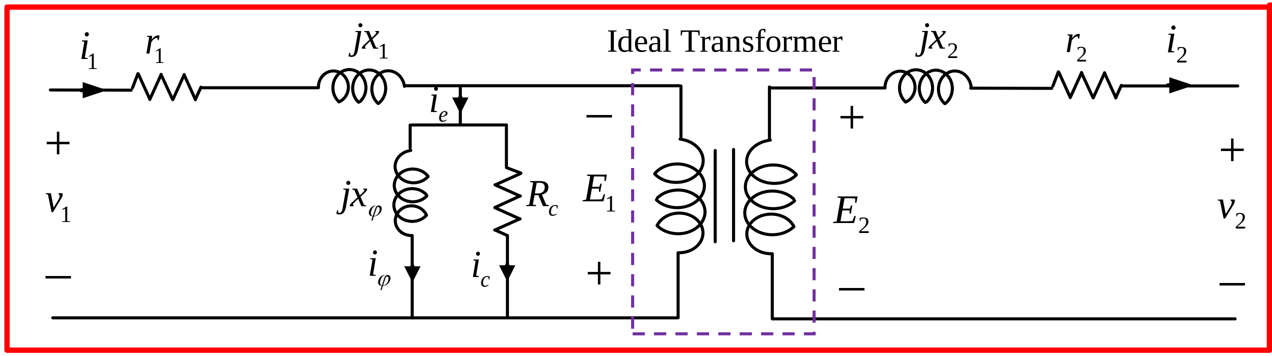
- The laminated sheets are covered by **epoxy** to provide **insulation** between the layers.

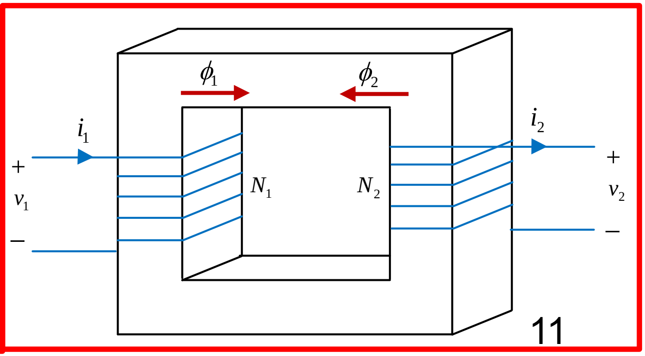
- H.V. winding has **higher number of turns** but the **thickness** of the wire is **lower** compared to those of L.V. winding

Ideal Transformers

An ideal transformer has the following characteristics:

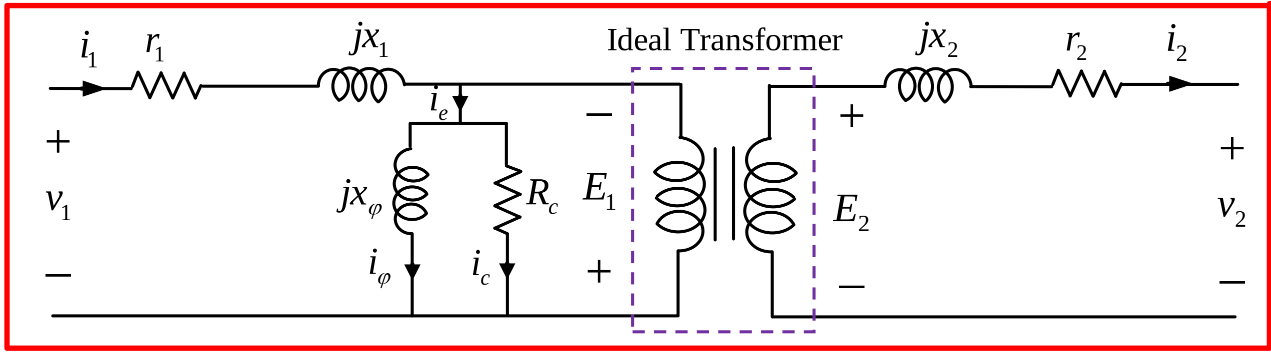
1. The **ohmic losses** due to the primary and secondary winding resistances are **neglected**. $r_1 = r_2 = 0$
2. The **core losses** are **neglected**. $R_c \to \infty$
3. The magnetizing curve of the transformer core is assumed to be linear.
4. The **leakage inductances** of the windings are **neglected**. $x_1 = x_2 = 0$
5. The **core permeability** goes to **infinity**. $\mu_c \to \infty$ ⇒ $x_\varphi \to \infty$

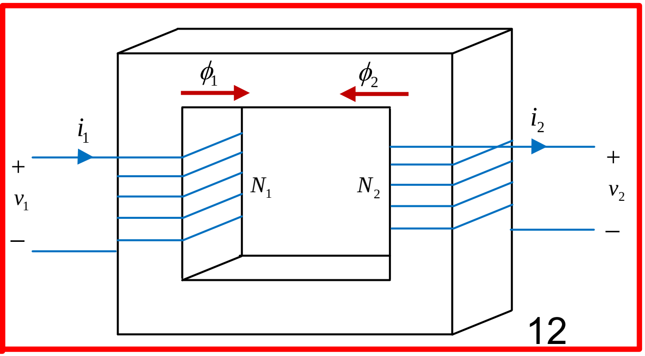




Ideal Transformers

To make the analysis of ideal transformers more realistic, we assume that the permeability of the core is a finite value. So x_φ is a finite value.





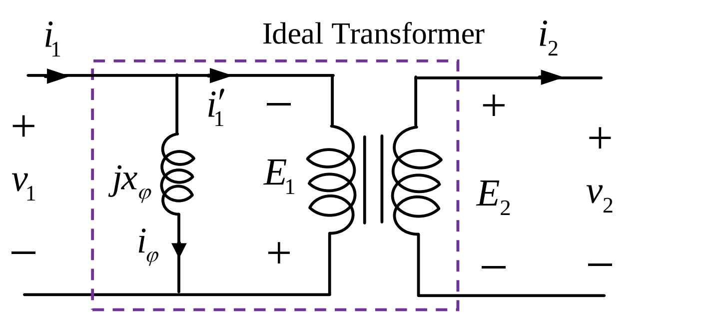
Ideal Transformers

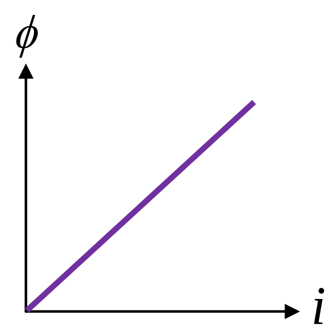
- Assume the magnetizing curve is linear.
- If current $i = I_m \sin \omega t$ flows in the windings, the flux in the core will be $\phi = \phi_m \sin \omega t$.
- The induced voltage in the primary and secondary windings will be

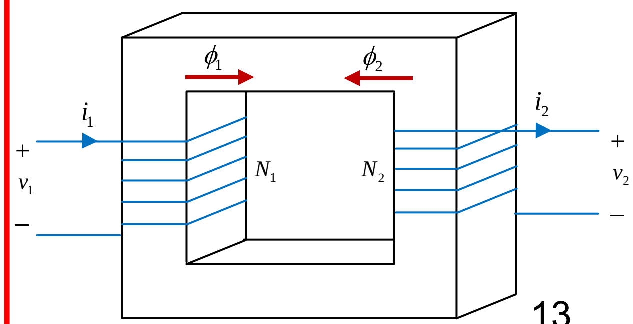
$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \omega \phi_m \cos \omega t \Rightarrow E_{m1} = N_1 \omega \phi_m$$

$$e_2 = -N_2 \frac{d\phi}{dt} = -N_2 \omega \phi_m \cos \omega t \Rightarrow E_{m2} = N_2 \omega \phi_m$$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

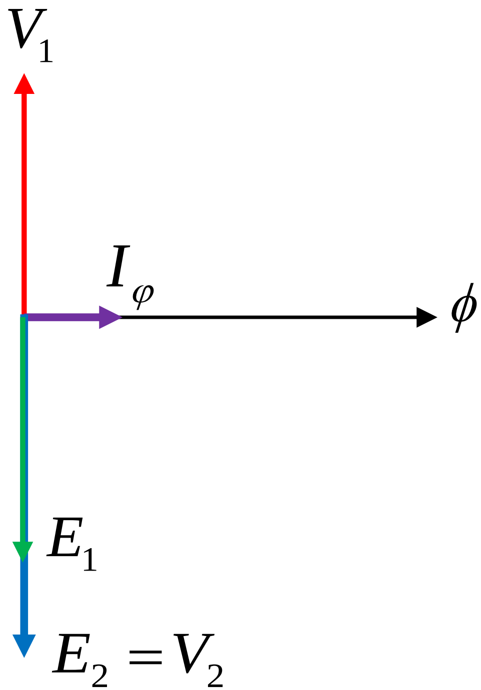




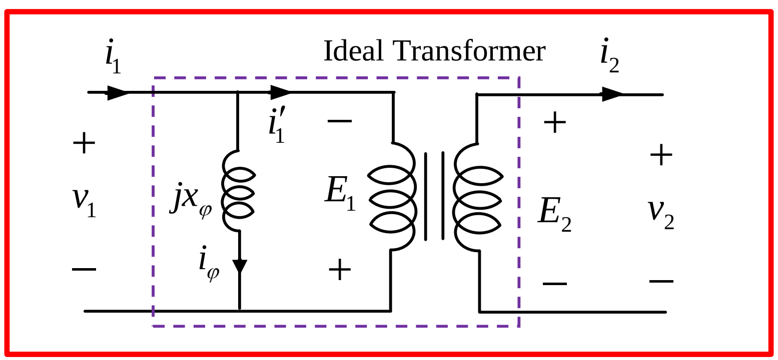


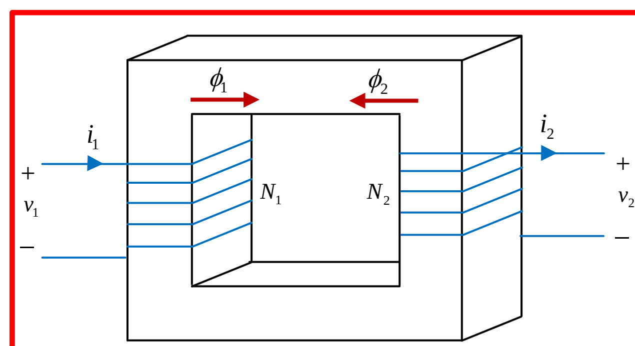
Phasor Diagram of Ideal Transformers

No-load



- The objective is to **draw** the **phasors of voltages and currents**.
- Note that **upper-case letters** are used to indicate that the quantities are in **phasor** form.
- At **no-load** $I_1 = I_\varphi$ $I_2 = 0$





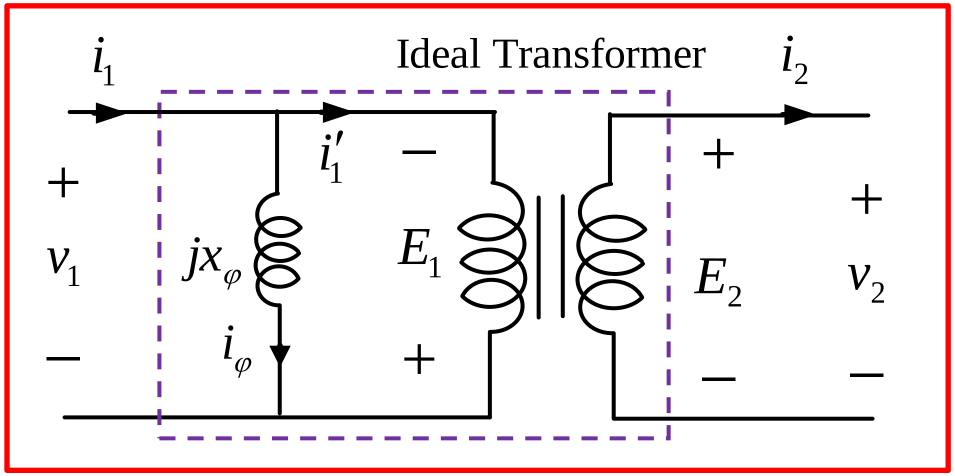
Ideal Transformers

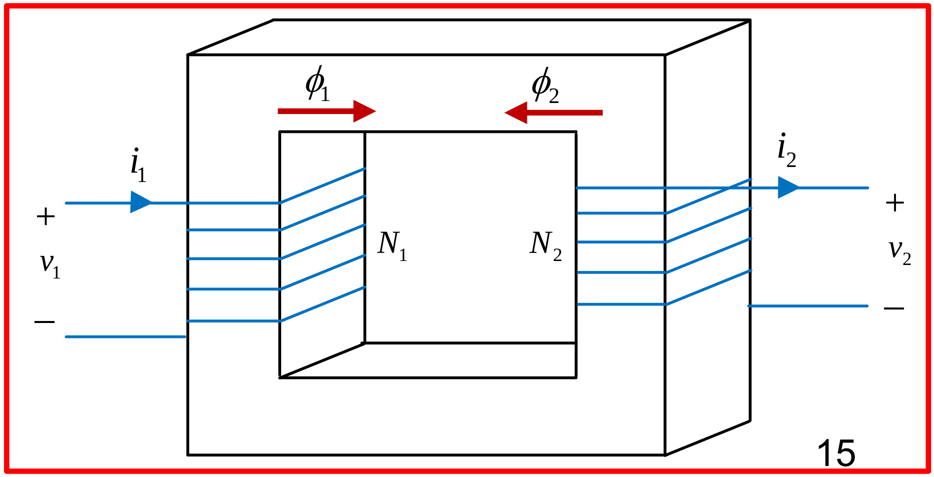
- From the primary winding view $\phi_1' = \dfrac{N_1 I_1'}{\Re}$
- From the secondary winding view $\phi_2 = \dfrac{N_2 I_2}{\Re}$

$\Rightarrow \phi_1' = -\phi_2$

$\Rightarrow \dfrac{I_1'}{I_2} = -\dfrac{N_2}{N_1}$

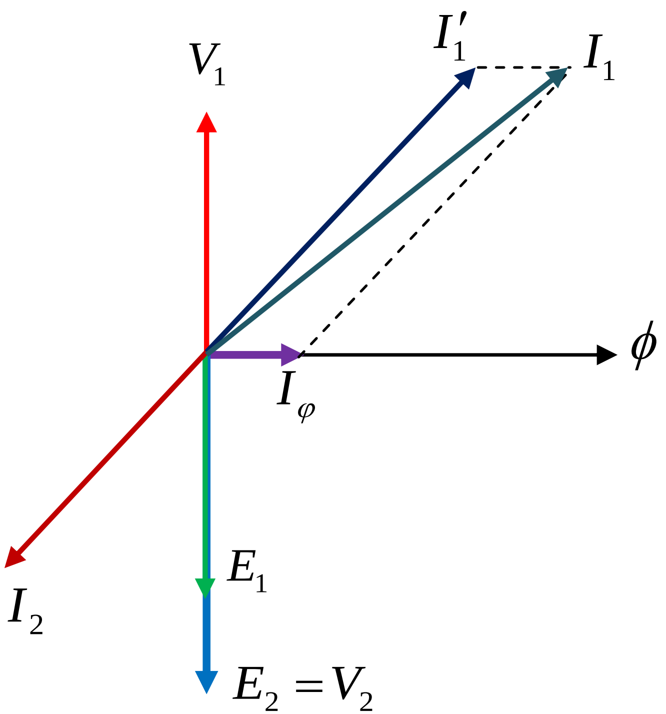
- Where $\Re$ is the magnetic reluctance of the core.





Phasor Diagram of Ideal Transformers

Under-load

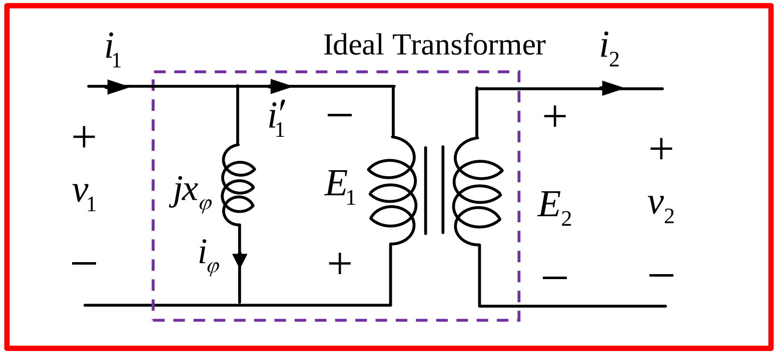


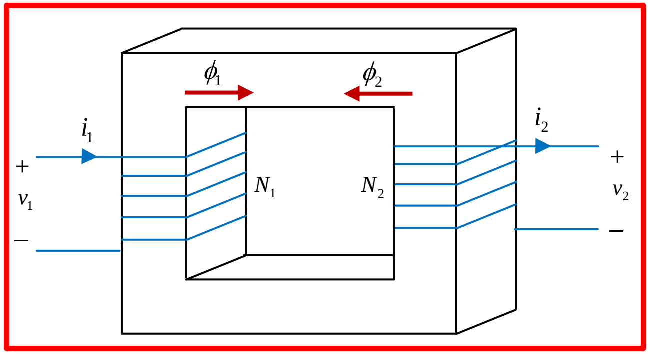
- Assume the load is **resistive-inductive**.
- The objective is to **draw** the **phasors of voltages and currents**.

$$\frac{I_1'}{I_2} = -\frac{N_2}{N_1}$$

$$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

$$I_1 = I_1' + I_\varphi$$

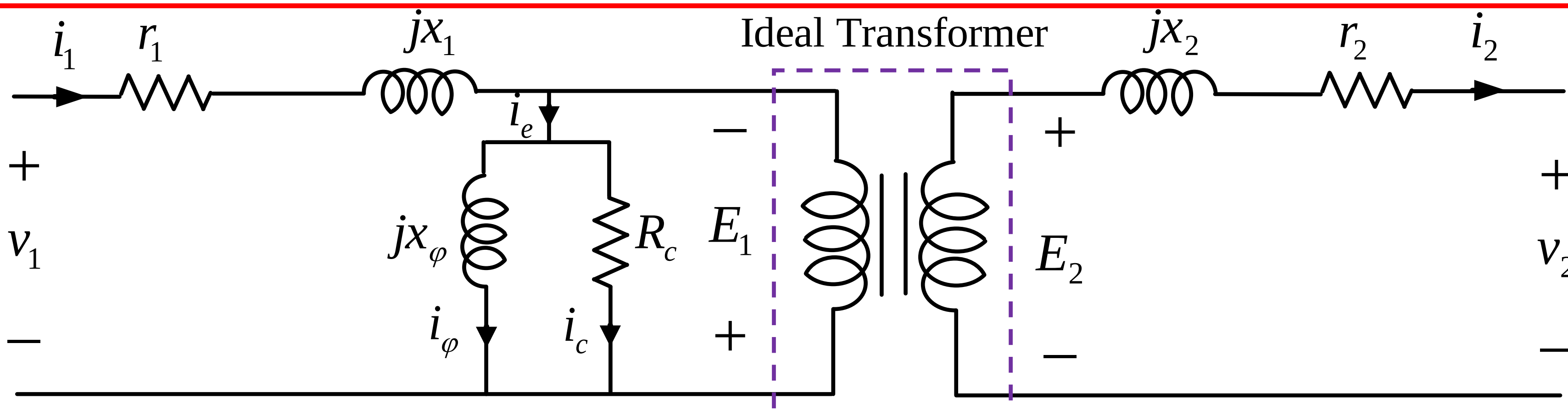




Non-Ideal Transformers

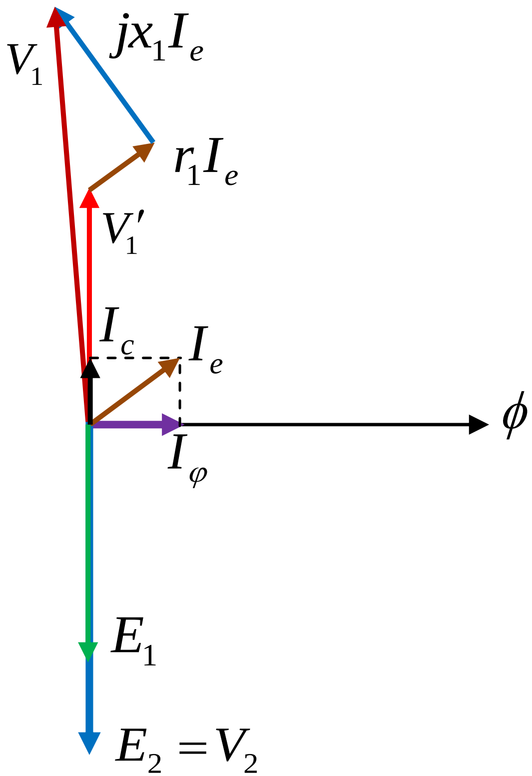
A non-ideal transformer has the following characteristics:

1. The **ohmic losses** are **considered and modelled by** the primary and secondary winding resistances. r_1 r_2
2. The **core losses** are **considered** and modelled by a resistance. R_c
3. The **magnetizing reactance** is **considered**. x_φ
4. The **flux leakage** of the windings are **considered** and modelled by two leakage reactances. x_1 x_2



Phasor Diagram of Non-Ideal Transformers

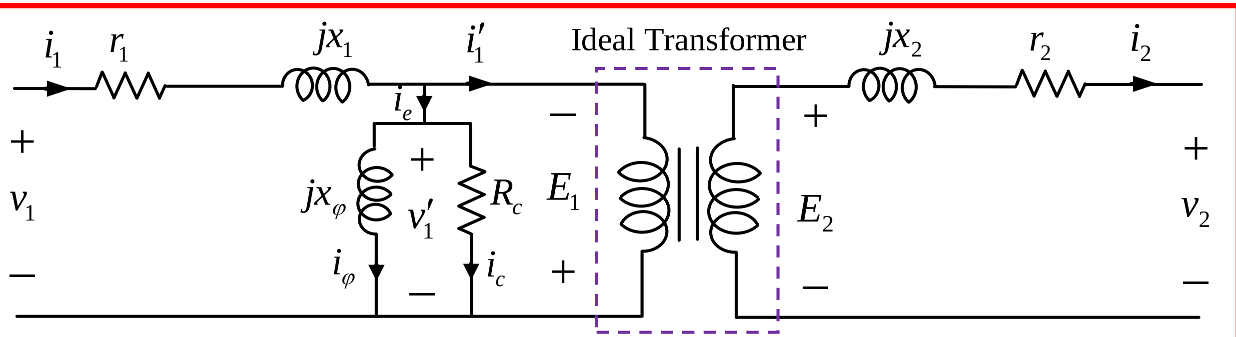
No-load

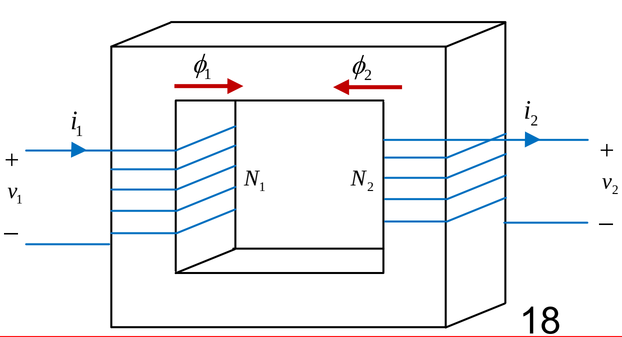


At no-load $I_2 = 0$ $\vec{I}_1 = \vec{I}_e$

$$\vec{I}_e = \vec{I}_\varphi + \vec{I}_c$$

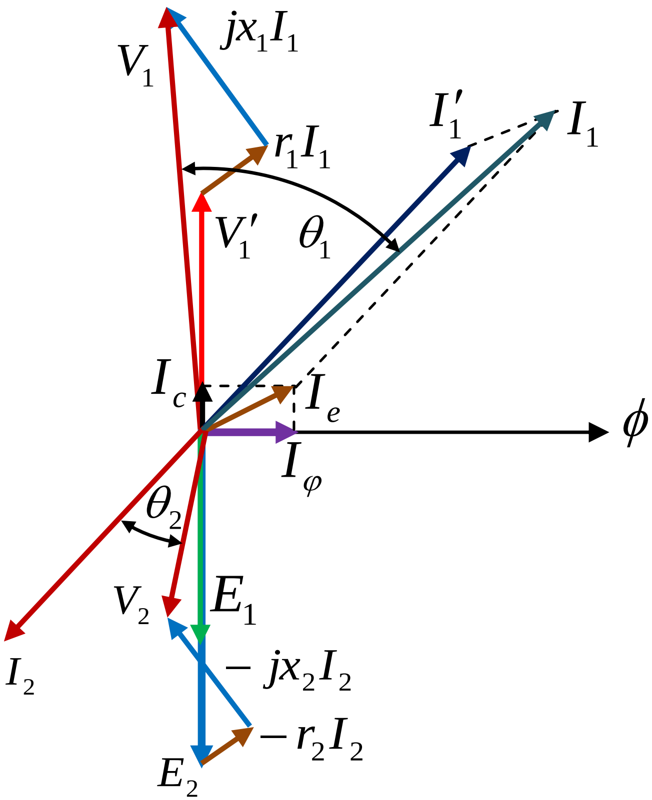
$$\vec{V}_1 = \vec{V}_1' + r_1 \vec{I}_e + jx_1 \vec{I}_e$$





Phasor Diagram of Non-Ideal Transformers

Under-load



Assume the load is resistive-inductive (RL).

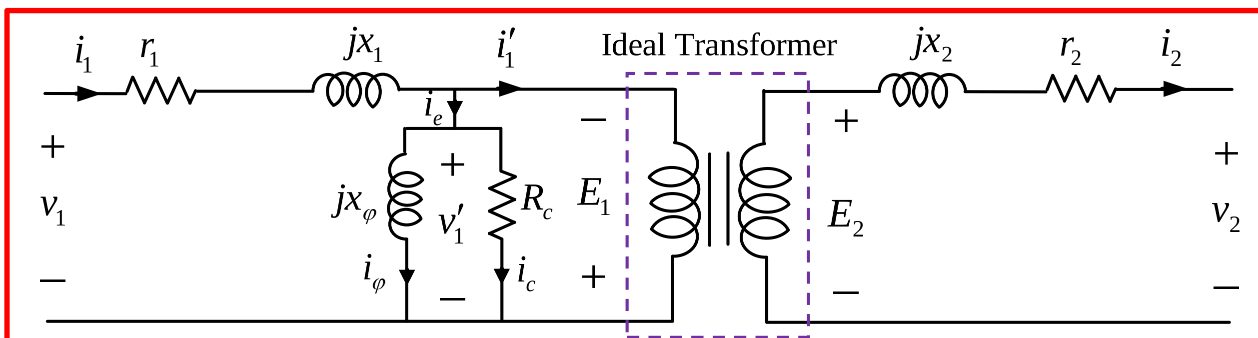
$$\vec{V}_1 = \vec{V}_1' + r_1\vec{I}_1 + jx_1\vec{I}_1$$

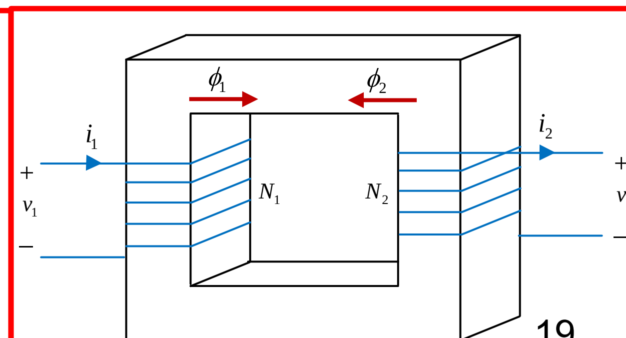
$$\vec{I}_e = \vec{I}_\varphi + \vec{I}_c$$

$$\vec{V}_2 = \vec{E}_2 - r_2\vec{I}_2 - jx_2\vec{I}_2$$

$\cos\theta_2$ Load power factor

$\cos\theta_1$ Source power factor





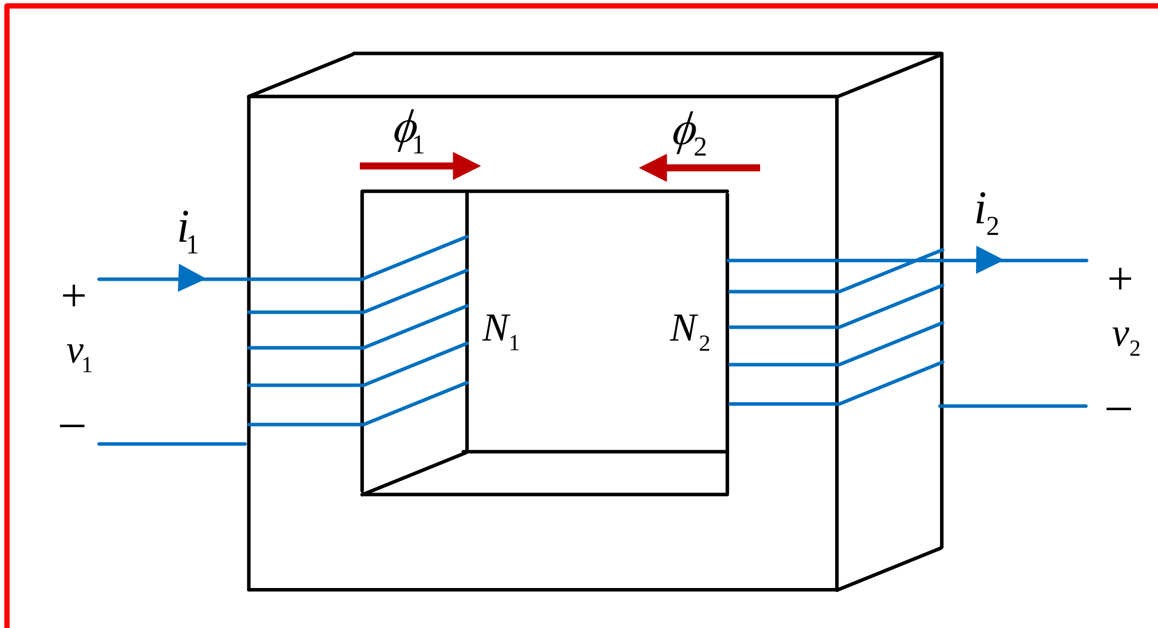
Nominal Values of Transformers

- The **nominal primary and secondary voltages**, the **nominal frequency** and the **nominal apparent power** are mentioned on the **name plate** of transformers.
- The **nominal primary and secondary currents** can be found as

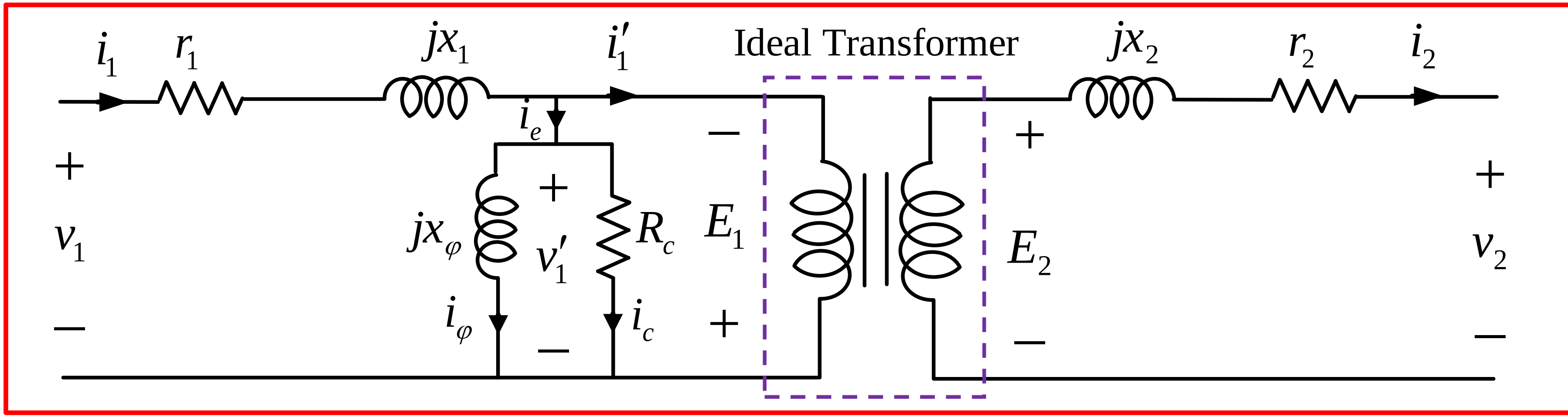
$$I_1 = \frac{S}{V_1} \qquad I_2 = \frac{S}{V_2}$$

Name plate

V_1 / V_2
f (Hz)
S (kVA)



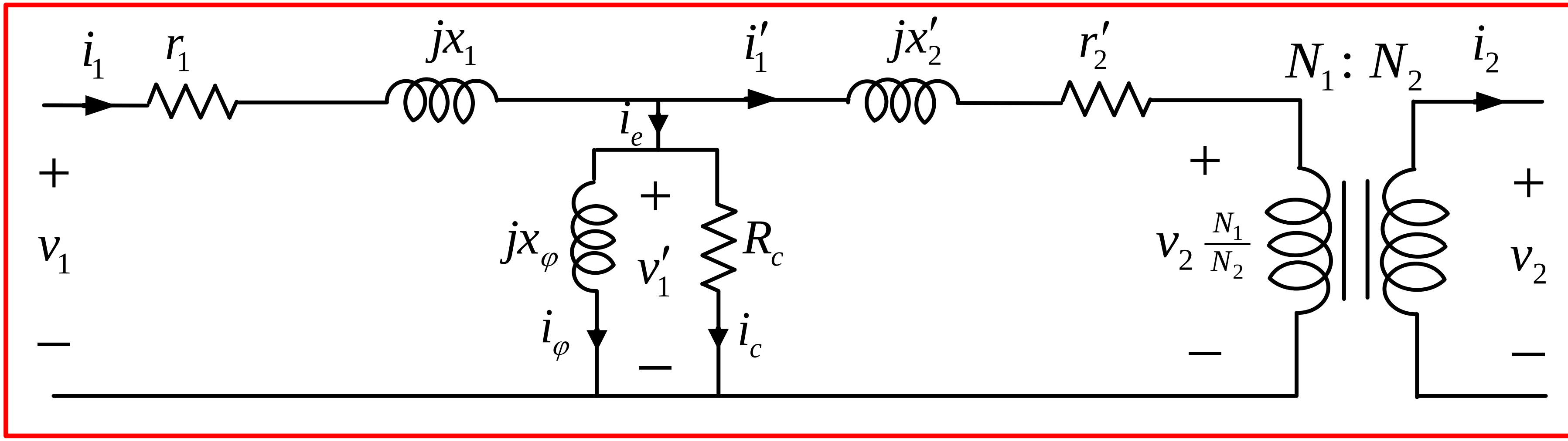
Equivalent Circuit of Transformers



r_1	The primary winding resistance.
r_2	The secondary winding resistance.
R_c	The resistance equivalent to core losses.
x_φ	The magnetizing reactance.
x_1	The reactance models the primary winding flux leakage.
x_2	The reactance models the secondary winding flux leakage.

$$x_\varphi = L_\varphi \omega \qquad L_\varphi = \frac{N_1^2}{\Re} \qquad P_c = \frac{V_1'^2}{R_c}$$

Equivalent Circuit Referred to Primary

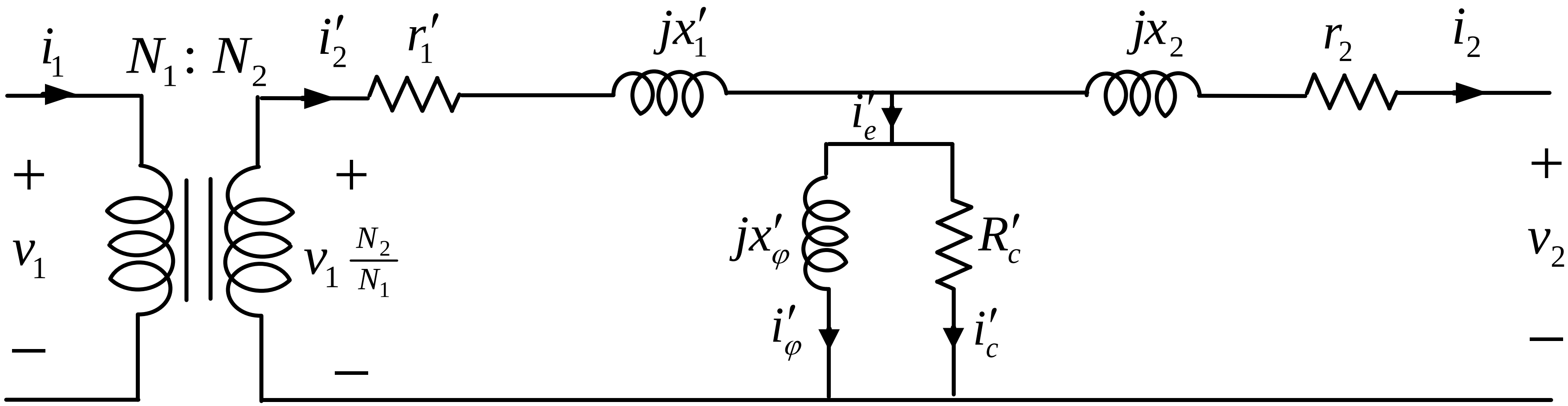


$$r_2' = r_2\left(\frac{N_1}{N_2}\right)^2$$

$$x_2' = x_2\left(\frac{N_1}{N_2}\right)^2$$

$$i_1' = i_2\frac{N_2}{N_1}$$

Equivalent Circuit Referred to Secondary



$$r_1' = r_1\left(\frac{N_2}{N_1}\right)^2$$

$$x_1' = x_1\left(\frac{N_2}{N_1}\right)^2$$

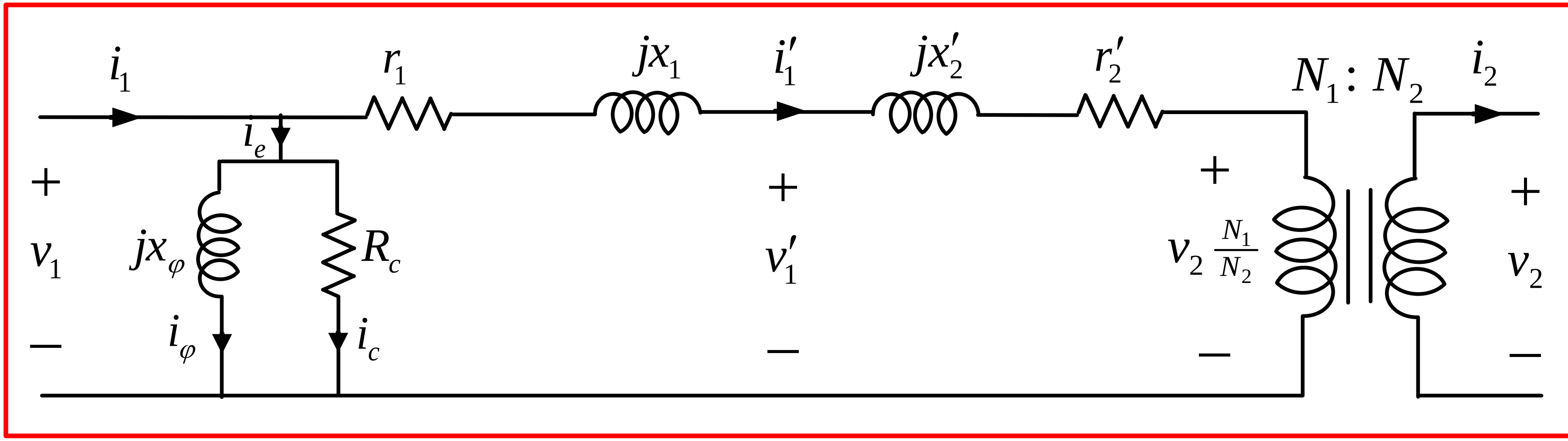
$$i_2' = i_1\frac{N_1}{N_2}$$

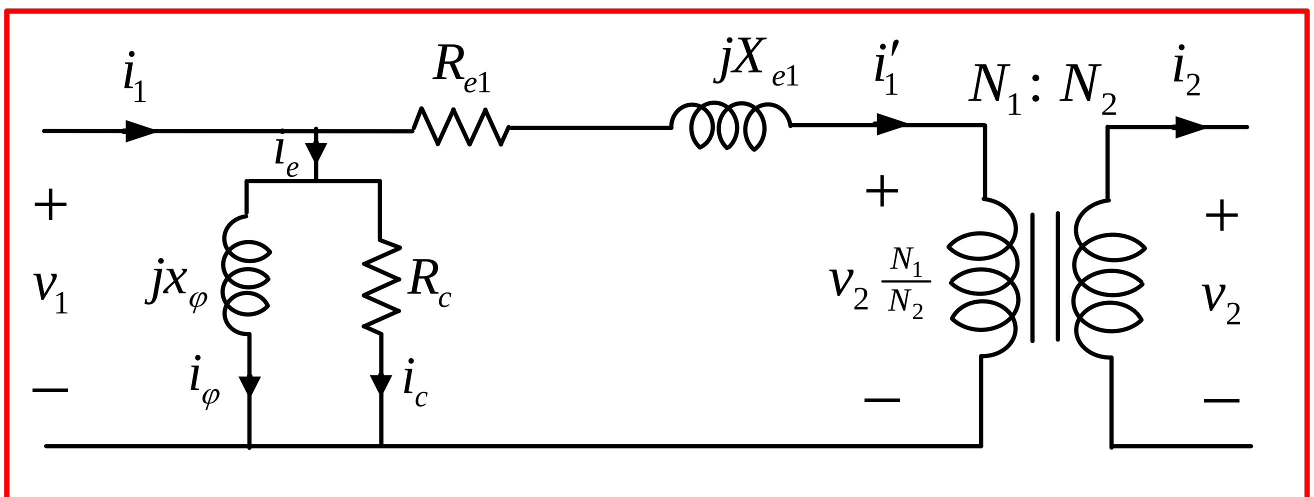
$$R_c' = R_c\left(\frac{N_2}{N_1}\right)^2$$

$$x_\varphi' = x_\varphi\left(\frac{N_2}{N_1}\right)^2$$

$$i_e' = i_e\frac{N_1}{N_2}$$

Approximated Equivalent Circuit Referred to Primary

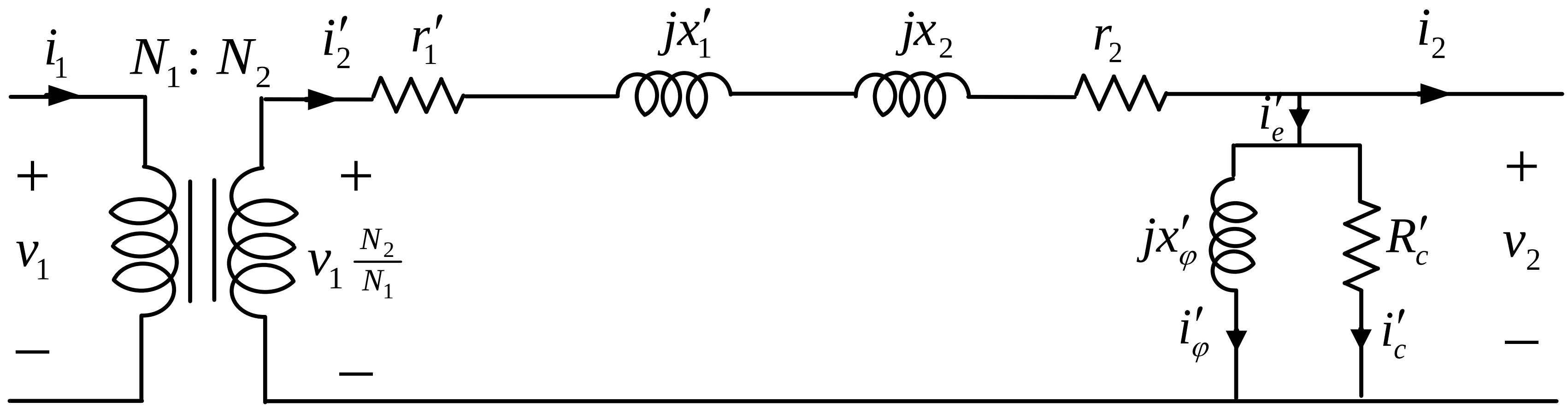


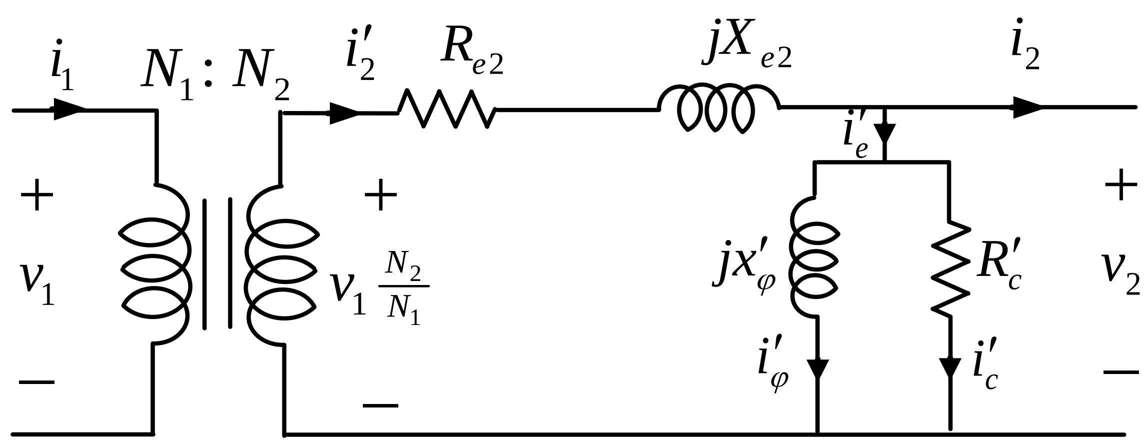


$$R_{e1} = r_1 + r_2'$$

$$X_{e1} = x_1 + x_2'$$

Approximated Equivalent Circuit Referred to Secondary



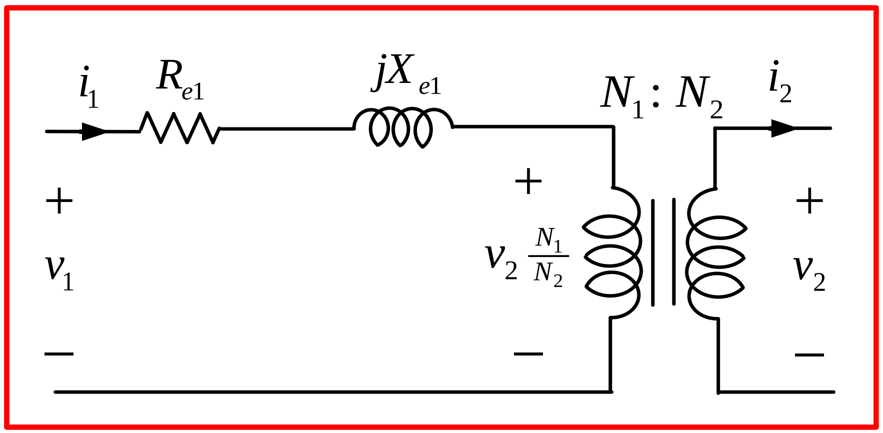


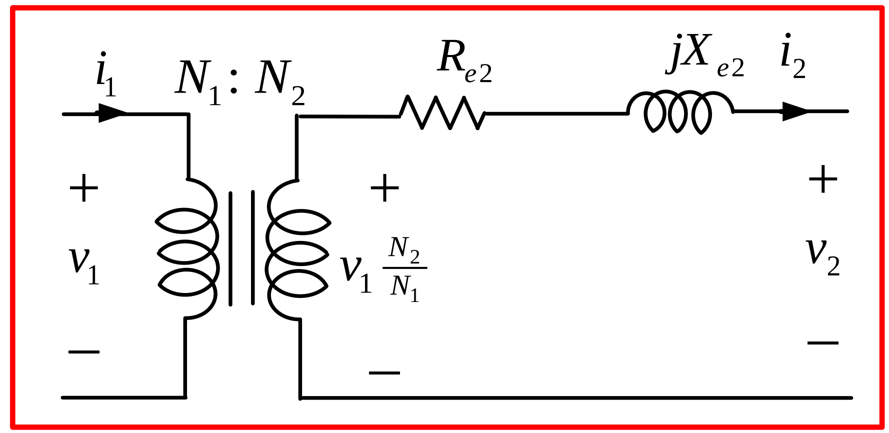
$$R_{e2} = r_1' + r_2$$

$$X_{e2} = x_1' + x_2$$

Approximated Equivalent Circuit

Neglecting the core losses and magnetizing reactance yields to the following approximated equivalent circuits





$$R_{e1} = r_1 + r_2'$$

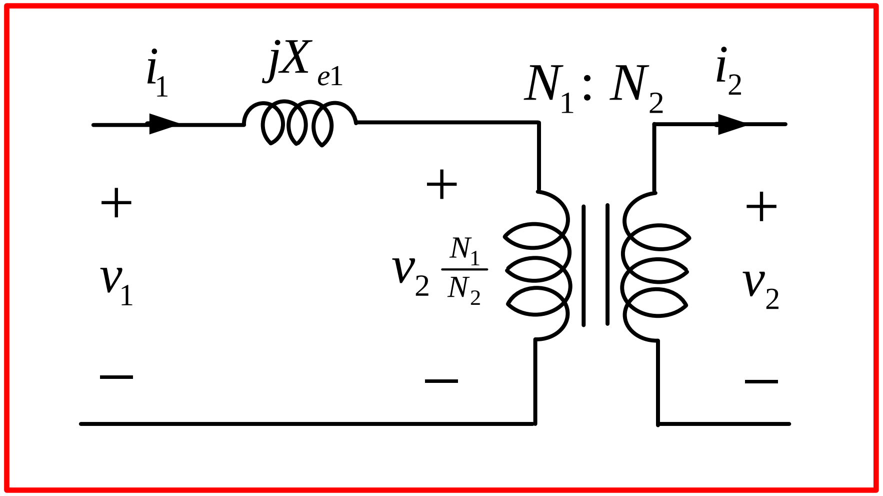
$$R_{e2} = r_1' + r_2$$

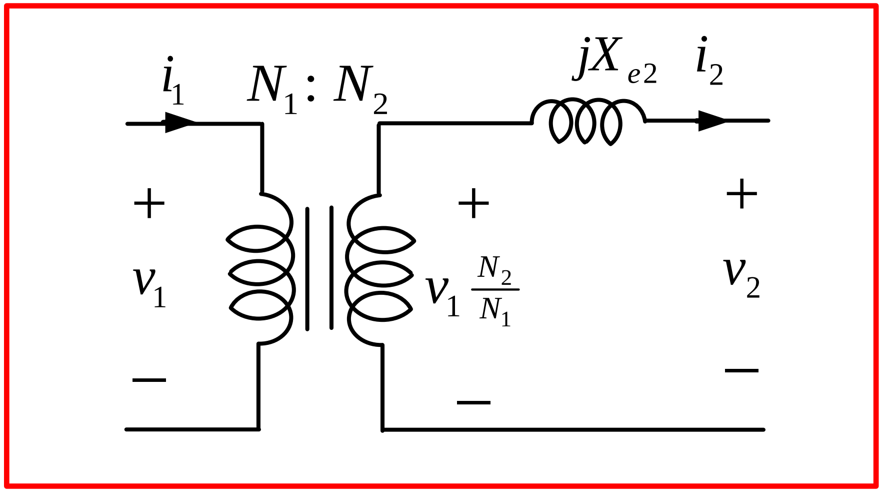
$$X_{e1} = x_1 + x_2'$$

$$X_{e2} = x_1' + x_2$$

Approximated Equivalent Circuit

Neglecting the core losses, magnetizing reactance and the winding resistances yields to the following approximated equivalent circuits





$$X_{e1} = x_1 + x_2'$$

$$X_{e2} = x_1' + x_2$$

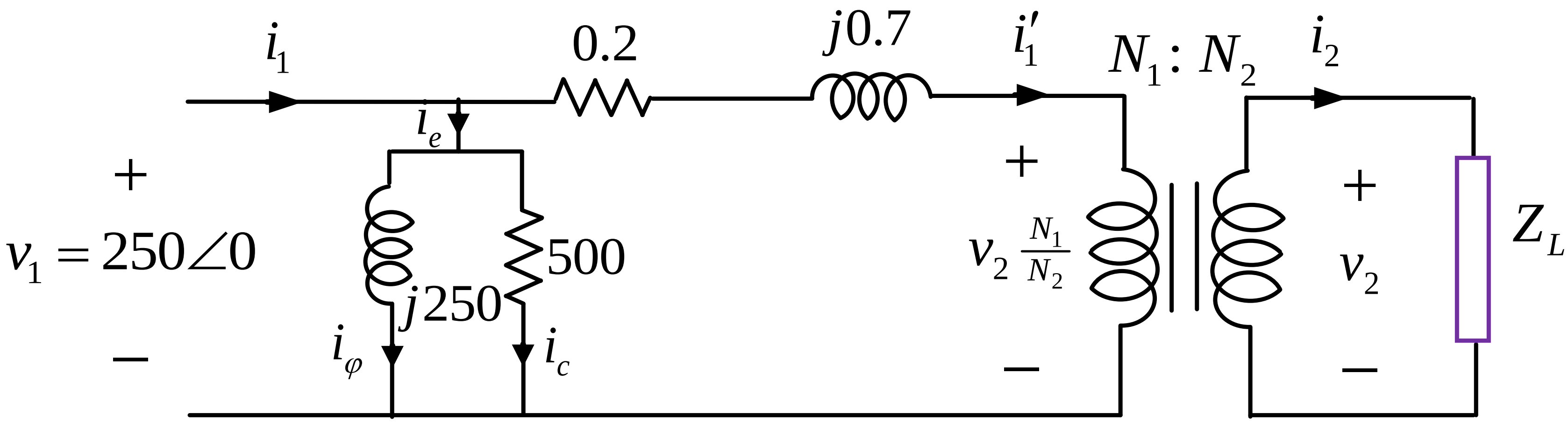
Transformers

Example 1: Following is the equivalent circuit of a single phase transformer referred to primary with **$250^V/2500^V$**. If a load with impedance of **380+j230** ohms is connected to the secondary terminal

a) Calculate V_2

b) Calculate I_1 and the **source power factor** and **load power factor**

c) Calculate the **output power** and the **efficiency**



Transformers

Solution 1: **part a** 250^{V}/2500^{V}, Z_L=380+j230 ohms $V_2 = ?$

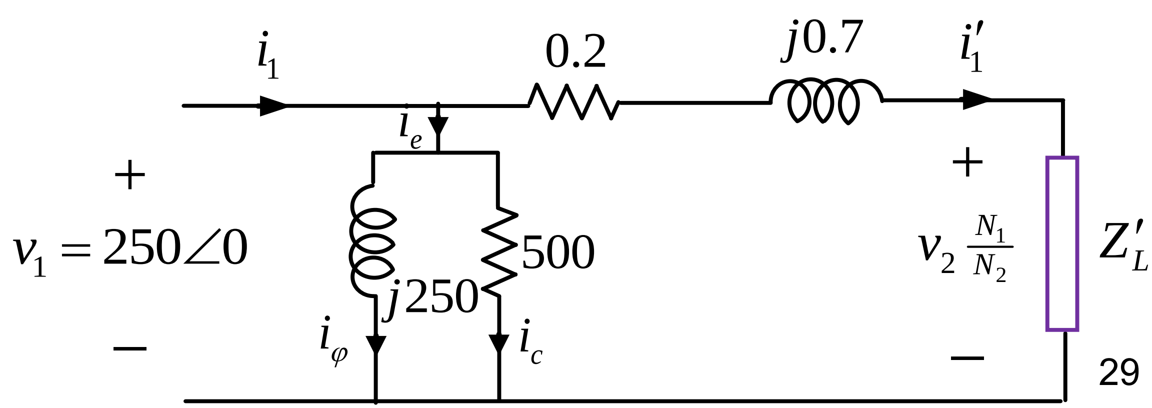
The load impedance referred to the primary is

$$Z'_L = (380 + j230)\left(\tfrac{250}{2500}\right)^2 = 3.8 + j2.3\ \ \Omega$$

$$I'_1 = \frac{250\angle 0}{(0.2+3.8)+j(0.7+2.3)} = 50\angle -37^\circ = 40 - j30\ \text{A}$$

$$V_2 \tfrac{N_1}{N_2} = I'_1 Z'_L \Rightarrow V_2 = \tfrac{N_2}{N_1} I'_1 Z'_L \Rightarrow V_2 = \tfrac{2500}{250}(40 - j30)(3.8 + j2.3)$$

$$V_2 = 2221\angle -5.7^\circ$$



Transformers

Solution 1: **part b** 250V/2500V, Z_L=380+j230 ohms $I_1 = ?$ $\cos\theta_1 = ?$

$\cos\theta_2 = ?$

$$\vec{I}_1 = \vec{I}_1' + \vec{I}_e$$

$$\vec{I}_e = \vec{I}_c + \vec{I}_\varphi$$

$$\vec{I}_1 = \vec{I}_1' + \vec{I}_c + \vec{I}_\varphi$$

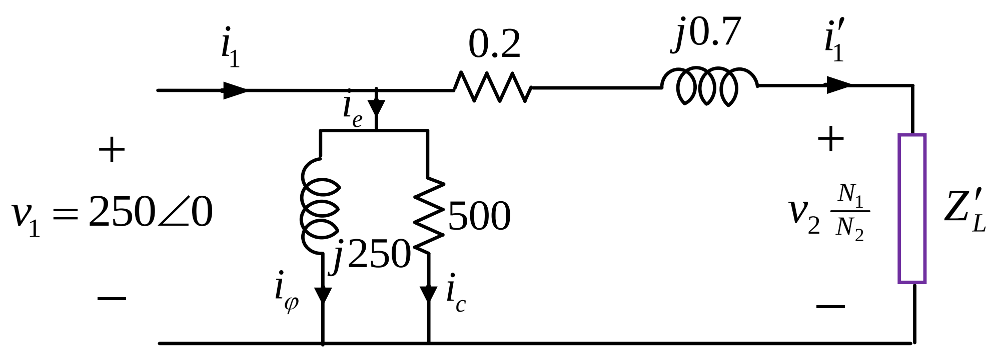
$$\vec{I}_1 = 40 - j30 + \frac{250}{500} + \frac{250}{j250}$$

$$\vec{I}_1 = 40.5 - j31 = 51\angle -37^\circ$$

$$\cos\theta_1 = \cos\left(0 - (-37^\circ)\right) = 0.794$$

$$\cos\theta_2 = \frac{R_L}{|Z_L|}$$

$$\cos\theta_2 = \frac{3.8}{\sqrt{3.8^2 + 2.3^2}} = 0.85$$



Transformers

Solution 1: **part c** 250^{V}/2500^{V}, Z_L=380+j230 ohms $P_{out}=?$ $\eta=?$

$$P_{out}=V_2 I_2\cos\theta_2$$

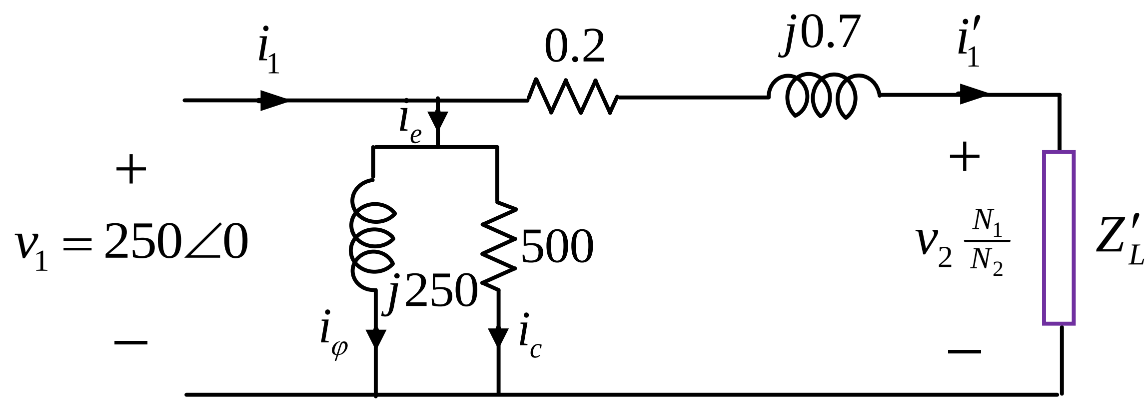
$$I_2=\frac{N_1}{N_2}I_1'$$

$$\Rightarrow P_{out}=2221\times 50\times\tfrac{250}{2500}\times 0.85=9439\ \text{W}$$

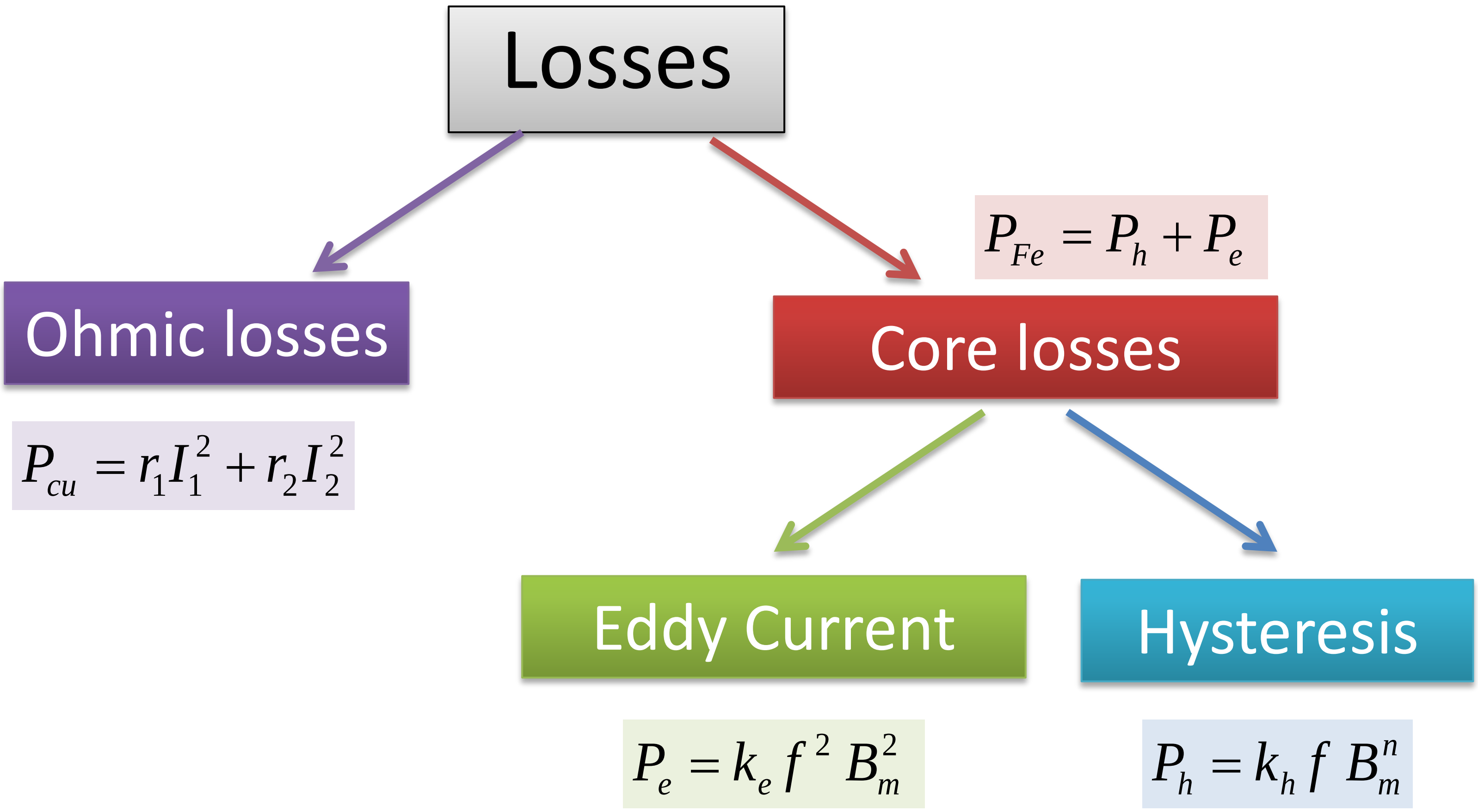
$$P_{in}=V_1 I_1\cos\theta_1 \Rightarrow P_{in}=250\times 51\times 0.794=10123\ \text{W}$$

$$\eta=\frac{P_{out}}{P_{in}}\times 100$$

$$\eta=93.2\ \%$$



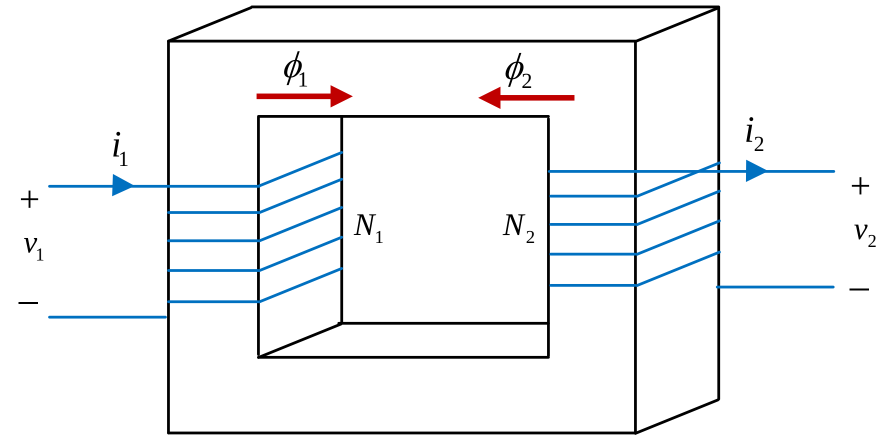
Losses and Efficiency



Hysteresis Losses

Hysteresis losses are due to residual flux in the ferromagnetic core and defined as:

$$P_h = k_h f B_m^n \qquad 1.5 \le n \le 2.5$$



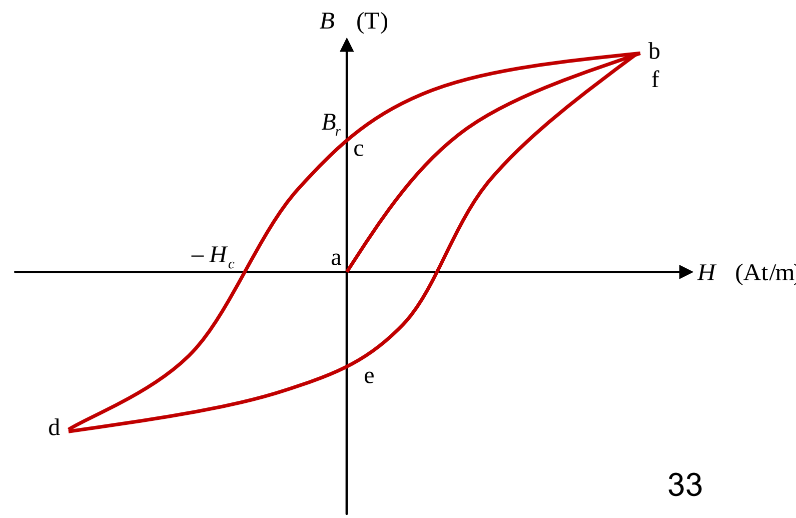
where

n Steinmetz constant

f frequency

B_m maximum flux density

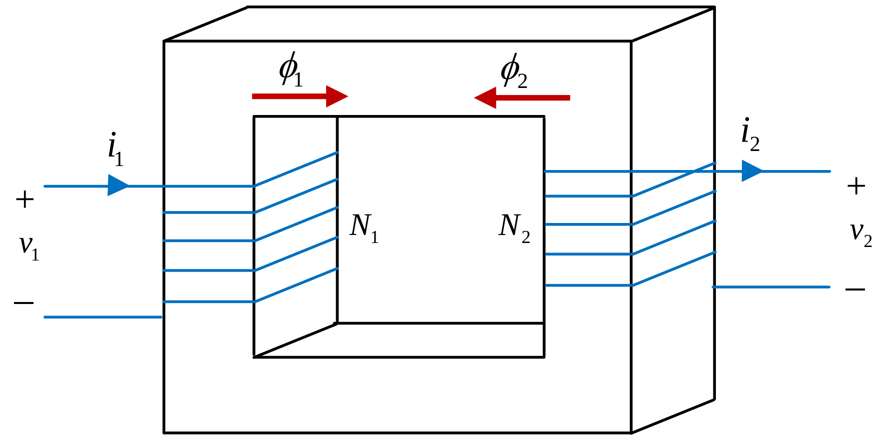
k_h constant depends of the type and volume of the core



Eddy Current Losses

Eddy current losses are due to current circulating in the ferromagnetic core and defined as:

$$P_e = k_e f^2 B_m^2$$



where

f frequency

B_m maximum flux density

k_e constant depends of the type and thickness of the core

The total core (magnetic) losses are defined as

$$P_{Fe} = P_c = P_h + P_e$$

Relation Between Core Losses and Input Voltage

$$V_1 = E_1 = \frac{N_1 \phi_m \omega}{\sqrt{2}} = \frac{N_1 A B_m 2\pi f}{\sqrt{2}} = \sqrt{2}\pi f N_1 A B_m \Rightarrow B_m = \frac{V_1}{\sqrt{2}\pi f N_1 A}$$

$$P_h = k_h f B_m^n = k_h f \left(\frac{V_1}{\sqrt{2}\pi f N_1 A} \right)^n \Rightarrow P_h = k_1 \frac{V_1^n}{f^{n-1}} \qquad k_1 = k_h \left(\frac{1}{\sqrt{2}\pi N_1 A} \right)^n$$

$$P_e = k_e f^2 B_m^2 = k_e f^2 \left(\frac{V_1}{\sqrt{2}\pi f N_1 A} \right)^2 \Rightarrow P_e = k_2 V_1^2 \qquad k_2 = k_e \left(\frac{1}{\sqrt{2}\pi N_1 A} \right)^2$$

Core Losses

Example 2: In a single phase transformer the core losses is 52 W at the frequency of 40 Hz; the core losses increases to 90 W at the frequency of 60 Hz. Both cases are at the same maximum flux density. Calculate the eddy current and hysteresis losses at the frequency of 50 Hz.

$f_1 = 40 \text{ Hz}$ ⇨ $P_{c1} = 52 \text{ W}$

$B_{m1} = B_{m2} = B_m$

$f_2 = 60 \text{ Hz}$ ⇨ $P_{c2} = 90 \text{ W}$

$f_3 = 50 \text{ Hz}$ ⇨ $P_{e3} = ?$ $\quad P_{h3} = ?$

Core Losses

Solution 2: $f_1 = 40\ \text{Hz} \Rightarrow P_{c1} = 52\ \text{W}$

$B_{m1} = B_{m2} = B_m$

$f_2 = 60\ \text{Hz} \Rightarrow P_{c2} = 90\ \text{W}$

$f_3 = 50\ \text{Hz} \Rightarrow P_{e3} = ?\qquad P_{h3} = ?$

$$P_c = P_e + P_h = k_e f^2 B_m^2 + k_h f B_m^n$$

$$\Rightarrow \begin{cases} P_{c1} = k_e f_1^2 B_{m1}^2 + k_h f_1 B_{m1}^n \\ P_{c2} = k_e f_2^2 B_{m2}^2 + k_h f_2 B_{m2}^n \end{cases} \Rightarrow \begin{cases} P_{c1} = k_1 f_1^2 + k_2 f_1 \\ P_{c2} = k_1 f_2^2 + k_2 f_2 \end{cases}$$

Core Losses

Solution 2: $f_1 = 40\ \text{Hz}$ ⇒ $P_{c1} = 52\ \text{W}$

$f_2 = 60\ \text{Hz}$ ⇒ $P_{c2} = 90\ \text{W}$ $\qquad B_{m1} = B_{m2} = B_m$

$f_3 = 50\ \text{Hz}$ ⇒ $P_{e3} = ?$ $\qquad P_{h3} = ?$

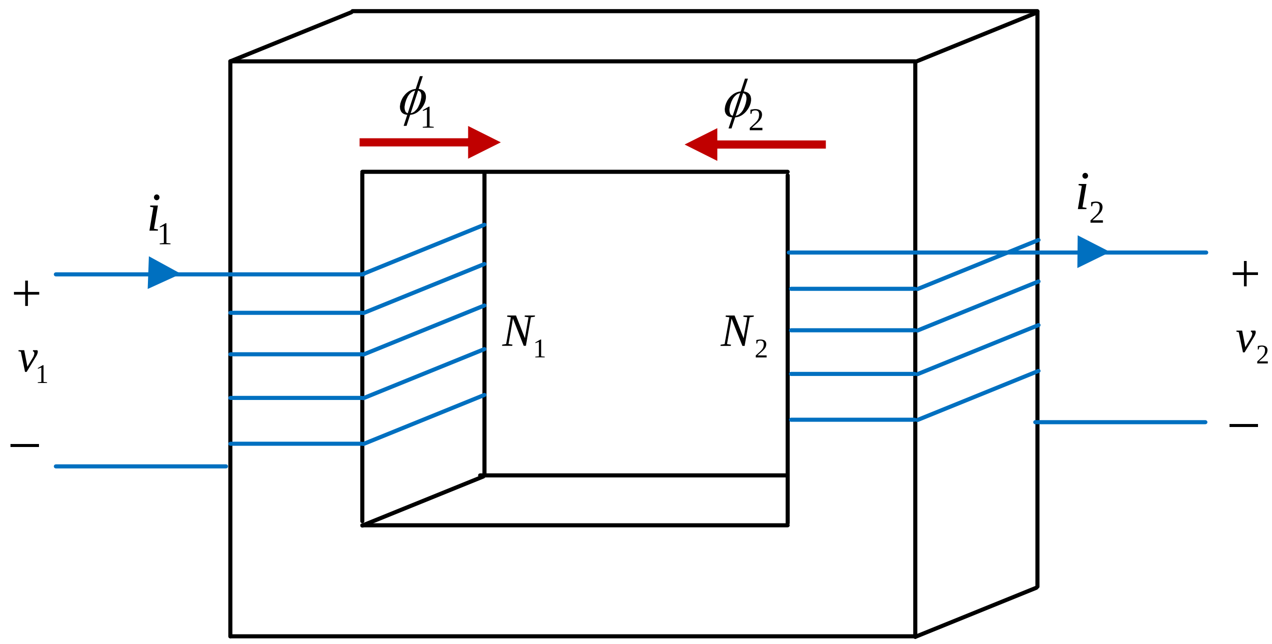
$$\Rightarrow \begin{cases} P_{c1} = k_1 f_1^2 + k_2 f_1 \\ P_{c2} = k_1 f_2^2 + k_2 f_2 \end{cases} \Rightarrow \begin{cases} 52 = 1600k_1 + 40k_2 \\ 90 = 3600k_1 + 60k_2 \end{cases}$$

$$\Rightarrow \begin{cases} k_1 = 0.01 \\ k_2 = 0.9 \end{cases} \Rightarrow \begin{cases} P_{e3} = k_1 f_3^2 \\ P_{h3} = k_2 f_3 \end{cases} \xrightarrow{f_3 = 50\ \text{Hz}} \begin{cases} P_{e3} = 25\ \text{W} \\ P_{h3} = 45\ \text{W} \end{cases}$$

Efficiency

$$\eta = \frac{\text{Output power}}{\text{Input power}} \times 100 = \frac{\text{Output power}}{\text{Output power} + \text{Losses}} \times 100$$

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_{cu} + P_{Fe}} \times 100$$



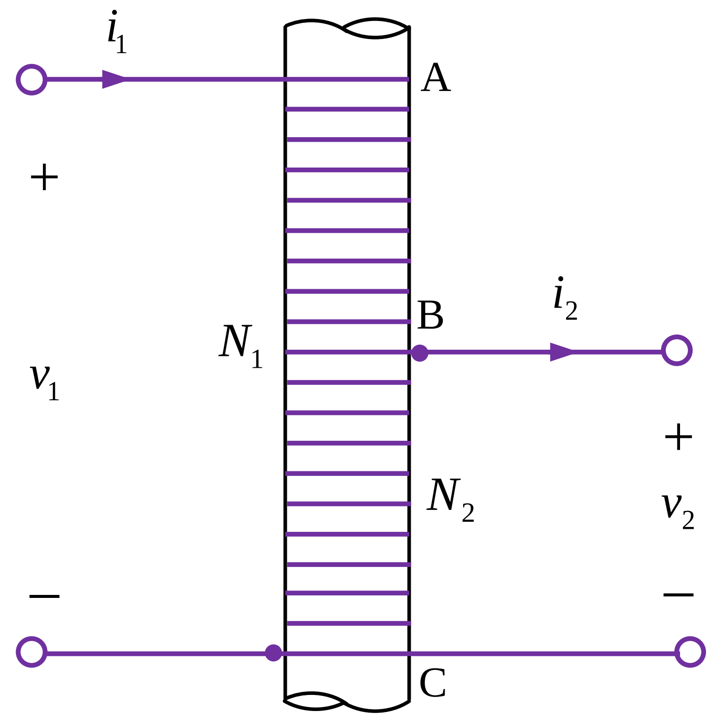
Auto-Transformers

- The voltage over one turn is $\frac{V_1}{N_1}$
- Therefore $V_2 = N_2 \frac{V_1}{N_1}$ ⇒ $\frac{V_2}{V_1} = \frac{N_2}{N_1}$
- Assuming an ideal auto-transformer

$$P_1 = P_2 \Rightarrow V_1 I_1 \cos\theta_1 = V_2 I_2 \cos\theta_2$$

$$\& \quad \cos\theta_1 \approx \cos\theta_2$$

$$\Rightarrow V_1 I_1 = V_2 I_2 \Rightarrow \frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$$



Step-Down Auto-Transformers

$$MMF_{AB} = I_1(N_1 - N_2) = I_1N_1 - I_1N_2$$

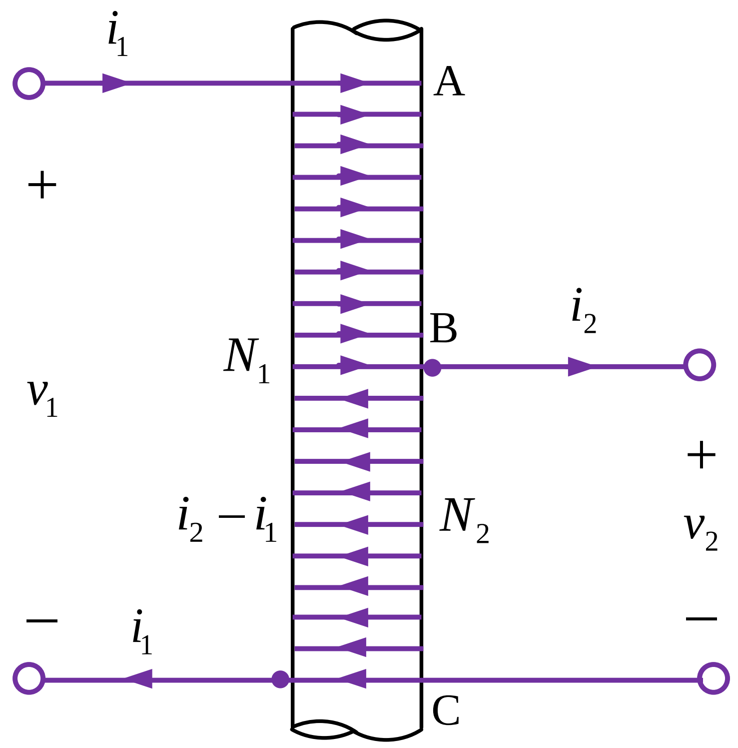
$$= I_2N_2 - I_1N_2 = N_2(I_2 - I_1) = MMF_{BC}$$

$$MMF_{AB} = MMF_{BC}$$

$$S_{AB} = I_1(V_1 - V_2) = I_1V_1 - I_1V_2$$

$$= I_2V_2 - I_1V_2 = V_2(I_2 - I_1) = S_{BC}$$

$$S_{AB} = S_{BC}$$



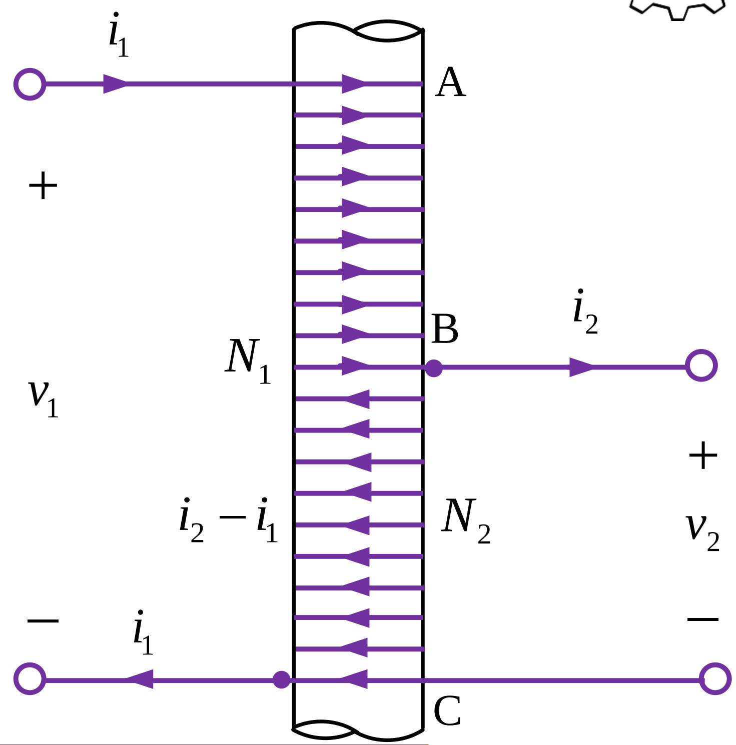
$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Step-Down Auto-Transformers

$$\frac{\text{Inductive power}}{\text{Total input power}} = \frac{I_1(V_1 - V_2)}{I_1 V_1} = 1 - \frac{V_2}{V_1}$$

$$\frac{\text{Inductive power}}{\text{Total input power}} = 1 - \frac{N_2}{N_1} = 1 - k$$

$$k = \frac{N_2}{N_1} < 1$$



$$\text{Conductive power} = \text{Total input power} - \text{Inductive power}$$

$$\text{Conductive power} = V_1 I_1 - I_1(V_1 - V_2) = V_2 I_1$$

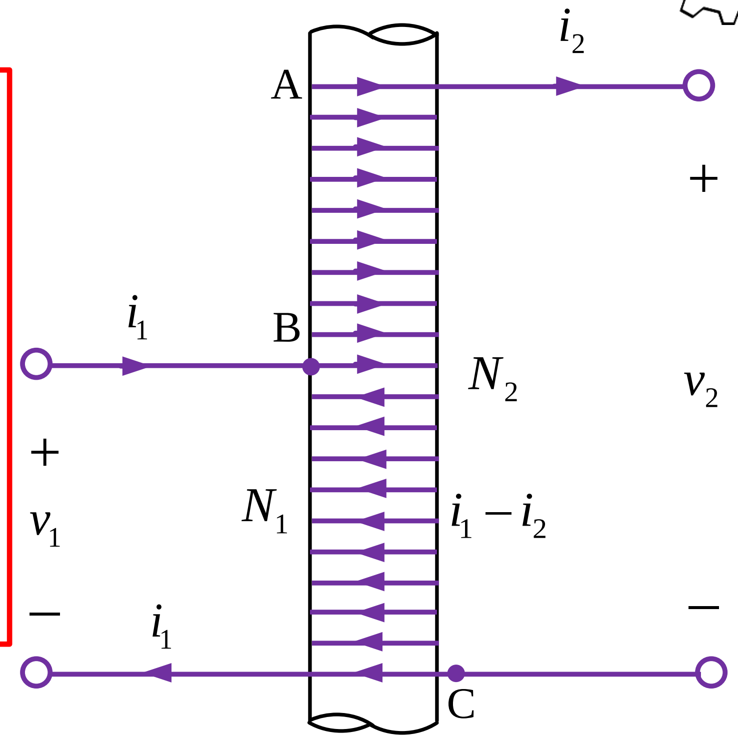
$$\frac{\text{Conductive power}}{\text{Total input power}} = \frac{V_2 I_1}{V_1 I_1} = \frac{N_2}{N_1} = k$$

Step-Up Auto-Transformers

$$\frac{\text{Inductive power}}{\text{Total input power}} = \frac{V_1(I_1 - I_2)}{I_1 V_1} = 1 - \frac{I_2}{I_1}$$

$$\frac{\text{Inductive power}}{\text{Total input power}} = 1 - \frac{N_1}{N_2} = 1 - k$$

$$k = \frac{N_1}{N_2} < 1$$



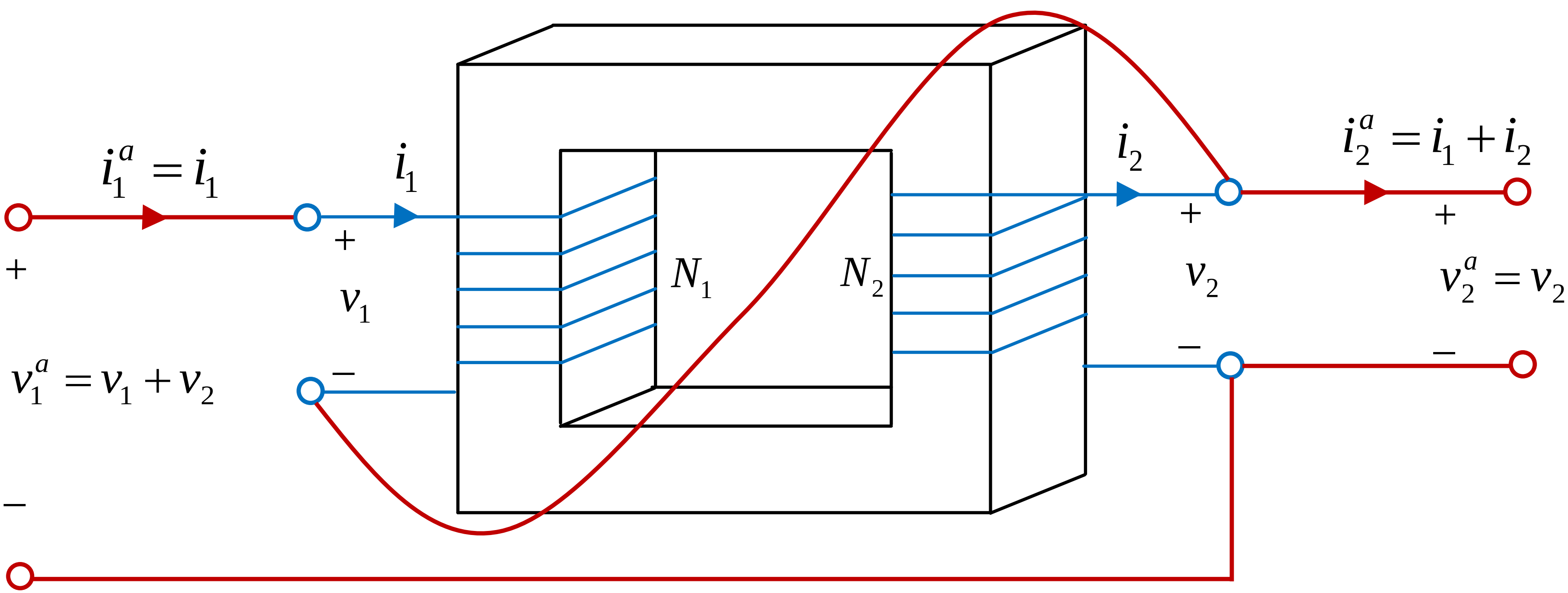
$$\text{Conductive power} = \text{Total input power} - \text{Inductive power}$$

$$\text{Conductive power} = V_1 I_1 - V_1(I_1 - I_2) = V_1 I_2$$

$$\frac{\text{Conductive power}}{\text{Total input power}} = \frac{V_1 I_2}{V_1 I_1} = \frac{N_1}{N_2} = k$$

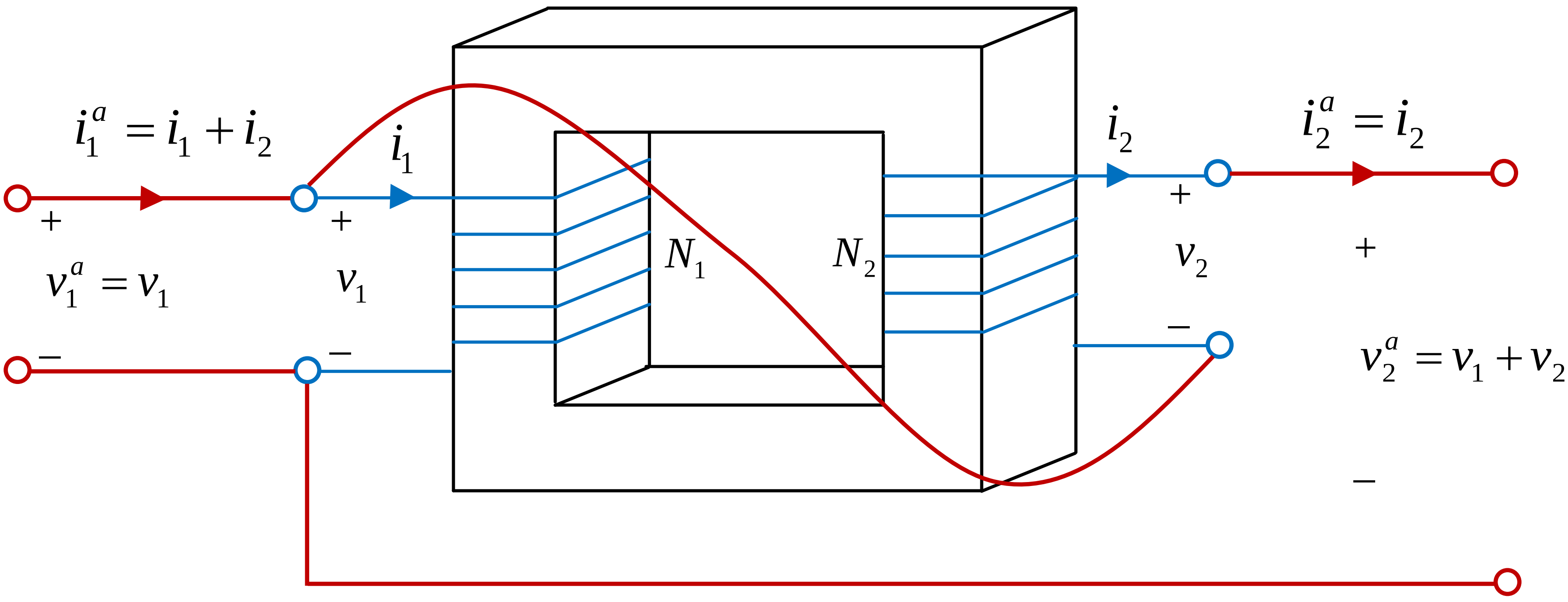
Auto-Transformers From Two-Winging Transformers

Step-Down



Auto-Transformers From Two-Winging Transformers

Step-Up



Auto-Transformers

Example 3: A two winding transformer with 2300/230 and $S=20\ \mathrm{kVA}$ is used as an auto-transformer. The voltage source of the auto-transformer is 2300 V.

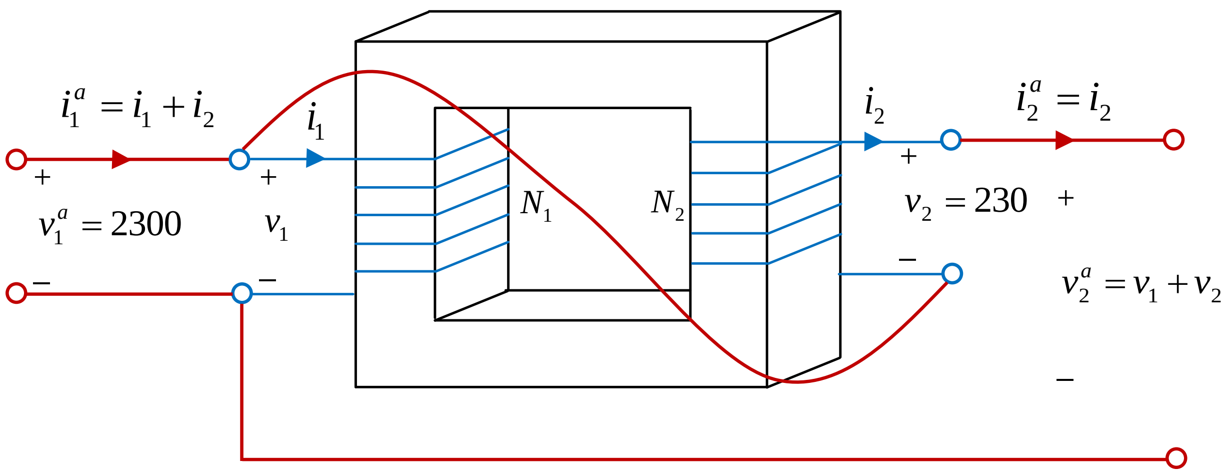
a) If the load power factor is unity, calculate the output power, inductive and conductive power.

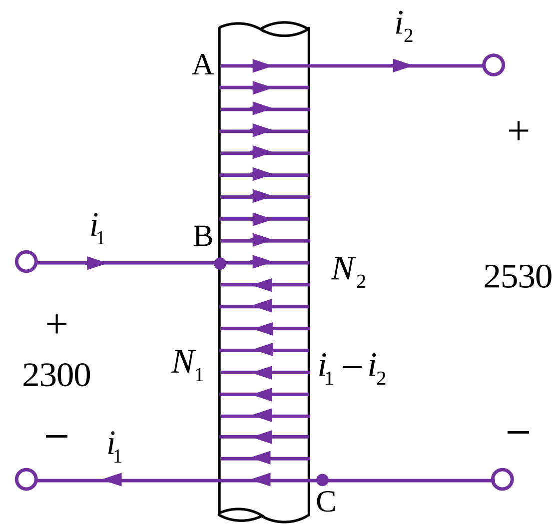
b) If the efficiency of the two-winding transformer at nominal load and power factor of 0.6 is 96%, calculate the efficiency of the auto-transformer at the same power factor.

Auto-Transformers

Solution 3: A two winding transformer with 2300/230 and S=20 kVA is used as an auto-transformer. The voltage source of the auto-transformer is 2300 V.

a) $PF=1$ $P_{out}=?$ $P_{ind}=?$ $P_{con}=?$





Auto-Transformers

Solution 3: A two winding transformer with 2300/230 and S=20 kVA is used as an auto-transformer. The voltage source of the auto-transformer is 2300 V.

a) $PF=1$ $\quad P_{out}=?$ $\quad P_{ind}=?$ $\quad P_{con}=?$

$$I_2=\frac{20000}{230}=86.9 \text{ A}$$

$$I_1=\frac{20000}{2300}=8.69 \text{ A}$$

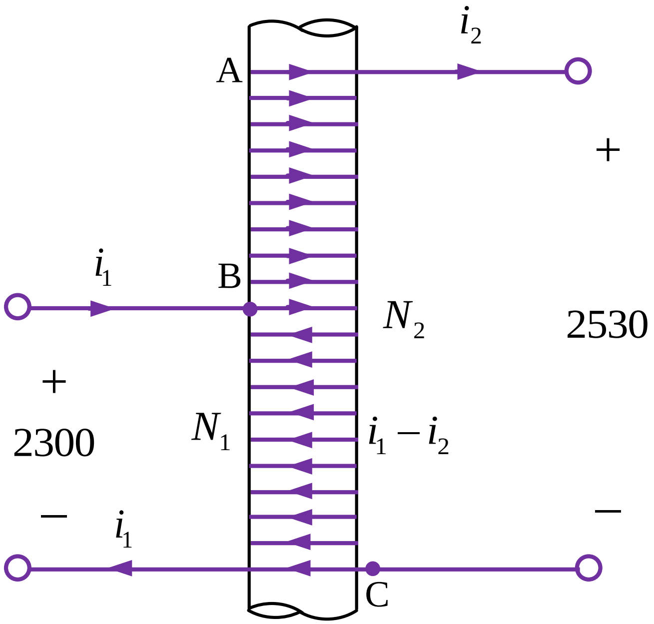
$$I_2^a=I_2=86.9 \text{ A}$$

$$I_1^a=I_1+I_2=95.59 \text{ A}$$

$$P_{out}=V_2^a I_2^a=2530\times 86.9=220 \text{ kVA}$$

$$P_{ind}=I_2^a\left(V_2^a-V_1^a\right)=86.9\times 230=20 \text{ kVA}$$

$$P_{con}=P_{out}-P_{ind}=200 \text{ kVA}$$



Auto-Transformers

Solution 3: A two winding transformer with 2300/230 and S=20 kVA is used as an auto-transformer. The voltage source of the auto-transformer is 2300 V.

b) $PF = 0.6$ at nominal load $\eta = 96\%$ $\eta_a = ?$

$$\eta = \frac{P_{out}}{P_{out} + P_{losses}}$$

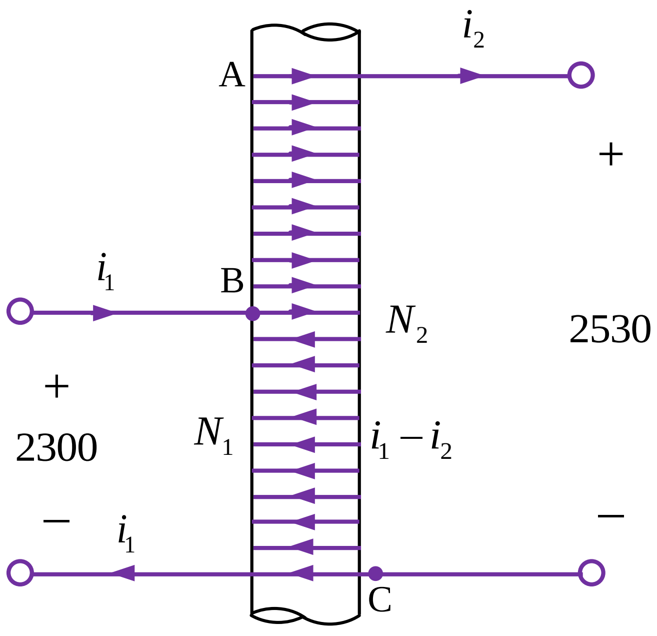
$$0.96 = \frac{20000 \times 0.6}{20000 \times 0.6 + P_{losses}}$$

$$P_{losses} = 500 \ \text{W}$$

$$\eta_a = \frac{P_{out}}{P_{out} + P_{losses}}$$

$$\eta_a = \frac{220000 \times 0.6}{220000 \times 0.6 + 500}$$

$$\eta_a = 0.9962$$



Maximum Efficiency of Transformers

$$\eta = \frac{P_{out}}{P_{out} + P_{cu} + P_{Fe}}$$

At a constant voltage

$$P_{out} \propto I$$

$$P_{cu} \propto I^2$$

$$P_{Fe} = cte$$

$$\Rightarrow \eta = \frac{k_1 I}{k_1 I + k_2 I^2 + k_3}$$

$$\frac{d\eta}{dI} = 0 \Rightarrow k_3 = k_2 I^2$$

For maximum efficiency the **core losses** should be **the same** as **ohmic losses**.